Integration

1. Simplify Using Algebra:
   a. Can I put it into power rule form by expanding or distributing terms?
   b. Can I factor the denominator to help cancel things out?

2. Numerator has a higher order polynomial than denominator:
   Use Polynomial Long Division to simplify

3. Denominator has a higher order polynomial than numerator:
   a. Use partial fractions to break up integral into multiple fractions
   b. If the denominator can't be factored complete the square

4. You have a function inside a function
   Try substitution: let \( w = \) the inside function

5. You have two functions multiplied
   Try integration by parts:
   - usually \( u = \) polynomial or \( \ln \)
   - \( dv = \sin x \cos x \, e^x \)
   - can use for powers of \( \sin \) and \( \cos \)
     or \( \sin xe^x \cos xe^x \) — make a copy
Integration Practice

Answers.

By Parts
\[ \int x \sin(x) \, dx \]
\[ = -x \cos(x) + \sin(x) + C \]

\[ \int (t + 2)^{3/2} \, dt \]
\[ = \frac{2}{9} \left( t + 2 \right)^{5/2} + C \]

By Parts
\[ \int y \ln(y) \, dy \]
\[ = \frac{1}{2} y^2 \ln|y| - \frac{1}{4} y^2 + C \]

Polynomial Division
\[ \int \frac{x^2 + 7x^2 + 10x + 1}{x^2 + 7x + 10} \, dx \]
\[ = \frac{1}{2} x^2 + \frac{1}{3} \ln|\frac{x+2}{x+5}| + C \]

Partial Fraction
\[ \int \frac{x+1}{6x+x^2} \, dx \]
\[ = \frac{1}{6} \ln|x| + \frac{5}{6} \ln|x+6| + C \]

Factor then u-sub
\[ \int \frac{1}{x^2 + 4x + 4} \, dx \]
\[ = \frac{1}{2} \ln|x+2| + C \]

u-sub
\[ \int \frac{x}{1+x^2} \, dx \]
\[ = -\frac{1}{2} \ln|1+x^2| + C \]

Simplification
\[ \int \frac{x^3 \sin(x)}{x \sin(x)} \, dx \]
\[ = \frac{1}{3} x^3 + C \]

u-sub
\[ \int x^2(1+2x^3)^2 \, dx \]
\[ = \frac{1}{6} \left[ \frac{1}{3} (1+2x^3)^3 \right] + C \]

u-sub
\[ \int e^x \cos(x) \, dx \]
\[ = \frac{1}{2} e^x \left[ \sin(x) + \cos(x) \right] + C \]

Hint: Integrate by parts twice to make a copy of the integral on the right hand side.

Completing the Square
\[ \int \frac{1}{x^2 + 4x + 5} \, dx \]
\[ = \arctan(x+2) + C \]