Consider a 2-D metal. It has two distinct freedoms: the conduction electrons and the lattice vibrations, i.e. phonons. Assume that the particles are confined to move within the plane, so there are only two dimensions in which the phonons or electrons can propagate and there are only 2 possible phonon polarizations.

Answer the following questions.

1. **Partition Functions**

a.) What is the Partition Function of one of the electron states, in terms of e_{e-s} (electron state energy) m, and b (i.e. 1/kT)?

b.) What is the Partition Function of one of the phonon states, in terms of e_{p-s} (phonon state energy) and b (i.e. 1/kT)? Recall, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for x < 1.

2. **Average Occupancy**

c.) What is the average occupancy of one of the electron states, in terms of e_{e-s} (electron state energy) m, and b (i.e. 1/kT)?

d.) What is the average occupancy of one of the phonon states, in terms of e_{p-s} (phonon state energy) and b (i.e. 1/kT)?

But, for phonons, there is no chemical potential, μ=0.

3. **Density of States**

e.) What is the density of states for the electrons, $g_e(e)$, in terms of *m, L, h* and possibly *e*?

f.) What is the density of states for the phonons, $g_p(e)$, in terms of c_S (wave speed), *L, h* and possibly *e*?

4. **Energy Scales**

g.) What is the Fermi Energy of the electrons, e_F?

h.) What is the Debey Energy of the phonons, $e_D = kT_D$? Since I biffed this in class, I'll point you in the right direction: you follow the *exact* same approach as you did to find the Fermi Energy, except in place of N is N×(# of dimensions in which the waves can propagate).

i.) Re-express $g_e(e)$ and $g_p(e)$ in terms of e_F and e_D, respectively.

5. **Thermal Energy**

j.) *Set up* the general integrals for the thermal energy of the Phonons and of the electrons, U_p and U_e. For the electrons, by making the substitution $e = (e - m) + m$ in the numerator, you can rewrite your integral as two integrals. Now, you can get your integrals for U_p and U_e in terms of an appropriate variable of integration, x.

For the next two parts, the following facts may be useful.

$$\int_{x_{min}}^{x_{max}} \frac{1}{e^x+1}\,dx = -\ln\left(1+e^{-x}\right)\Big|_{x_{min}}^{x_{max}}$$

$$\int_{x_{min}}^{x_{max}} \frac{x^n}{e^x+1}\,dx = \frac{1}{n+1}\frac{x^{n+1}}{e^x+1}\Bigg|_{x_{min}}^{x_{max}} + \frac{1}{n+1}\int_{x_{min}}^{x_{max}} \frac{x^{n+1}e^x}{\left(e^x+1\right)^2}\,dx = \frac{1}{n+1}\frac{x^{n+1}}{e^x+1}\Bigg|_{x_{min}}^{x_{max}} + \frac{1}{n+1}\int_{x_{min}}^{x_{max}} \frac{x^{n+1}}{\left(e^{x/2}+e^{-x/2}\right)^2}\,dx$$

$$\int_0^\infty \frac{x}{e^x+1}\,dx = \frac{p^2}{12} \qquad \int_0^\infty \frac{x}{e^x-1}\,dx = \frac{p^2}{6}$$

$$\int_0^\infty \frac{x^2}{e^x-1}\,dx = 1.803 \qquad \int_0^\infty \frac{x^2}{e^x-1}\,dx = 2.404$$

$$\int_0^\infty \frac{x^3}{e^x+1}\,dx = \frac{7}{8}\frac{p^4}{15} \qquad \int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{p^4}{15}$$

k.) Low T limit: Evaluate the thermal energies (U_p and U_e) in the low T limit. (not when T=0 mind you, just when it's small).

l.) One of the two integrals in your U_e expression can be analytically solved. This is evident if you do a second change of variables $y = e^{-x}$ Low T Limit: Evaluate the integrals in the low temperature limitWithout actually evaluating these integrals, take the appropriate derivative to get the integrals for heat capacity.

a. Use $e = (e - m) + m$, rewrite the $C_{v\text{-}e}$ integral first as two integrals, and then change variables to $x = (e - m)b$. Similarly, change variables for the phonons' integral.