Mon. Tues.	6.3, 6.4 Auxiliary Field & Linear Media	 HW9
Mon. Wed. Fri.	7.1.1-7.1.3 Ohm’s Law & Emf 7.1.3-7.2.2 Emf & Induction 7.2.3-7.2.5 Inductance and Energy of B	

Magnetization

$$\vec{M} \equiv \frac{d\vec{m}}{d\tau'}$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\vec{M} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \qquad \vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\vec{J}_b d\tau'}{\mathscr{r}} + \frac{\mu_o}{4\pi} \oint \frac{\vec{K}_b da'}{\mathscr{r}}$$

Recall: The Electric "Displacement"

Quite Generally, Gauss's law says

$$\vec{\nabla}\cdot\vec{E} = \tfrac{1}{\varepsilon_o}\rho_{(all)}$$

Now we also relate "bound" charge (due to variation in density of dipoles)

$$-\vec{\nabla}\cdot\vec{P} = \rho_b$$

So, if you have a region with dipoles *and* free charges,

$$\rho_{(all)} = \rho_{free} + \rho_{bound}$$

or,

$$\rho_{free} = \rho_{(all)} - \rho_{bound}$$

$$\rho_{free} = \varepsilon_o\vec{\nabla}\cdot\vec{E} - \left(-\vec{\nabla}\cdot\vec{P}\right)$$

$$\rho_{free} = \vec{\nabla}\cdot\left(\varepsilon_o\vec{E} + \vec{P}\right)$$

$$\varepsilon_o\vec{E} + \vec{P} \equiv \vec{D}$$ Electric Displacement

So Gauss's Law for *free* charge and Electric *Displacement*

$$\rho_{free} = \vec{\nabla}\cdot\vec{D}$$

and

$$Q_{free} = \int \rho_{free}\, d\tau = \int \vec{D}\cdot d\vec{a}$$

Now: The Magnetic "Auxiliary"

Quite Generally, Ampere's law says

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}_{(all)}$$

Now we also relate "bound" charge (due to variation in density of dipoles)

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

So, if you have a region with dipoles *and* free current,

$$\vec{J}_{(all)} = \vec{J}_{free} + \vec{J}_{bound}$$

or,

$$\vec{J}_{free} = \vec{J}_{(all)} - \vec{J}_{bound}$$

$$\vec{J}_{free} = \tfrac{1}{\mu_o} \vec{\nabla} \times \vec{B} - \left(\vec{\nabla} \times \vec{M}\right)$$

$$\vec{J}_{free} = \vec{\nabla} \times \left(\tfrac{1}{\mu_o} \vec{B} - \vec{M}\right)$$

$$\tfrac{1}{\mu_o} \vec{B} - \vec{M} \equiv \vec{H}$$ Magnetic auxiliary field

So Ampere's Law for *free* current and the auxiliary field

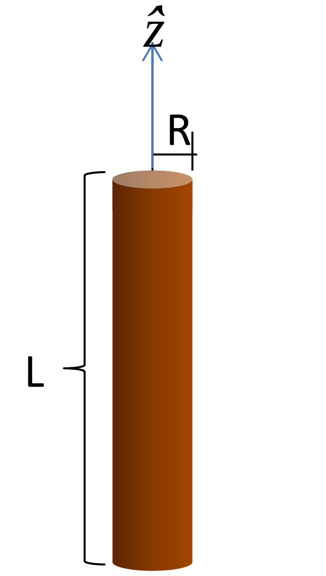
$$\vec{J}_{free} = \vec{\nabla} \times \vec{H}$$

and

$$I_{free} = \int \vec{J}_{free} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l}$$

The Magnetic "Auxiliary"

$$I_{free} = \int \vec{J}_{free} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l}$$



Example 6.1: A long copper rod of radius R caries a uniformly distributed (free) current I. Find the auxiliary field, H, every where.

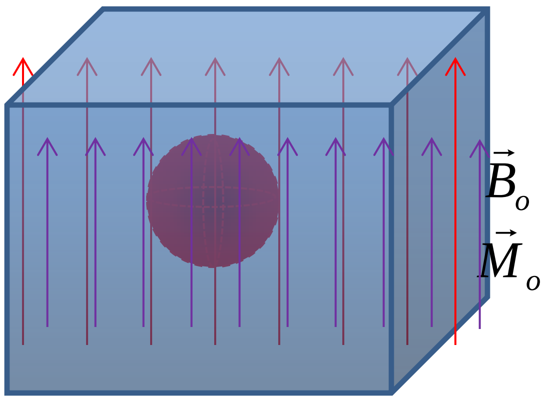
[redacted]

Magnetization & Auxiliary Magnetic field

Example

$$\vec{H} \equiv \tfrac{1}{\mu_o}\vec{B} - \vec{M}$$

Consider a huge slab of magnetized material initially in the presence of a uniform field $\vec{B}_o$ (partly due to itself, partly due to other sources in the environment) and corresponding uniform polarization and electric displacement $\vec{H}_o = \tfrac{1}{\mu_o}\vec{B}_o - \vec{M}_o$.



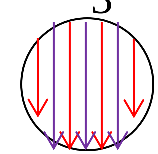
You cut a small spherical hole out of it. What is the field in its center in terms of $\vec{B}_o$ and $\vec{M}_o$?

For illustrative purposes only, take the Magnetization to be parallel to the field, and imagine both to be in the z direction.

By Superposition Principle, cutting out a sphere is the same as inserting a sphere of opposite magnitization.

Quoting Example 6.1 (which in turn builds on 5.11), the field *inside* a uniformly polarized sphere is $\vec{B}_{added.sphere} = \frac{2}{3}\mu_o \vec{M}_{added.sphere}$

So, we 'add in ' a sphere of polarization $-\vec{M}_o$



Adding field $\vec{B}_{added} = -\frac{2}{3}\mu_o \vec{M}_o$

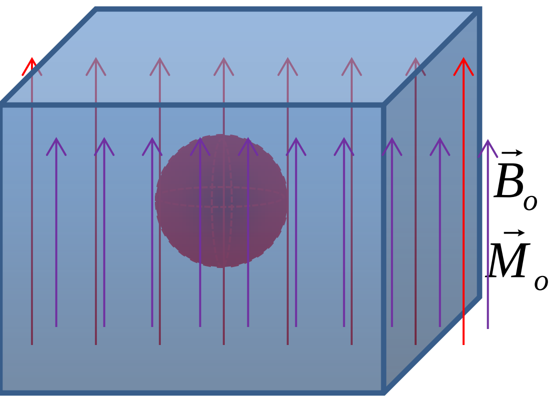
$$\vec{B}_{in.sphere} = \vec{B}_o + \vec{B}_{added} = \vec{B}_o - \tfrac{2}{3}\mu_o \vec{M}_o$$

Magnetization & Auxiliary Magnetic field

Example

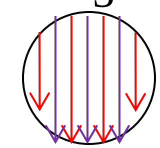
$$\vec{H} \equiv \tfrac{1}{\mu_o}\vec{B} - \vec{M}$$

Consider a huge slab of magnetized material initially in the presence of a uniform field $\vec{B}_o$ (partly due to itself, partly due to other sources in the environment) and corresponding uniform polarization and electric displacement $\vec{H}_o = \tfrac{1}{\mu_o}\vec{B}_o - \vec{M}_o$.



You cut a small spherical hole out of it. What is the field in its center in terms of $\vec{B}_o$ and $\vec{M}_o$?

$$\vec{B}_{added.sphere} = \frac{2}{3}\mu_o \vec{M}_{added.sphere}$$



$$\vec{B}_{in.sphere} = \vec{B}_o - \tfrac{2}{3}\mu_o \vec{M}_o$$

What is the auxiliary field in its center in terms of $\vec{H}_o$ and $\vec{M}_o$?

$$\vec{H}_{in.sphere} = \tfrac{1}{\mu_o}\vec{B}_{in.sphere} - \vec{M}_{in.sphere}$$

There is no material in the sphere, so

$$\vec{H}_{in.sphere} = \tfrac{1}{\mu_o}\left(\vec{B}_o - \tfrac{2}{3}\mu_o \vec{M}_o\right)$$ where $\vec{B}_o = \mu_o\left(\vec{H}_o + \vec{M}_o\right)$

so

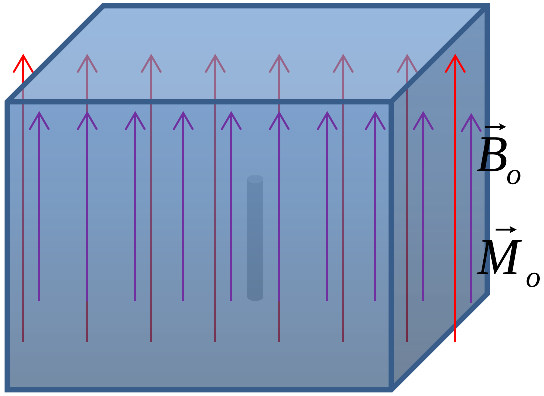
$$\vec{H}_{in.sphere} = \tfrac{1}{\mu_o}\left(\mu_o\left(\vec{H}_o + \vec{M}_o\right) - \tfrac{2}{3}\mu_o \vec{M}_o\right) = \left(\vec{H}_o + \tfrac{1}{3}\vec{M}_0\right)$$

Magnetization & Magnetic Auxiliary

$$\vec{M} \equiv \frac{d\vec{m}}{d\tau'} \qquad \vec{J}_b = \vec{\nabla} \times \vec{M} \qquad \vec{K}_b = \vec{M} \times \hat{n}$$

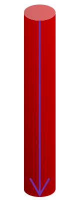
$$\vec{H} \equiv \tfrac{1}{\mu_o}\vec{B} - \vec{M} \qquad I_{free} = \int \vec{J}_{free} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l}$$

Exercise: Consider a huge slab of magnetic material initially with uniform field, $\vec{B}_o$ and corresponding uniform polarization and magnetic auxiliary $\vec{H}_o = \tfrac{1}{\mu_o}\vec{B}_o - \vec{M}_o$



You cut out a tall and narrow cavity along $\vec{M}_o$.
What is the field in its center in terms of $\vec{B}_o$ and $\vec{M}_o$?

Hint: Think of *inserting* the appropriate rod of opposite magnetization, determining its bound current, and the field of that configuration of current.



What is the electric displacement in its center in terms of $\vec{H}_o$ and $\vec{M}_o$?

The Magnetic "Auxiliary"

Recall for D:

$$\vec{\nabla}\times\vec{D}=\vec{\nabla}\times\left(\varepsilon_o\vec{E}+\vec{P}\right)=0+\vec{\nabla}\times\vec{P}$$

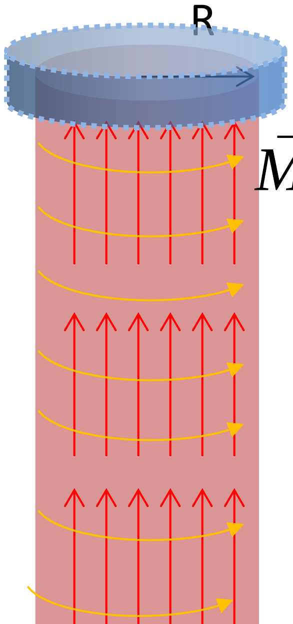
So D isn't necessarily analogous to E, unless P happens to be curl-less

Now for H:

$$\vec{\nabla}\cdot\vec{H}=\vec{\nabla}\cdot\left(\tfrac{1}{\mu_o}\vec{B}-\vec{M}\right)=0-\vec{\nabla}\cdot\vec{M}$$

So H isn't necessarily analogous to B, unless M happens to be divergence-less

Example: Magnetization parallel to cylinder's axis:



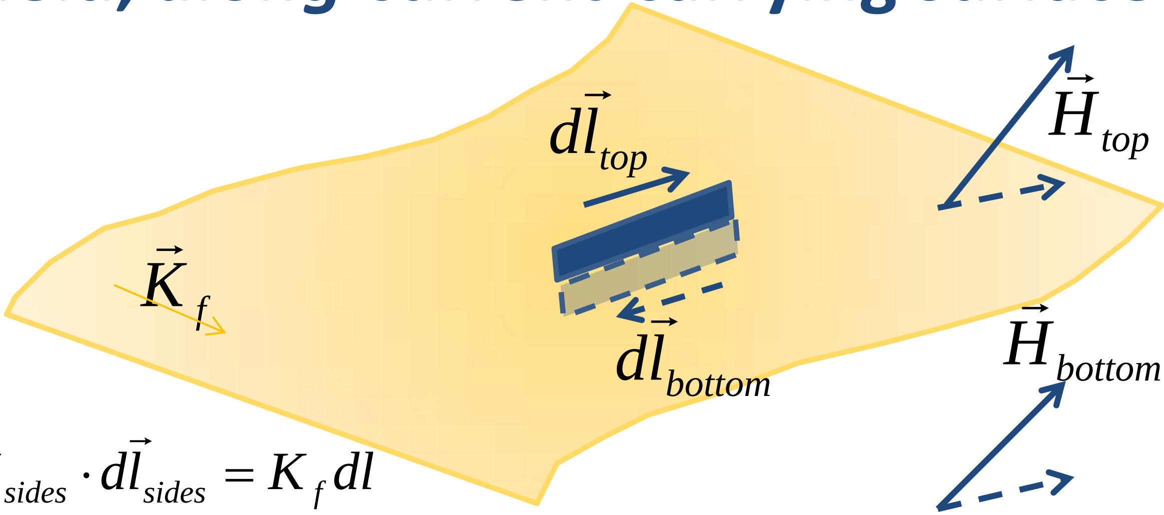
Amperian box

$\vec{M}\equiv M\hat{z}$

There is flux in the bottom, but none out the other sides.

$$\vec{\nabla}\cdot\vec{H}=-\vec{\nabla}\cdot\vec{M}\neq 0$$

Boundary Conditions
Magnetic Auxiliary field, *along* current-carrying surface



$$\oint \vec{H}\cdot d\vec{l} = I_{free.enc} = K_f\, dl$$

$$\int \vec{H}_{top}\cdot d\vec{l}_{top} + \int \vec{H}_{bottom}\cdot d\vec{l}_{bottom} + \int \vec{H}_{sides}\cdot d\vec{l}_{sides} = K_f\, dl$$

Send side height to 0

$$\int \vec{H}_{top}\cdot d\vec{l}_{top} + \int \vec{H}_{bottom}\cdot d\vec{l}_{bottom} = K_f\, dl$$

$$H_{\|top}L + H_{\|bottom}L(-1) = K_f L$$

$$H_{\|top} - H_{\|bottom} = K_f$$

With H perpendicular to K, we can relate the directions by

$$\vec{H}_{\|top} - \vec{H}_{\|bottom} = \vec{K}_f \times \hat{n}$$

As for B, it depends on *all* current (bound and free)

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{enc}$$

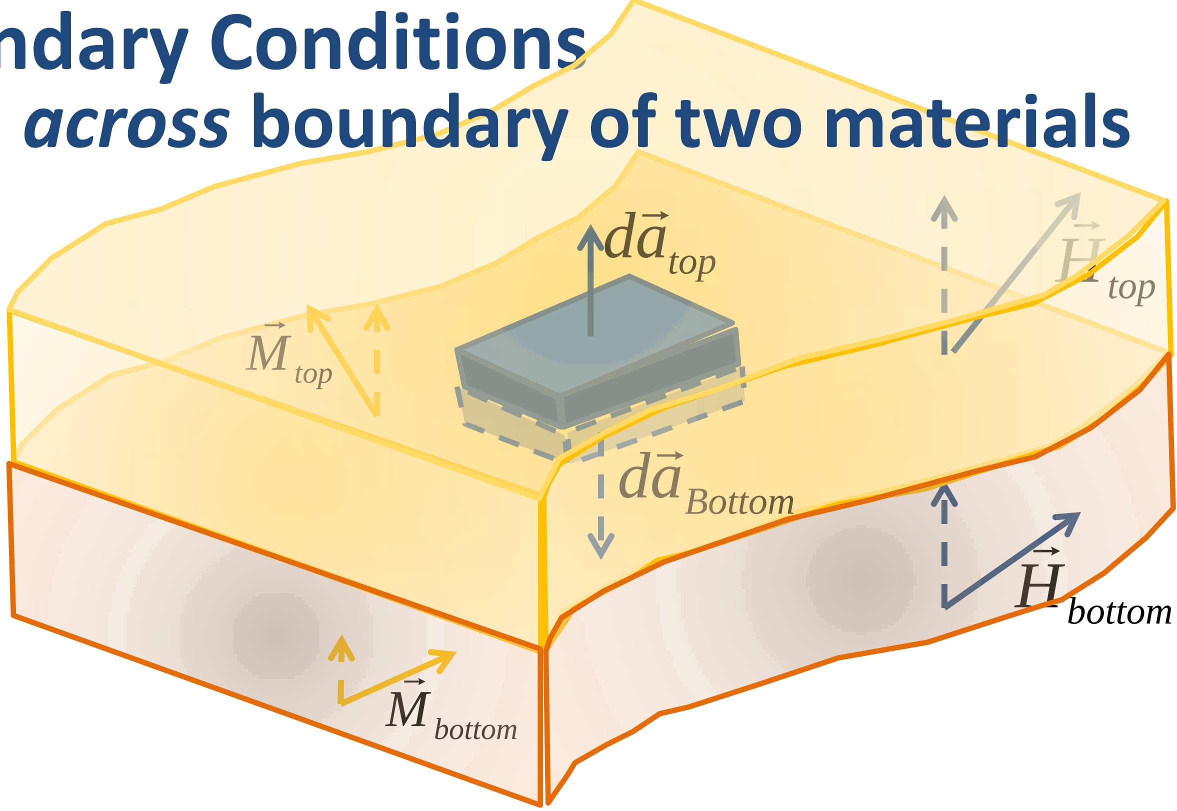
.

.

.

$$\vec{B}_{\|top} - \vec{B}_{\|bottom} = \vec{K}_{(all)} \times \hat{n}$$

Boundary Conditions
Magnetic Auxiliary, *across* boundary of two materials



$$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$$

$$\oint \vec{H}\cdot d\vec{a} = -\oint \vec{M}\cdot d\vec{a}$$

$$\int \vec{H}_{top}\cdot d\vec{a}_{top} + \int \vec{H}_{bottom}\cdot d\vec{a}_{bottom} + \int \vec{H}_{sides}\cdot d\vec{a}_{sides} = \int \vec{M}_{top}\cdot d\vec{a}_{top} + \int \vec{M}_{bottom}\cdot d\vec{a}_{bottom} + \int \vec{M}_{sides}\cdot d\vec{a}_{sides}$$

Send side height / area to 0

$$\int \vec{H}_{top}\cdot d\vec{a}_{top} + \int \vec{H}_{bottom}\cdot d\vec{a}_{bottom} = \int \vec{M}_{top}\cdot d\vec{a}_{top} + \int \vec{M}_{bottom}\cdot d\vec{a}_{bottom}$$

$$H_{\perp top}A + H_{\perp bottom}A(-1) = M_{\perp top}A + M_{\perp bottom}A(-1)$$

$$H_{\perp top} - H_{\perp bottom} = M_{\perp top} - M_{\perp bottom}$$

As for B, it's got no divergence

$$\vec{\nabla}\cdot\vec{B} = 0$$

.

.

.

$$B_{\perp top} - B_{\perp bottom} = 0$$

Boundary Conditions
Magnetic and Auxiliary fields

$$\vec{H} \equiv \frac{1}{\mu_o}\vec{B} - \vec{M}$$

Along

$$\vec{B}_{\parallel top} - \vec{B}_{\parallel bottom} = \vec{K}_{(all)} \times \hat{n}$$

$$\vec{H}_{\parallel top} - \vec{H}_{\parallel bottom} = \vec{K}_{f} \times \hat{n}$$

$$\vec{K} - \vec{K}_{free} = \vec{K}_b = \vec{M} \times \hat{n}$$

Across

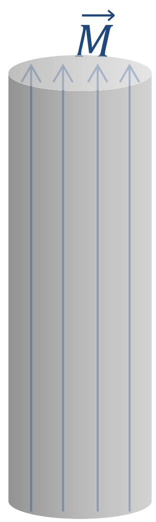
$$B_{\perp top} - B_{\perp bottom} = 0$$

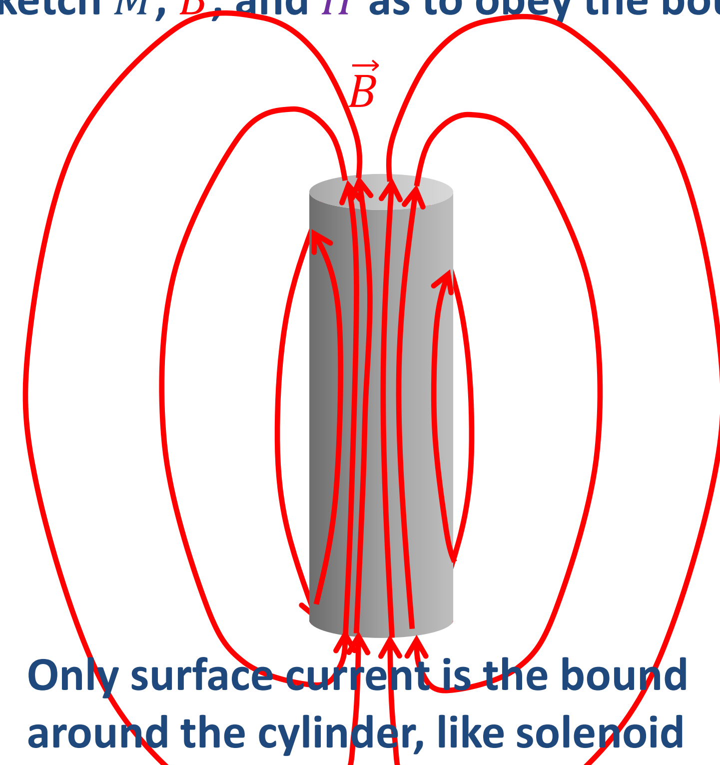
$$H_{\perp top} - H_{\perp bottom} = M_{\perp top} - M_{\perp bottom}$$

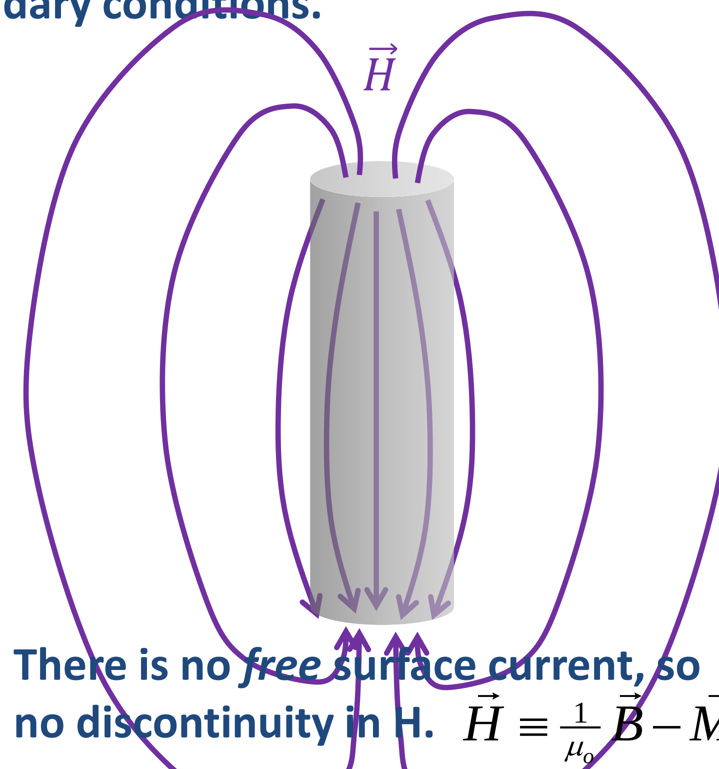
Exercise

Bar Magnet: uniform M along axis

Sketch $\vec{M}$, $\vec{B}$, and $\vec{H}$ as to obey the boundary conditions.



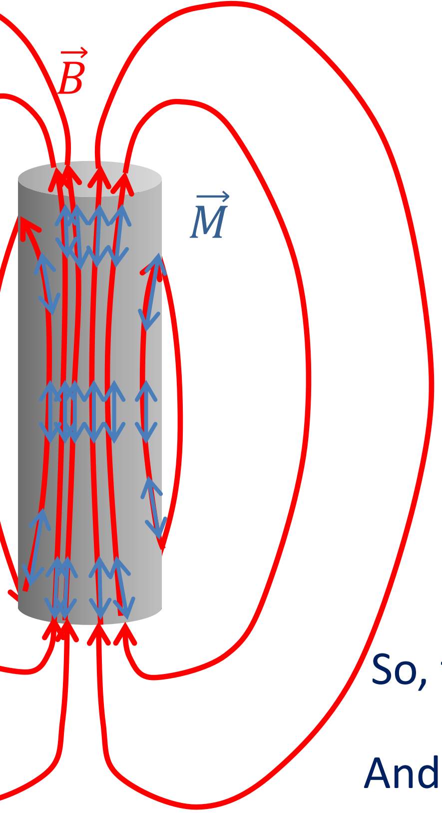




Only surface current is the bound around the cylinder, like solenoid

There is no *free* surface current, so no discontinuity in H. $\vec{H} \equiv \frac{1}{\mu_o}\vec{B} - \vec{M}$

Linear Para/Dia-magnetic



$$\vec{m}_{elect} = -\frac{e}{2m_e}\vec{L} - \frac{e^2 s^2}{4m_e}\vec{B}$$

Diamagnetic is clearly linearly dependent on B

While an *individual* electron's paramagnetic term *isn't* linearly dependent on B, the fraction of a large population of atoms that get their electrons aligned is fairly linearly dependent on B.

So, for many materials, to a good approximation, $\vec{M} \propto \vec{B}$

And since $\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right)$ we can also say $\vec{M} \propto \vec{H}$

Putting a name to the proportionality constant: $\vec{M} = \chi_m \vec{H}$

↑ **Magnetic Susceptibility**

Paramagnetic > 0
Diamagnetic < 0

Combining things: $\vec{B} = \mu_o\left(\vec{H} + \chi_m \vec{H}\right)$

or $\vec{B} = \mu_o\left(1 + \chi_m\right)\vec{H}$

or $\vec{B} = \mu\vec{H}$ $\mu \equiv \mu_o\left(1 + \chi_m\right)$

↑ **Permeability**

(Inconsistency Warning: for polarization and electric field, the constant was defined between P and *E*, but this is between M and *H*.)

Linear Diamagnetic

Example: Ball-park value for diamagnetic contribution to χ_m. We'll think about that atomic scale.

$$M = \chi_m H \quad \text{and} \quad M = \frac{dm}{d\tau} \approx \frac{m_{elect}}{\tau_{elect}} \quad \text{For which} \quad m_{elect.dia} = -\frac{e^2 s^2}{4m_e} B$$

This atomic dipole's personal space, τ, is that of an atom, in a cubic lattice with atoms distance *d* apart, that's $\tau_{elect.dia} = d^3$

The orbit of the electron is of this same order, so $s^2 \approx d^2$

so

$$M \approx \left(-\frac{e^2}{4m_e d} \right) B$$

But χ is defined as the proportionality constant between M and H, not M and B, so rephrasing with the help of $B = \mu_o (1 + \chi_m) H$

$$M = \frac{1}{\mu_o \left(1 + \frac{1}{\chi_m}\right)} B$$

Comparing,

$$\left(-\frac{e^2}{4m_e d} \right) \approx \frac{1}{\mu_o \left(1 + \frac{1}{\chi_m}\right)} \quad \text{or} \quad \chi_m \approx \left(1 - \frac{4m_e d}{\mu_o e^2} \right)^{-1}$$

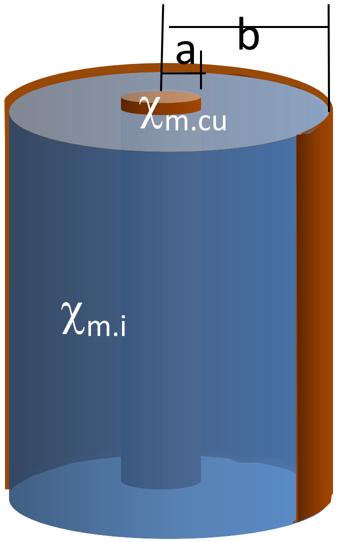
Plugging in,

$$\chi_m \approx \left(1 - \frac{4\left(9.11\times 10^{-31} kg\right)\left(2\times 10^{-10} m\right)}{4\pi \times 10^{-7} \frac{N}{A} \left(1.6\times 10^{-19} C\right)^2} \right)^{-1} \approx -4.4\times 10^{-5}$$

Right ballpark for values in Table 6.1

Linear Para/Dia-magnetic

$$I_{free} = \int \vec{J}_{free} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l}$$ $$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right)$$ $$\vec{M} = \chi_m \vec{H}$$



Example/Exercise: A coaxial cable consists of a copper wire of radius *a* surrounded by a concentric copper sheath of radius *b*. Copper has a magnetic susceptibility $\chi_{m.cu}$. The space between is filled with an insulating material of susceptibility $\chi_{m.i}$. If a current *I* flows up the inner wire (uniformly distributed across the wire's cross-section) and down the outer sheath,

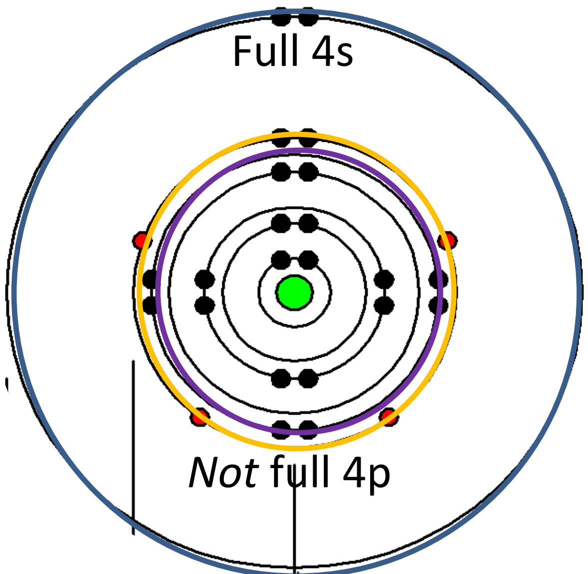
a) find the Auxiliary field everywhere
b) Find the Magnetization everywhere
c) Find the Magnetic field everywhere

I'll do for s<a (inside coper wire), you'll do b<s<a and s<b

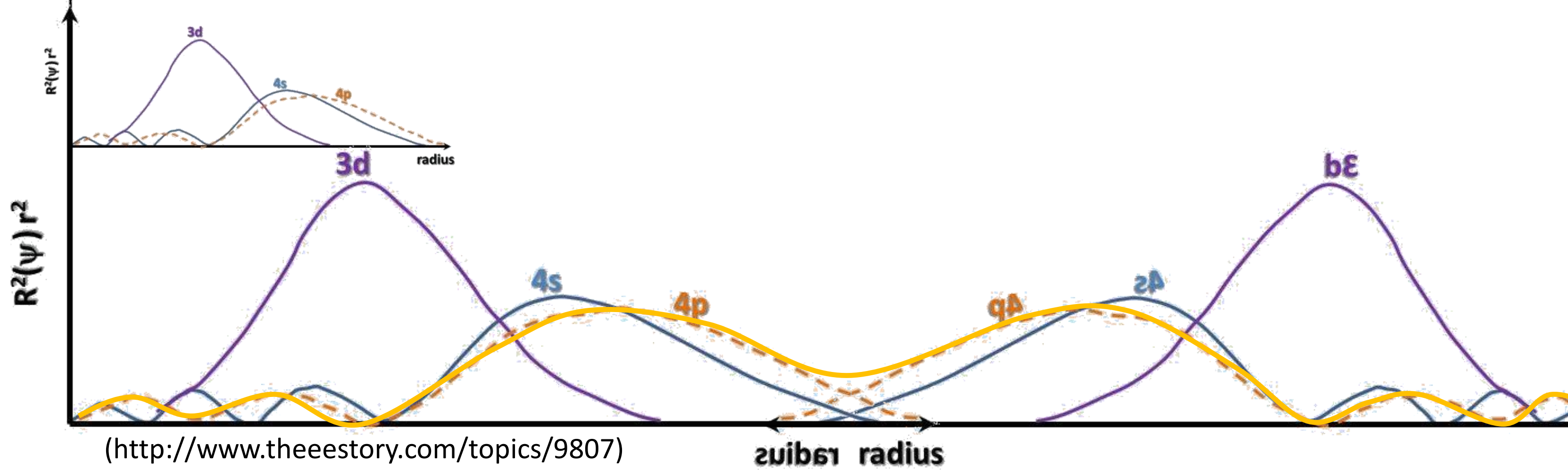
The other magnetism

Ferromagnetism

Fe: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$

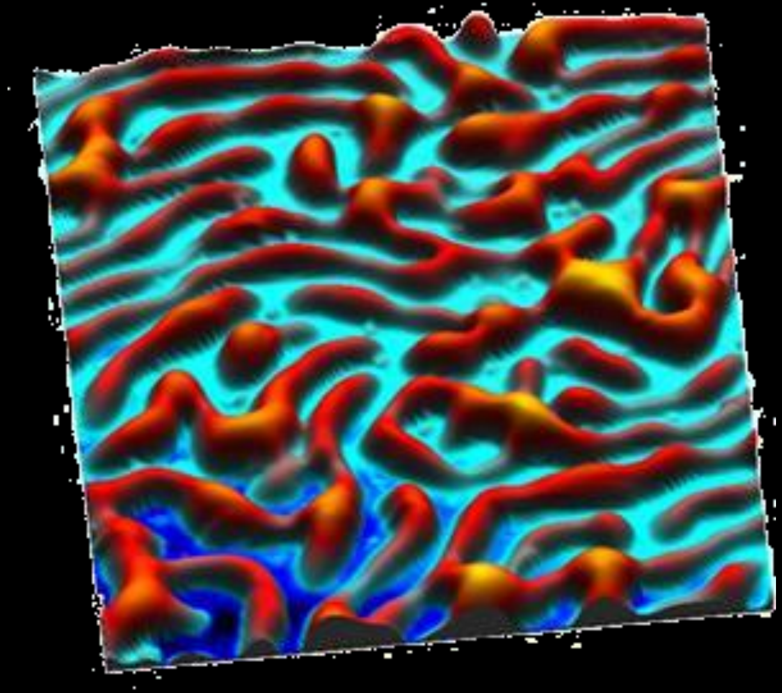


The 3d level is actually higher energy than the on-average larger 4s (which has 4 radial peaks, one closer to the nucleus than the 3d's inner radial peaks), so Iron and its neighbors have filled 4s but only partially filled 3d which is too far in to covalently bond but far enough out to overlap with neighboring irons' 3d's and form a conduction band. So they can share extended wave functions and long wavelength's it's energetically favorable for electrons in this band to be spin-aligned!

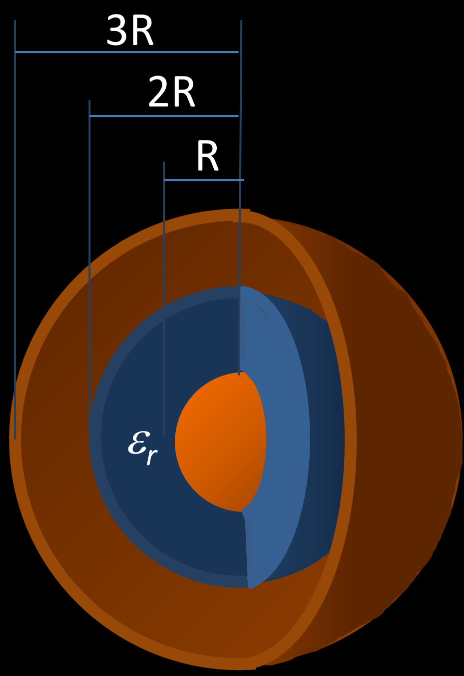


(http://www.theeestory.com/topics/9807)

Mon.	6.3, 6.4 Auxiliary Field & Linear Media	
Tues.		HW9
Mon.	7.1.1-7.1.3 Ohm's Law & Emf	
Wed.	7.1.3-7.2.2 Emf & Induction	
Fri.	7.2.3-7.2.5 Inductance and Energy of B	



Exercise: There are two spinning metal spherical shells with radii R and 3R and equal and opposite surface charge. There is material with a susceptability χ_m between radii R and 2R. What is the magnetic field everywhere?



Boundary Conditions & Linear Material Magnetic fields

$$\vec{H} \equiv \tfrac{1}{\mu_o}\vec{B} - \vec{M} \qquad \vec{M} = \chi_m \vec{H} \qquad \mu \equiv \mu_o(1+\chi_m) \qquad \vec{B} = \mu\vec{H}$$

Along

$$\vec{B}_{\parallel top} - \vec{B}_{\parallel bottom} = \vec{K}_{(all)} \times \hat{n}$$

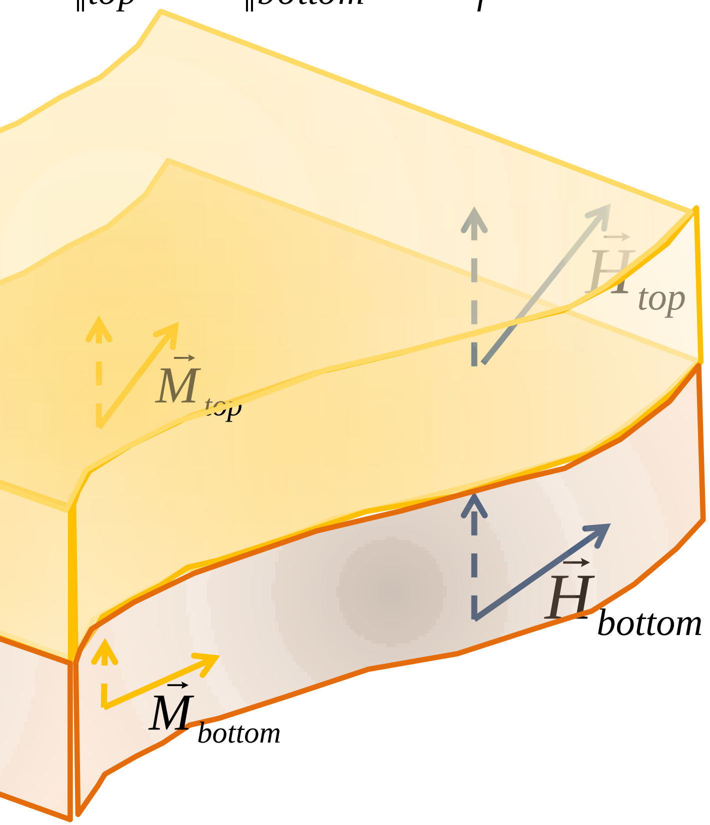
$$\vec{H}_{\parallel top} - \vec{H}_{\parallel bottom} = \vec{K}_{f} \times \hat{n}$$

$$\vec{K} - \vec{K}_{free} = \vec{K}_b = \vec{M} \times \hat{n}$$

Across

$$B_{\perp top} - B_{\perp bottom} = 0$$

$$H_{\perp top} - H_{\perp bottom} = M_{\perp top} - M_{\perp bottom}$$



Exercise: Problem 6.27