Mon. 9/16 Wed. 9/18 Thurs 9/19 Fri. 9/20	*(C 16)* **2.3.4-.3.5** Electric Potential **2.4.1-.4.2** Work & Energy in Electrostatics **T3** Contour Plots **2.4.3-4.4** Work & Energy in Electrostatics	HW2
Mon. 9/23 Wed. 9/25	**2.5** Conductors ***Summer Science Research Poster Session****: Hedco7pm~9pm*	HW3

Materials

Last Time

Met the Electric Potential Difference
Explored its relation to E.

- **Electric Potential Difference.**
  - $\Delta V_1 \equiv \frac{\Delta P.E._{1,2}}{q_2} = -\int_a^b \vec{E}_1(\vec{r}_2)\cdot d\vec{\ell}$
  - Or $\boxed{V(P) \equiv -\int_{ref}^{P} \vec{E}\cdot d\vec{\ell}}$
  - $\vec{E} = -\vec{\nabla} V$

Met its relation to charges:

Poisson's Equation is written as

$$\boxed{\nabla^2 V = -\rho/\varepsilon_0}$$

In regions where there is no charge, ***Laplace's Equation*** holds

$$\nabla^2 V = 0.$$

Today:

Potential of a Localized Charge Distribution

For a single point charge at the origin, the potential is

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r},$$

which only depends on the distance from the charge.

If a point charge is not at the origin, but the location $\vec{r}\,'$, the potential at the location $\vec{r}$ is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\mathscr{r}}, \qquad [2.26]$$

where $\mathscr{r}$ is the distance from the charge.

Because of the Superposition Principle, the potential for a collection of charges $q_1, q_2, \ldots$ at $\vec{r}_1, \vec{r}_2, \ldots$ is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}. \qquad [2.27]$$

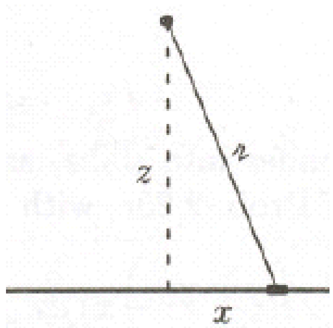
Start with the summation, <u>not</u> the integral equation [2.29], even if the distribution is continuous!

<u>Example</u>:

Problem 2.25(b) – may have to look up an integral

Find the potential a distance *z* from the center of a line of length 2*L* with a uniform charge density λ. Check that $\vec{E} = -\vec{\nabla}V$ gives the right result (see Ex. 2.1).

The potential due to a segment of length *dx* located at *x* (see the diagram below) is



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{\mathscr{r}_i} = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda\, dx)}{\sqrt{x^2+z^2}}.$$

Add (integrate) up the contributions from all parts of the line to get

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{dx}{\sqrt{x^2+z^2}}.$$

Look up the integral (I used the Wolfram website):

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x+\sqrt{x^2+z^2}\right)\right]_{-L}^{L} = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L+\sqrt{L^2+z^2}\right) - \ln\left(-L+\sqrt{L^2+z^2}\right)\right].$$

Look in L>>z limit, agree with previous example?

Check the answer. (This works because $\partial V/\partial x = \partial V/\partial y = 0$ when *z* = 0. In other words, we know by symmetry that the electric field only has a *z* component.)

Exercise: Take gradient and get electric field.

$$
\begin{aligned}
-\vec{\nabla} V &= -\frac{\partial V}{\partial z}\hat{z} \\
&= -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{L+\sqrt{L^2+z^2}}\left(\tfrac{1}{2}\frac{1}{\sqrt{L^2+z^2}}2z\right)-\frac{1}{\left(-L+\sqrt{L^2+z^2}\right)}\left(\tfrac{1}{2}\frac{1}{\sqrt{L^2+z^2}}2z\right)\right]\hat{z} \\
&= -\frac{\lambda}{4\pi\varepsilon_0}\frac{z}{\sqrt{L^2+z^2}}\left[\frac{\left(-L+\sqrt{L^2+z^2}\right)-\left(L+\sqrt{L^2+z^2}\right)}{L^2-\left(L^2+z^2\right)}\right]\hat{z} = \frac{1}{4\pi\varepsilon_0}\frac{2L\lambda}{z\sqrt{L^2+z^2}}\hat{z}
\end{aligned}
$$

This agrees with Example 2.1.

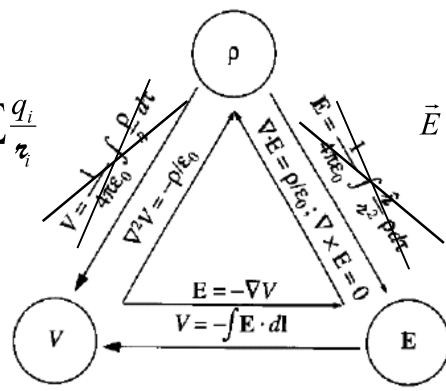
Exercise: What about the potential due to a uniform disc of charge (just set up and generalize integral)

Worth Setting up Example 2.7 to see where the expression for r comes from. (p. 85)

Summary of Electrostatics

Figure 2.35 is a good summary, but the summation equations for V and $\vec{E}$ are much more useful than the integrals ones. We've used all of the relations, except for Poisson's Equation, $\nabla^2 V = -\rho/\varepsilon_0$.

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$$



Only 1 Relation's worth of info. Griffith's points out that , while there are 7 different relations here, all the essential information is contained in just one: Coulomb's Law, the others follow from it.

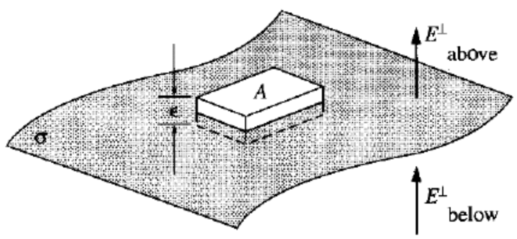
Boundary Conditions

Intro: future use in solving Diff. Eq's. Half of the relations in the above triangle of logic are *differential equations.* We will eventually get in the business of *solving differential equations.* You may remember from past experience solving differential equations that a key step is applying *boundary conditions* – those take you from general to specific solutions to the differential equation. Though we're not now going to solve these differential equations, we do have the tools now to say a few things about how E and V behave across boundaries.

Suppose there is a surface with a charge density σ (may depend on position). How do the electric field and the potential differ across the boundary? Be careful to be consistent about what direction is called positive.

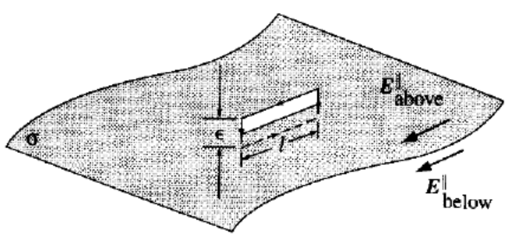
Apply Gauss's Law for a thin pillbox that extends across the boundary. Let the thickness ε go to zero to get

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\varepsilon_0}\sigma .$$



Apply $\oint \vec{E}\cdot d\vec{\ell} = 0$ to a thin rectangular loop that extends across the boundary. Let the thickness ε go to zero to get

$$\vec{E}^{\parallel}_{\text{above}} = \vec{E}^{\parallel}_{\text{below}} .$$



The results for the two components can be summarized as

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{n} ,$$

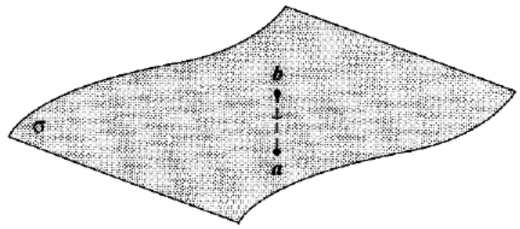
where $\hat{n}$ is a unit vector pointing from "below" to "above."

Find the potential difference for a short path across the boundary.

$$\Delta V = -\int_a^b \vec{E}(\vec{r}_2)\cdot d\vec{\ell}$$

Shrink the length of the path to get

$$V_{\text{above}} = V_{\text{below}}$$



Of course, since

$$\vec{E} = -\vec{\nabla} V \quad \text{and} \quad \vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{n} \quad \text{we have} \quad \begin{aligned} \vec{\nabla} V_a - \vec{\nabla} V_b &= -\frac{\sigma}{\varepsilon_0}\hat{n} \\ \tfrac{\partial}{\partial n} V_a - \tfrac{\partial}{\partial n} V_b &= -\frac{\sigma}{\varepsilon_0} \end{aligned}$$

Don't worry about "normal derivatives."

Ex. Boundary Conditions at a Charged Spherical Shell

Confirm that the electric field for a spherical shell of radius R with a charge Q satisfies the boundary conditions stated above.

$$\vec{E}(\vec{r}) = \begin{cases} 0 & r < R \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{r} & r > R \end{cases}$$

Let's call the outside of the sphere "above" and the inside "below." There is no component of the electric field parallel to the boundary, so $\vec{E}^{\parallel}_{\text{above}} = \vec{E}^{\parallel}_{\text{below}}$ is trivial.

The normal is $\hat{n} = \hat{r}$. The charge per area is

$$\sigma = \frac{Q}{4\pi R^2}.$$

The two sides of the second condition are

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} - 0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2},$$

$$\frac{\sigma}{\varepsilon_0} = \frac{\left(Q/4\pi R^2\right)}{\varepsilon_0},$$

which are equal as they should be.

<u>Preview</u>

For Wednesday, you'll read about work and energy for electrostatics. We'll also work on a 3rd Tutorial.

"I'm not sure how eqn 2.37 was derived, can we go over that?"
Jessica Hide responses Post a response
Admin

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I was also not very comfortable with the derivation in this section and would appreciate going over that in some better detail.

Casey McGrath

Yes, Griffiths mentions a discontinuity in the gradient of the potential, which I just cant seem to picture the right way in my head because I can't see where the discontinuity is coming from. And then he continues to build off of that in that normal derivative at the end, but by then he had lost me a bit.

Rachael Hach

Does the discontinuity only occur when we are dealing with a uniform charge distribution so at the surface of the distribution there's a change in the direction of the field that we can't really deal with mathematically?

Ben Kid

"I feel that I understood the first section in the reading more than I did the second on boundary conditions, so I'm hoping we could spend more time on that part of the reading."
Sam Post a response
Admin

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"Why can we switch from regular "r" (equation before 2.26) to script "r" in equation 2.26? Before it was saying that we went from infinity to the point where our test charge is located - by changing to script "r," doesn't that mess up the derivation?"
Casey McGrath Hide response Post a response
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Before equation 2.26, he was specifying that the point charge was at the origin, and with that, he derived V(r) at the bottom of page 84. He switches to script r in 2.26 because he's talking about the general case now, where the point charge is no longer specified to be at the origin.

Freeman, Nappleton

"And then additionally, in Example 2.8, Griffiths mentions a need for identifying and taking the positive root and dropping the other, relative to your position inside our outside the surface. Could we go into that a little?"
Rachael Hach Hide response Post a response
Admin

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I guess I was a little confused about why we get rid of the negative root entirely. Do we know not to use it just because it will give us a non-real answer?

Ben Kid

"Can we go over one of the problems in 2.3.4? Maybe 2.26, using the new method for finding V."
Spencer Post a response
Admin