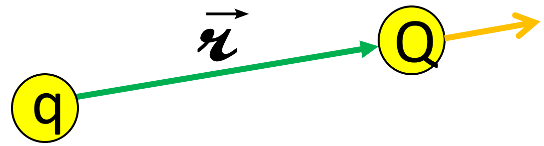
Mon. Wed Thurs. Fri.	(C 17) 12.1.1-.1.2, 12.3.1 E to B; 5.1.1-.1.2 Lorentz Force Law: fields and forces (C 17) 5.1.3 Lorentz Force Law: currents (C 17) 5.2 Biot-Savart Law	 HW6

Force between stationary charges

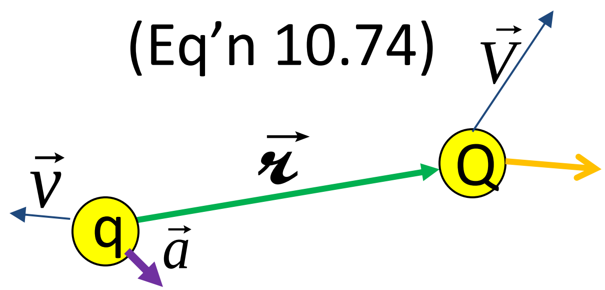
(Coulomb's Law: Eq'n 2.1)



$$\vec{F}_{q\to Q} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{\mathscr{r}^2_{q\to Q}}\hat{\mathscr{r}} = \frac{qQ}{4\pi\varepsilon_o}\frac{\vec{\mathscr{r}}}{(\mathscr{r})^3}$$

Force between moving charges

(Eq'n 10.74)

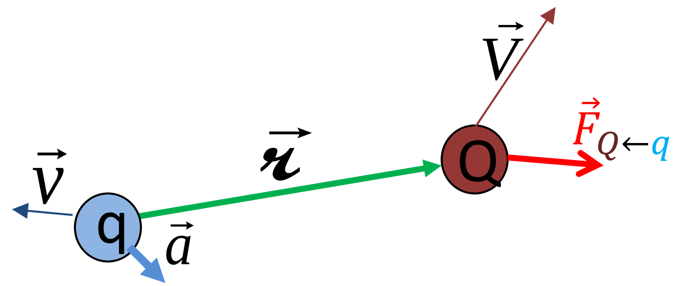


$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left\{\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right] + \frac{\vec{V}}{c}\times\left[\hat{\mathscr{r}}\times\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]\right]\right\}$$

"The entire theory of classical electrodynamics is contained in that equation…but you see why I preferred to start out with Coulomb's law." - Griffiths

Force between moving charges



$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left\{\underbrace{\left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]}_{\text{Electric}} + \underbrace{\frac{\vec{V}}{c} \times \left[\hat{\mathscr{r}} \times \left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]\right]}_{\text{Magnetic}}\right\}$$

Electric

Depends on *observer's* perception of *source* charge's velocity and acceleration

Magnetic

Also depends on *observer's* perception of *recipient* charge's velocity

But who's moving how fast is all *relative*

Note: extremely asymmetric between two charges; reciprocity (Newton's 3rd) does not hold

Crash Course in Special Relativity

Principle: Laws of Physics should be the same in all inertial frames of reference

~~**Old frame-transformation math:**~~

$$\vec{v}_{ball-you} = \vec{v}_{ball-me} + \vec{v}_{me-you}$$

$$\Delta\vec{x}_{ball-you} = \Delta\vec{x}_{ball-me} + \vec{v}_{me-you} \cdot \Delta t$$

Observations:

- laws of E&M referenced an absolute speed, not speed relative to preferred frame
- Charge moving near stationary magnet feels magnetic force; charge stationary near moving magnet feels…?

One of the three must go

Key to new frame-transformation math:

Speed of light is measured to be the same in all reference frames

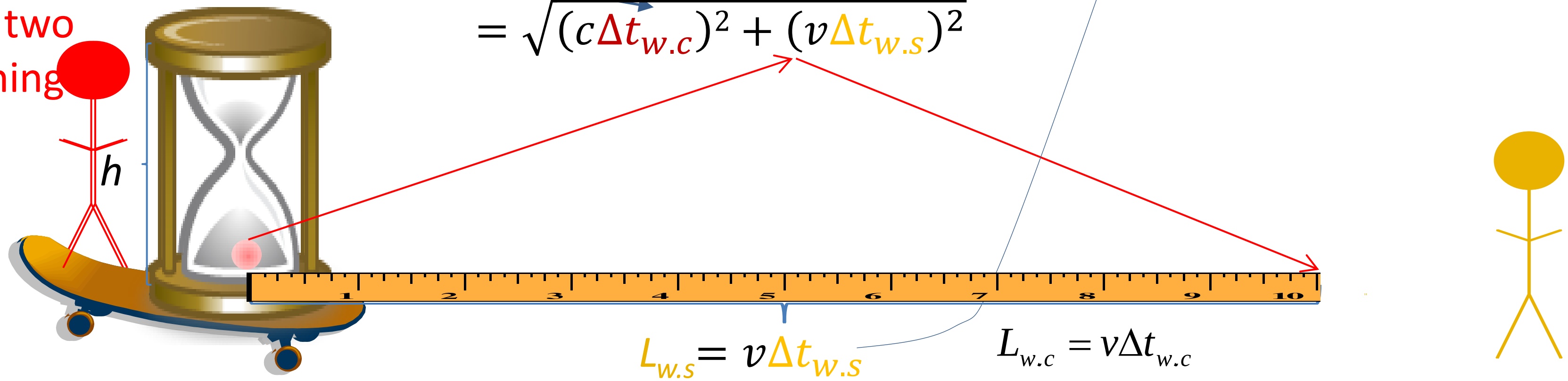
Crash Course in Special Relativity

New frame-transformation math derived – light clocks and meter sticks

with clock Measured in frame *at rest* with two events defining interval

$$\Delta t_{w.c} = 2\frac{h}{c} \qquad d = \sqrt{(2h)^2 + L_{w.s}^2} = \sqrt{(2h)^2 + (v\Delta t_{w.s})^2}$$

$$= \sqrt{(c\Delta t_{w.c})^2 + (v\Delta t_{w.s})^2}$$



$$L_{w.c} = v\Delta t_{w.c}$$

$$L_{w.c} = v\Delta t_{w.s}\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$L_{w.c} = L_{w.s}\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$L_{w.s} = \frac{L_{w.c}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma L_{w.c}$$

With meter stick Measured in frame at rest with two *locations* of defining events.

$$\frac{d}{\Delta t_{w.s}}\Delta t_{w.s} = \sqrt{(c\Delta t_{w.c})^2 + (v\Delta t_{w.s})^2}$$

$$c\Delta t_{w.s} = \sqrt{(c\Delta t_{w.c})^2 + (v\Delta t_{w.s})^2}$$

$$(c\Delta t_{w.s})^2 = (c\Delta t_{w.c})^2 + (v\Delta t_{w.s})^2$$

$$\left(1-\left(\frac{v}{c}\right)^2\right)(\Delta t_{w.s})^2 = (\Delta t_{w.c})^2$$

Clock's "proper" time: $\Delta t_{w.c} = \Delta t_{w.s}\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{1}{\gamma}\Delta t_{w.s}$

: Stick's "proper" length

Agreeing about both c and v means disagreeing about t and L.

Crash Course in Special Relativity

New frame-transformation math derived – light clocks and meter sticks

Example

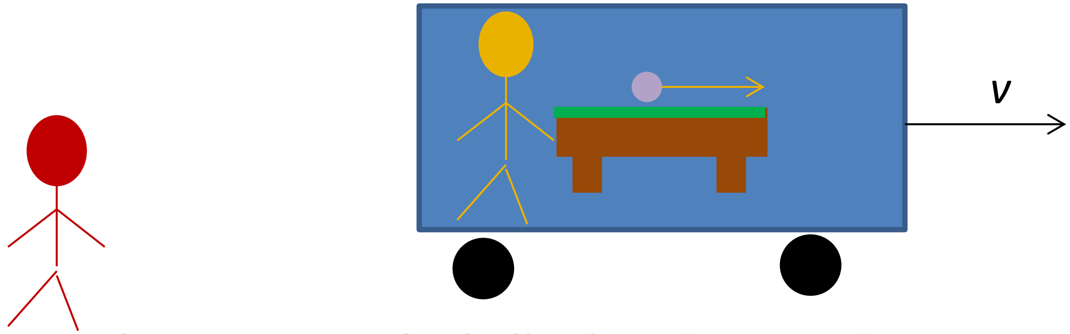
$$L_{w.s} = \frac{L_{w.c}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\, L_{w.c}$$

$$\Delta t_{w.c} = \Delta t_{w.s}\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{1}{\gamma}\Delta t_{w.s}$$

Think of the ladder and barn problem. Say the Farmer's got hold of the ladder and he's going to run at the barn.

Crash Course in Special Relativity

Galilean Transformation Corrected



I'm in a train watching a pool game. I see the ball role a distance $\Delta x_{ball.table}$ to the pocket. I see it's taking time $\Delta t_{ball.table}$ to get there. Meanwhile, the train and I are rolling through the station in the same direction at speed *v*. You, standing in the station, see all this happen. *Classically*, *you'd* imagine that with *my* yard stick, *I'd* measure the ball rolling a distance

$$"\Delta x_{ball.table}" = \Delta x_{ball.station} - v\Delta t_{ball.station}$$

But to phrase that in terms of what *I* in the train would measure, $"\Delta x_{ball.table}" = \frac{1}{\gamma}\Delta x_{ball.table}$

so, $\frac{1}{\gamma}\Delta x_{ball.table} = \Delta x_{ball.station} - v\Delta t_{ball.station}$ **or,** $\Delta x_{ball.table} = \gamma(\Delta x_{ball.station} - v\,\Delta t_{ball.station})$

Then again, *I'd* imagine with *your* yardstick *you'd* measure the ball rolling a distance

$$"\Delta x_{ball.station}" = \Delta x_{ball.table} + v\Delta t_{ball.table}$$

But in terms of what *you* in the station would measure, $"\Delta x_{ball.station}" = \frac{1}{\gamma}\Delta x_{ball.station}$

so, $\Delta xball_{station} = \gamma(\Delta x_{ball.table} + v\Delta t_{ball.table})$

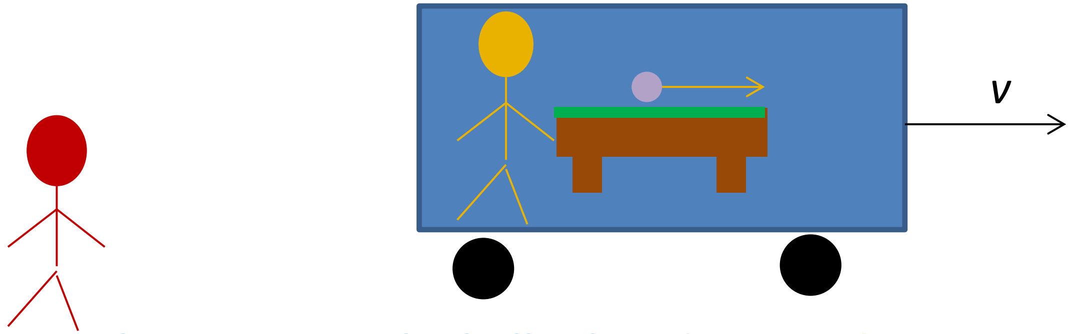
Consistent only if $\Delta t_{ball.station} = \gamma(\Delta t_{ball.table} + \frac{v}{c^2}\Delta x_{ball.table})$

$$\Delta t_{ball.table} = \gamma(\Delta t_{ball.station} - \frac{v}{c^2}\Delta x_{ball.station})$$

Note: both observers measure distances as *'proper'* lengths of their sticks, but neither is a equidistant to the two events- neither measures proper *time*

Crash Course in Special Relativity

Galilean Transformation Corrected



I'm in a train watching a pool game. I see the ball role a distance $\Delta x_{ball.table}$ to the pocket. I see it's taking time $\Delta t_{ball.table}$ to get there. Meanwhile, the train and I are rolling through the station in the same direction at speed ***v***. You, standing in the station, see all this happen.

$$v_{ball.station} = \frac{\Delta x_{ball.station}}{\Delta t_{ball.station}} = \frac{\gamma(\Delta x_{ball.table} + v\Delta t_{ball.table})}{\gamma\left(\Delta t_{ball.table} + \frac{v}{c^2}\Delta x_{ball.table}\right)}$$

Some algebra later,

$$v_{ball.station} = \frac{v_{ball.table} + \mathrm{v}}{1 + \frac{v v_{ball.table}}{c^2}}$$

Or renaming **v = v**$_{\textbf{table.station}}$

$$v_{ball.station} = \frac{v_{ball.table} + v_{table.station}}{1 + \frac{v_{table.station} v_{ball.table}}{c^2}}$$

Crash Course in Special Relativity

New frame-transformation math derived – light clocks and meter sticks

Example

$$v_{ball\_station} = \frac{v_{ball\_table} + v_{table\_station}}{1 + \frac{v_{table.station}\, v_{ball.table}}{c^2}}$$

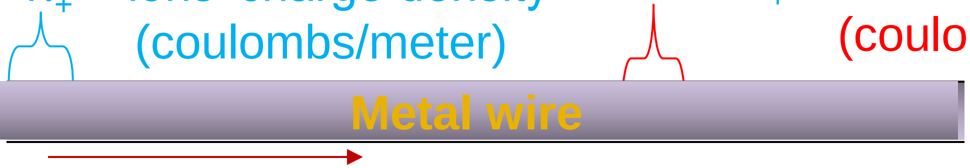
Transition / Transformation from E to M

Lab frame: you and I see an electrically neutral wire (all be it, with the electrons moving)

$$\lambda_+ = \frac{e}{\Delta x_{lab}} \qquad \lambda_- = -\frac{e}{\Delta x_{lab}}$$

λ_+ = ions' charge density (coulombs/meter)

$\lambda_- = -\lambda_+$ = electrons' charge density (coulombs/meter)



● = ionic atomic core
● = electron

v_e = electron velocity measured by us in the "lab frame"

Stationary charge

q

$$\vec{F}_{q\leftarrow wire} = q\vec{E}_{wire}$$

$$\vec{E}_{wire} = \vec{E}_+ + \vec{E}_- = \tfrac{1}{4\pi\varepsilon_o}\frac{2\lambda_+}{r} + \tfrac{1}{4\pi\varepsilon_o}\frac{2\lambda_-}{r} = \tfrac{1}{4\pi\varepsilon_o}\frac{2\lambda_+}{r} + \tfrac{1}{4\pi\varepsilon_o}\frac{-2\lambda_+}{r} = 0$$

Transition / Transformation from E to M

Lab frame: you and I see an electrically neutral wire (all be it, with the electrons moving)

$$\lambda_+ = \frac{e}{\Delta x_{lab}} \qquad \lambda_- = -\frac{e}{\Delta x_{lab}}$$

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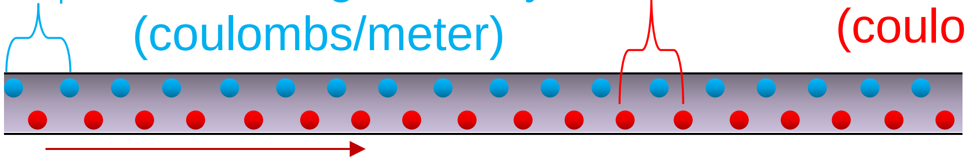


● = ionic atomic core

● = electron

v_e = electron velocity measured by us in the "lab frame"

Moving charge

$$\vec{F}_{q\leftarrow wire} = ?$$

Transition / Transformation from E to M

Charge's frame:

Chain of positive atoms moving backwards at $v_{atoms}{}' = -v_q$

So spacing seen by charge is related to their stationary, "proper" separation (as seen in lab) by

$$\Delta x_{atom}{}' = \frac{\Delta x_{atom.proper}}{\gamma_{q'}} = \frac{\Delta x_{lab}}{\gamma_{q'}} = \Delta x_{lab}\sqrt{1-\left(\frac{v_q}{c}\right)^2}$$

Or charge density appears compressed to

$$\lambda_+{}' = \frac{\gamma_{q'} e}{\Delta x_{lab}}$$

Chain of electrons moving forward at only $v_e{}' = \dfrac{v_e - v_q}{1-\frac{v_e v_q}{c^2}}$

So spacing seen by charge is related to their stationary, "proper" separation (*not* seen in lab) by

$$\Delta x_e{}' = \frac{\Delta x_{e.proper}}{\gamma_{e'}} \qquad \text{where} \qquad \gamma_{e'} = \frac{1}{\sqrt{1-\left(\frac{v_e{}'}{c}\right)^2}}$$

Similarly, separation in *lab* frame relates to "proper" separation (seen in electrons' rest frame)

$$\Delta x_{e.proper} = \gamma_e \Delta x_{lab} \quad \text{where } \gamma_e = \frac{1}{\sqrt{1-\left(\frac{v_e}{c}\right)^2}}$$

Combined: $\Delta x_e{}' = \dfrac{\gamma_e \Delta x_{lab}}{\gamma_{e'}}$

So, $\lambda_-{}' = \dfrac{-\gamma_{e'} e}{\gamma_e \Delta x_{lab}} = \dfrac{-\gamma_{e'}}{\gamma_e \gamma_{q'}} \lambda_+{}'$

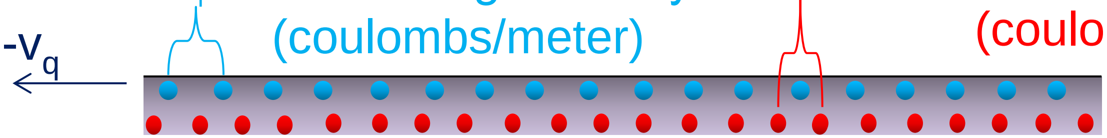
Transition / Transformation from E to M

Charge's frame:

$$\lambda_+' = \frac{\gamma_{q'} e}{\Delta x_{lab}} \qquad \lambda_-' = \frac{-\gamma_{e'} e}{\gamma_e \Delta x_{lab}} = \frac{-\gamma_{e'}}{\gamma_e \gamma_{q'}} \lambda_+'$$

λ_+' = ions' charge density (coulombs/meter)

λ_-' = electrons' charge density (coulombs/meter)



→ v_e' = electron velocity measured by charge

$$\vec{F}_{q\leftarrow wire}' = q\vec{E}_{wire}'$$

q

$$\vec{E}_{wire}' = \vec{E}_+' + \vec{E}_-' = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_+'}{r} + \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_-'}{r} = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_+'}{r} + \frac{1}{4\pi\varepsilon_o}\frac{2\left(\frac{-\gamma_{e'}}{\gamma_e\gamma_{q'}}\lambda_+'\right)}{r} = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_+'}{r}\left(1 - \frac{\gamma_{e'}}{\gamma_e\gamma_{q'}}\right) = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_+\gamma_{q'}}{r}\left(1 - \frac{\gamma_{e'}}{\gamma_e\gamma_{q'}}\right.$$

where $\gamma_q = \dfrac{1}{\sqrt{1-\left(\frac{v_q}{c}\right)^2}}$ $\quad \gamma_e = \dfrac{1}{\sqrt{1-\left(\frac{v_e}{c}\right)^2}}$ $\quad \gamma_{e'} = \dfrac{1}{\sqrt{1-\left(\frac{v_e'}{c}\right)^2}}$ $\quad v_e' = \dfrac{v_e - v_q}{1 - \frac{v_e v_q}{c^2}}$

A bit of algebra later, $E_{wire}' = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_+\gamma_{q'}}{r}\frac{v_e v_q}{c^2}$ Define $I = \lambda_+ v_e$ and $\mu_o \equiv \frac{1}{\varepsilon_o c^2}$ so $E_{wire}' = \frac{\mu_o}{4\pi}\frac{2I\gamma_{q'}}{r}v_q$

$$\vec{F}_{q\leftarrow wire}' = qv_q\frac{\mu_o}{4\pi}\frac{2I}{r}\gamma_{q'}$$

Hand waving: F~ distance/time^2

Transformation from rest frame requires factor of $\gamma/\gamma^2 = 1/\gamma$

Finally, we observe:

$$\vec{F}_{q\leftarrow wire} = qv_q\frac{\mu_o}{4\pi}\frac{2I}{r}$$

Jumping in to Magnetism

$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left\{\underbrace{[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})]}_{\text{Electric}} + \underbrace{\frac{\vec{V}}{c} \times [\hat{\mathscr{r}} \times [(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})]]}_{\text{Magnetic}}\right\}$$

Depends on *observer's* perception of *source* charge's velocity and acceleration

Also depends on *observer's* perception of *recipient* charge's velocity

Magnetic force

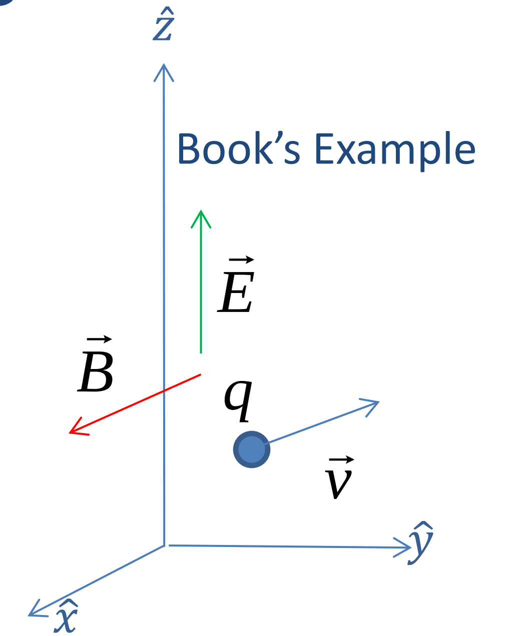
$$\vec{F}_{Q\leftarrow q.mag} = Q\vec{V} \times \underbrace{\frac{q}{4\pi c\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\{[\hat{\mathscr{r}} \times [(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})]]\}}_{\text{Magnetic Field,B}}$$

$$\vec{F}_{Q\leftarrow q.mag} = Q\vec{V} \times \vec{B}$$

Cyclotron Motion in a Uniform Magnetic Field

$$\frac{d\vec{p}}{dt} = \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



In general

$$m\begin{Bmatrix} dv_x/dt \\ dv_y/dt \\ dv_z/dt \end{Bmatrix} = q\left(\begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} + \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}\right) = q\begin{Bmatrix} E_x + v_y B_z - v_z B_y \\ E_y + v_z B_x - v_x B_z \\ E_z + v_x B_y - v_y B_x \end{Bmatrix}$$

$$\frac{dv_y}{dt} = \frac{qB}{m}v_z \qquad \frac{dv_z}{dt} = \frac{qE}{m} - \frac{qB}{m}v_y$$

Take next derivative

$$\frac{d^2v_y}{dt^2} = \frac{qB}{m}\frac{dv_z}{dt} \qquad \frac{d^2v_z}{dt^2} = -\frac{qB}{m}\frac{dv_y}{dt}$$

Cross substitute

$$\frac{d^2v_y}{dt^2} = \frac{qB}{m}\left(\frac{qE}{m} - \frac{qB}{m}v_y\right) \qquad \frac{d^2v_z}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_z$$

$$\frac{d^2v_y}{dt^2} = \frac{q^2BE}{m^2} - \left(\frac{qB}{m}\right)^2 v_y$$

Guess Solution Forms

$$v_z(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$$

$$v_y(t) = C_3\cos(\omega t) + C_4\sin(\omega t) + \frac{E}{B}$$

Plug in

$$\frac{dv_y}{dt} = \frac{qB}{m}v_z$$

$$\omega\left(-C_3\sin(\omega t) + C_4\cos(\omega t)\right) = \frac{qB}{m}\left(C_1\cos(\omega t) + C_2\sin(\omega t)\right)$$

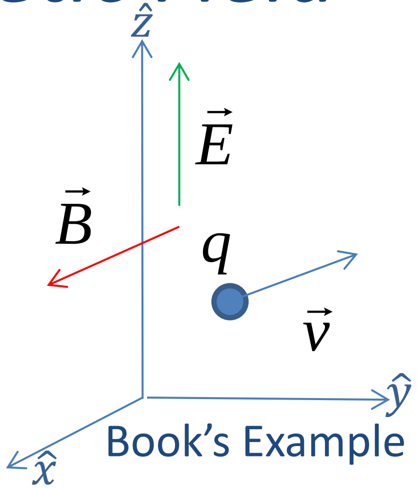
Compare terms and conclude

$$\omega = \frac{qB}{m} \qquad -C_3 = C_2 \qquad C_4 = C_1$$

Cyclotron Motion in a Uniform Magnetic Field

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$v_z(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) \qquad v_y(t) = C_1 \sin(\omega t) - C_2 \cos(\omega t) + \frac{E}{B}$$



where $\omega = \frac{qB}{m}$

For position components, integrate

$$z(t) = \frac{C_1}{\omega}\sin(\omega t) - \frac{C_2}{\omega}\cos(\omega t) + C_5 \qquad y(t) = -\frac{C_1}{\omega}\cos(\omega t) - \frac{C_2}{\omega}\sin(\omega t) + \frac{E}{B}t + C_6$$

Impose Initial Conditions

Start at origin

$$z(0) = -\frac{C_2}{\omega} + C_5 = 0 \qquad y(0) = -\frac{C_1}{\omega} + C_6 = 0$$

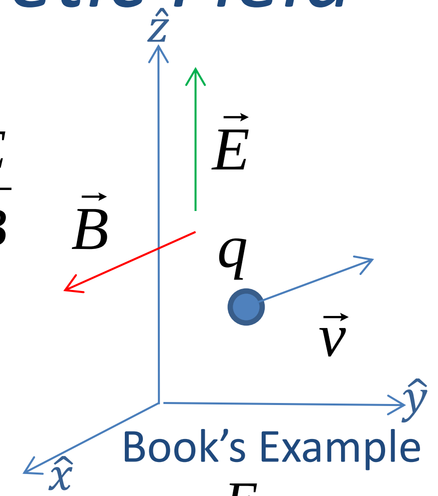
$$C_5 = \frac{C_2}{\omega} \qquad C_6 = \frac{C_1}{\omega}$$

$$z(t) = \frac{C_1}{\omega}\sin(\omega t) + \frac{C_2}{\omega}\left(1 - \cos(\omega t)\right) \qquad y(t) = \frac{C_1}{\omega}\left(1 - \cos(\omega t)\right) - \frac{C_2}{\omega}\sin(\omega t) + \frac{E}{B}t$$

Cyclotron Motion in a Uniform Magnetic Field

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$v_z(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) \qquad v_y(t) = C_1 \sin(\omega t) - C_2 \cos(\omega t) + \frac{E}{B}$$



where $\omega = \frac{qB}{m}$

Impose Initial Conditions

Initial position: (0,0)

$$z(t) = \frac{C_1}{\omega}\sin(\omega t) + \frac{C_2}{\omega}\left(1 - \cos(\omega t)\right) \qquad y(t) = \frac{C_1}{\omega}\left(1 - \cos(\omega t)\right) - \frac{C_2}{\omega}\sin(\omega t) + \frac{E}{B}t$$

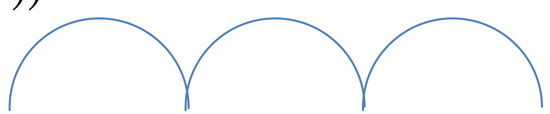
Initial Velocity: $\vec{v}(0) = 0$

$$v_z(0) = C_1 \cos(0) + C_2 \sin(0) \qquad v_y(0) = C_1 \sin(0) - C_2 \cos(0) + \frac{E}{B}$$

$$v_z(0) = C_1 = 0 \qquad v_y(0) = -C_2 + \frac{E}{B} = 0 \qquad C_2 = \frac{E}{B}$$

So.

$$v_z(t) = \frac{E}{B}\sin(\omega t) \qquad v_y(t) = \frac{E}{B}\left(1 - \cos(\omega t)\right)$$

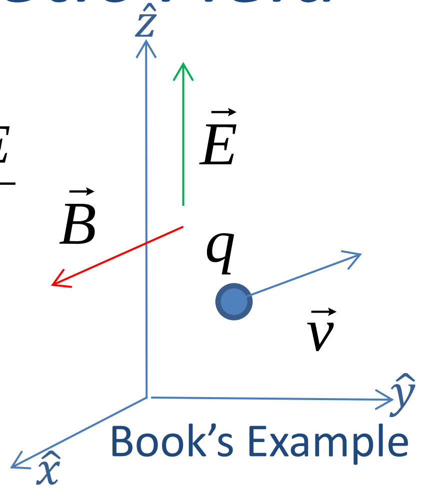


$$z(t) = \frac{E}{\omega B}\left(1 - \cos(\omega t)\right) \qquad y(t) = \frac{E}{\omega B}\left(\omega t - \sin(\omega t)\right)$$

Cyclotron Motion in a Uniform Magnetic Field

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$v_z(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) \qquad v_y(t) = C_1 \sin(\omega t) - C_2 \cos(\omega t) + \frac{mE}{\omega}$$



where $\omega = \frac{qB}{m}$

Impose Initial Conditions

Initial position: (0,0)

$$z(t) = \frac{C_1}{\omega} \sin(\omega t) + \frac{C_2}{\omega}\left(1 - \cos(\omega t)\right) \qquad y(t) = \frac{C_1}{\omega}\left(1 - \cos(\omega t)\right) - \frac{C_2}{\omega} \sin(\omega t) + \frac{mE}{\omega} t$$

Initial Velocity: $\vec{v}(0) = (E / B)\hat{y}$

$$v_z(0) = C_1 \cos(0) + C_2 \sin(0)$$

$$v_z(0) = C_1 = 0$$

$$v_y(0) = C_1 \sin(0) - C_2 \cos(0) + \frac{mE}{\omega}$$

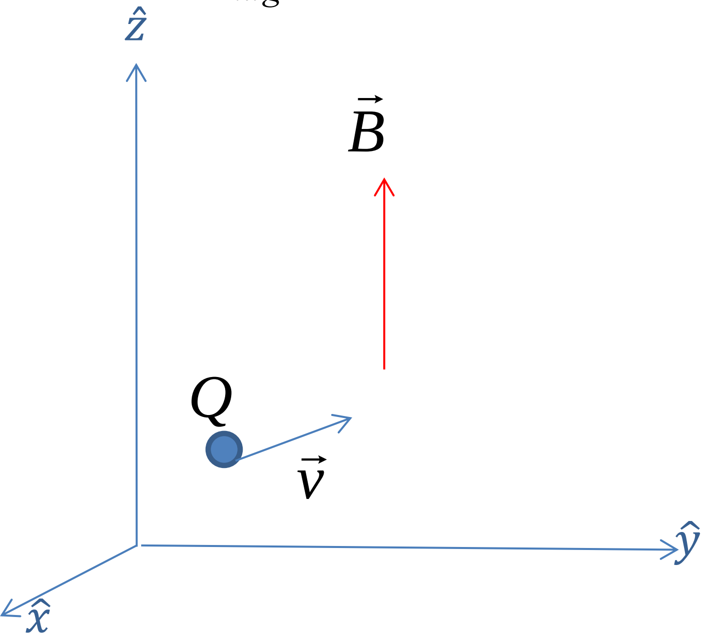
$$v_y(0) = -C_2 + \frac{mE}{\omega} = \frac{E}{B}$$

$$C_2 = \frac{mE}{\omega} - \frac{E}{B}$$

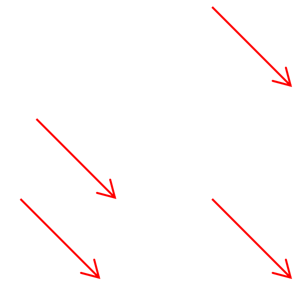
Mon. Wed Thurs. Fri.	(C 17) 12.1.1-.1.2, 12.3.1 E to B; 5.1.1-.1.2 Lorentz Force Law: fields and forces (C 17) 5.1.3 Lorentz Force Law: currents (C 17) 5.2 Biot-Savart Law	 HW6

Cyclotron Motion in a Uniform Magnetic Field

$$\vec{F}_{mag} = \vec{F}_B = Q\vec{v}\times\vec{B} \qquad \frac{d\vec{p}}{dt} = \vec{F}_{mag}$$



For $\vec{B} = B_z\hat{z}$



$$\frac{dv_y}{dt} = -\frac{qB_z}{m}v_x \qquad \frac{dv_x}{dt} = \frac{qB_z}{m}v_y \rightarrow \omega\left(-C_3\sin(\omega t)+C_4\cos(\omega t)\right) = \frac{qB_z}{m}\left(C_1\cos(\omega t)+C_2\sin(\omega t)\right)$$

$$\omega = \frac{qB_z}{m} \qquad -C_3 = C_2 \qquad C_4 = C_1$$

$$\frac{d^2v_y}{dt^2} = -\frac{qB_z}{m}\frac{dv_x}{dt}$$

Similarly for v_x:

$$\frac{d^2v_x}{dt^2} = -\left(\frac{qB_z}{m}\right)^2 v_x$$

$$\frac{d^2v_y}{dt^2} = -\left(\frac{qB_z}{m}\right)^2 v_y \rightarrow$$

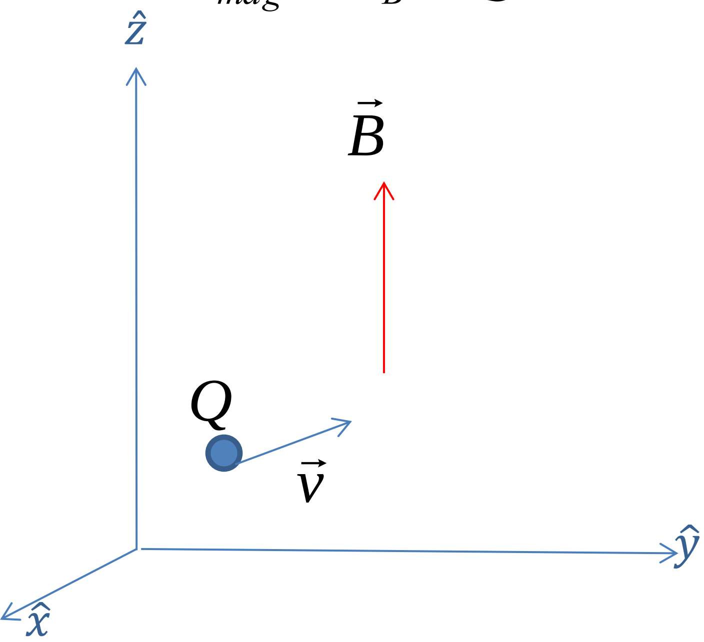
$$v_x(t) = C_3\cos(\omega t)+C_4\sin(\omega t)$$

$$v_y(t) = C_1\cos(\omega t)+C_2\sin(\omega t)$$

Plug in to

Cyclotron Motion in a Uniform Magnetic Field

$$\vec{F}_{mag} = \vec{F}_B = Q\vec{v} \times \vec{B}$$



$$v_x(t) = -C_2\cos(\omega t) + C_1\sin(\omega t)$$

$$\omega = \frac{qB_z}{m}$$

$$v_y(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$$

Integrate for positions:

$$\int_{x_o}^{x} dx = \int_{o}^{t} v_x(t)dt$$

$$x(t) - x_o = -\frac{C_2}{\omega}\sin(\omega t) - \frac{C_1}{\omega}(\cos(\omega t) - 1)$$

similarly:

$$y(t) - y_o = \frac{C_1}{\omega}\sin(\omega t) - \frac{C_2}{\omega}(\cos(\omega t) - 1)$$