Physics 233 **Lab 2**
September 2011 **The Rolling Sphere & Random Error**

Goals

- To investigate the motion of a sphere rolling down an inclined plane
- To study the distribution of a set of measurements
- To analyze the random uncertainties associated with the experimental measurements and the calculated results
- To practice writing the procedure section of a lab report.

Reading:

- Chapter 6 (Uncertainty of the Mean), chapter 7 (Propagation of Uncertainty), the supplement to chapter 7, and section 3.4 (The Procedure Section) of the lab reference manual
- Suppose that you make repeated measurements of a quantity *x* which can take on any value. If the differences in the measurements are due to random errors, the distribution of the measurements will typically follow a Gaussian distribution. If the mean of a series of measurements is $\bar{x}$ and the standard deviation is *s*, the probability density is

  $$P(x) = \frac{1}{s\sqrt{2\pi}} e^{-(x-\bar{x})^2/2s^2} .$$

  For a single measurement, the probability of measuring a value of *x* between *a* and *b* is

  $$\text{Prob}(a < x < b) = \int_a^b P(x)\, dx .$$

  In other words, the area under a graph of *P*(*x*) vs. *x* between *x* = *a* and *x* =*b* is equal to the probability of a measurement falling in that range. Since the total probability of measuring some value of *x* is one, we know that

  $$\int_{-\infty}^{+\infty} P(x)\, dx = 1 .$$

  For more details, see Philip R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969), pp. 43-49.

Pre-Lab Problems: (in WebAssign)

1. Exercise 6.2
2. Exercise 7.5 (from the Supplement)
3. Exercise 7.6
4. Find an expression for the uncertainty of the theoretical time t_{theory} that you expect a sphere to take to roll down the ramp. Your answer should be expressed in terms of the gravitational constant *g*, the distance *d* that the sphere rolls, the length *L* along the ramp, the height *H* of the ramp, and their uncertainties. (Hints: Put sin θ in terms of measured quantities instead of first solving for the angle θ. Do the calculations in steps, similar to Exercise 7.6.)

Lab Procedure:

- Using your “best” timing technique from last week, take a set of data consisting of 100 rolls of the sphere down the incline. Be sure to make appropriate measurements (with uncertainties) of the ramp, too.

Post-Lab Assignment:

1. Write the procedure section of a lab report. This should be typed.

2. Compare the result of the 100 measurements with the theoretical value:
   a. For the 100 time measurements, calculate the mean ($\bar{t}$), the standard deviation (s_t), and the uncertainty of the mean (U_m).
   b. If your experimental setup changed from the previous lab, recalculate the theoretical time t_{theory}.
   c. Using propagation of uncertainty, calculate the uncertainty in the theoretical time, U_{theory}. Assume that $g = 9.81 \pm 0.01$ m/s^2.
   d. Are the measurements consistent with the theoretical value? Explain.

3. Compare the distribution of your measurements to a Gaussian distribution:
   a. You will be making histograms using bins *between* values of t given by $\bar{t} \pm (m/4)s_t$ where $m = 1, 3, 5, \ldots$. Count the number of measurements in each bin. Calculate the fraction of measurements in each bin by dividing the number of measurements in each bin by the total number of measurements made.
   b. In a *height-normalized histogram*, the height represents the probability. Make a height-normalized histogram by plotting the fraction of measurements in each bin.
   In this type of graph, the heights of all of the bins add up to one. If you stacked all of the bins of the histogram, the height would be one and the width that of one bin. Therefore, the total area under this type of histogram is the width of a single bin ($s_t/2$, in the histogram that you made).
   c. In an *area-normalized histogram*, the area of a bin represents the probability. Make an area-normalized histogram by plotting the probability divided by ($s_t/2$). The total area under this histogram will be one so it can be compared directly to a Gaussian distribution, which also has that property (see the previous page).
   d. Plot a Gaussian probability density with the appropriate average and standard deviation on the same graph as the area-normalized histogram by doing the following. Evaluated the Gaussian distribution function at the centers of the time bins which are given by $\bar{t} \pm (n/2)s_t$ where $n = 0, 1, 2, \ldots$. Plot those points and draw in a smooth curve connecting them. This is the Gaussian distribution that best fits your data.
   e. How well does the area-normalized histogram match a Gaussian probability distribution? Do you think that the distribution was due to random errors? Explain.

   Note: Since you only made 100 measurements, the distribution of your data may not exactly match your expectations.