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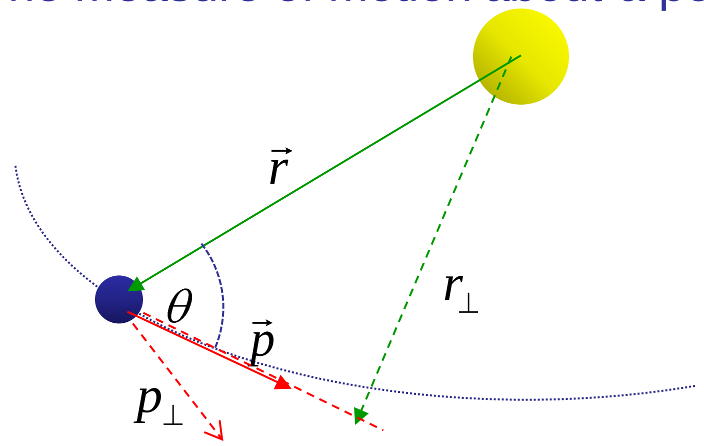
|---|---|---|
| Fri. | 11.10 Quantization, Quiz 11 Lecture Evals | RE 11.e |
| Mon. | Review for Final (1-11) | HW11: Pr’s 39, 57, 64, 74, 78 |
| Sat. | 9 a.m. Final Exam (Ch. 1-11) | |

# Using Angular Momentum

The measure of motion *about* a point




# Magnitude and Direction

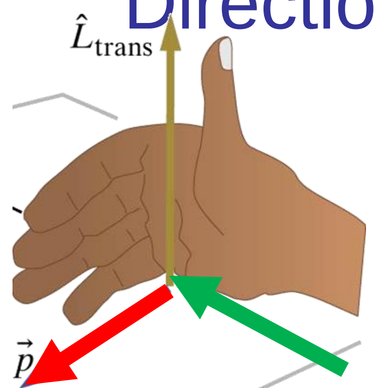
$\vec{\ } =$

$\vec{\ } = \langle \qquad \rangle$

## Magnitude

$$|L| = |p_\perp||r| = |p||r_\perp| = |p||r|\sin(\theta)$$

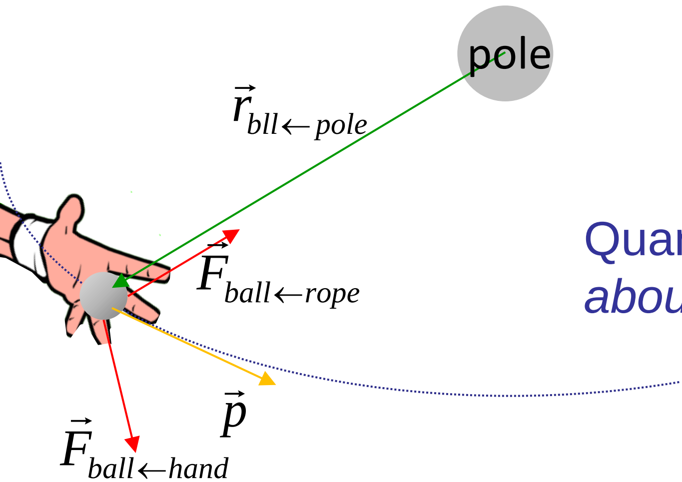
## Direction




Orient Right hand so fingers curl from the axis and with motion, then thump points in direction of angular momentum.

# Angular Momentum Principle

$$\frac{d}{dt}\vec{L}_{(about)A} = \sum_{net}\vec{\tau}_{(about)A}$$




Quantifies motion *about* a point

Interaction that changes motion *about* a point

## Torque

where, $\vec{\tau}_{(about)A} \equiv \vec{r}_{(from)A} \times \vec{F}$

## Magnitude

(yet another cross product)

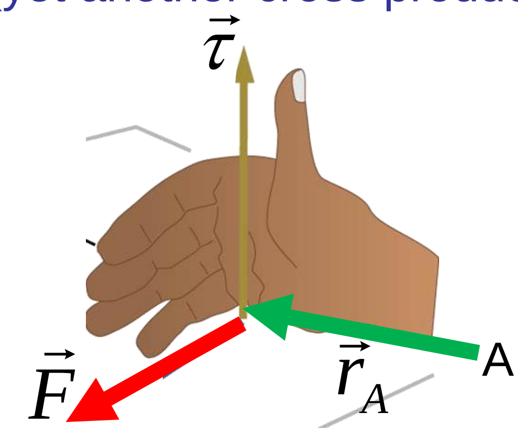
$$|\tau_A| = |r_A||F_\perp| \quad = |r_{A\perp}||F|$$

$$|\tau_A| = |r_A|\left(|F|\sin\theta\right) = \left(|r_A|\sin\theta\right)|F| = |r_A||F|\sin\theta$$

Making sense of the factors and cross-product

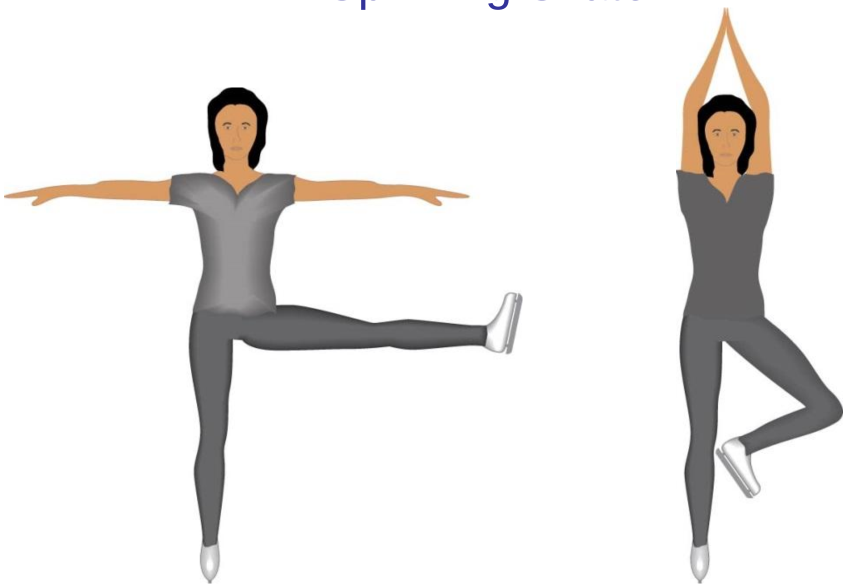
## Direction

(yet another cross product)



# Zero-Torque Systems

Spinning Skater



Initial

Final

$$\vec{L}_{rot.f} - \vec{L}_{rot.i} = \vec{\tau}_{ave}\Delta t$$

$$\vec{L}_{rot.f} - \vec{L}_{rot.i} \approx 0$$

$$\vec{L}_{rot.i} \approx \vec{L}_{rot.f}$$

$$I_i\vec{\omega}_i \approx I_f\vec{\omega}_f$$

mass farther from axis: $I_i$ larger
$\omega_i$ smaller

mass closer to axis:
$I_f$ smaller
$\omega_i$ larger

# Three Fundamental Principles

Angular Momentum:

$$\frac{d}{dt}\vec{L}_{(about)A} = \sum_{net} \vec{\tau}_{(about)A}$$

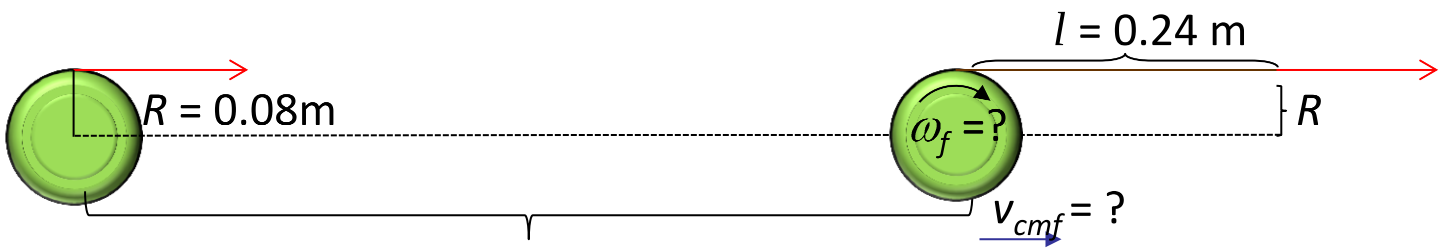
(linear) Momentum:

$$\frac{d}{dt}\vec{p} = \sum_{net} \vec{F}$$

Energy:

$$\Delta E = \sum_{net} W$$

Example all three together! Say we have a uniform 0.4 kg puck with an 8 cm radius. A 24 cm string is initially wrapped around its circumference. If it's on a frictionless surface and a 10 N force is applied to the end of the string until it's unwound…




$$I_{puck} = \tfrac{1}{2} m R^2 \qquad d = ? \qquad \Delta t = ?$$

a. What will be its rate of rotation when the string is fully unwound?

Energy Principle

$$\Delta E_{total} = W$$

$$\Delta K_{trans} + \Delta E_{int} = \vec{F} \cdot \Delta\vec{r}_F$$

$$\vec{F} \cdot \Delta\vec{r}_{cm} + \Delta K_{rot} = F\left(d + l\right)$$

$$Fd + \left(\tfrac{1}{2} I\omega_f^{\,2} - \tfrac{1}{2} I\omega_i^{\,2}\right) = Fd + Fl$$

$$\tfrac{1}{2} I\omega_f^{\,2} = Fl$$

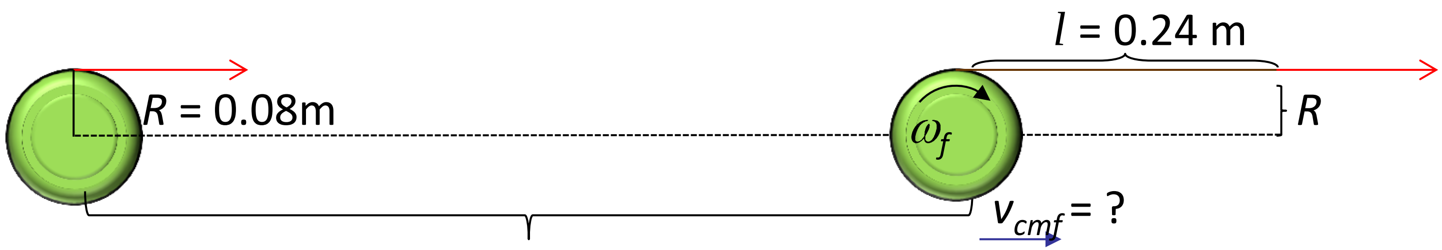
$$\omega_f = \sqrt{\frac{2Fl}{I}}$$

$$\omega_f = \sqrt{\frac{2Fl}{\tfrac{1}{2} R^2 m}} = \frac{2}{R}\sqrt{\frac{Fl}{m}}$$

# Three Fundamental Principles

| Angular Momentum: | (linear) Momentum: | Energy: |
|---|---|---|
| $\frac{d}{dt}\vec{L}_{(about)A} = \sum_{net} \vec{\tau}_{(about)A}$ | $\frac{d}{dt}\vec{p} = \sum_{net} \vec{F}$ | $\Delta E = \sum_{net} W$ |

Example all three together! Say we have a uniform 0.4 kg puck with an 8 cm radius. A 24 cm string is initially wrapped around its circumference. If it's on a frictionless surface and a 10 N force is applied to the end of the string until it's unwound…




$I_{puck} = \frac{1}{2} mR^2$ $\quad$ $d = ?$ $\quad$ $\Delta t = ?$

a. What will be its rate of rotation when the string is fully unwound? $\omega_f = \frac{2}{R}\sqrt{\frac{Fl}{m}}$

b. How long was the force applied? Angular Momentum Principle (axis through final location of cm)

$$\Delta \vec{L} = \int \vec{\tau} dt$$

$$\vec{L}_f - \vec{L}_i = \vec{\tau}\Delta t$$

$$(\vec{L}_{f.trans} + \vec{L}_{f.rot}) - (\vec{L}_{i.trans} + \vec{L}_{i.rot}) = \vec{\tau}\Delta t$$

$$\Rightarrow \Delta t = \frac{I\omega_f}{RF} = \frac{\frac{1}{2}R^2 m\omega_f}{RF} = \frac{Rm\omega_f}{2F} = \frac{Rm\frac{2}{R}\sqrt{\frac{Fl}{m}}}{2F} = \sqrt{\frac{ml}{F}}$$

$$I\vec{\omega}_f = \left| \vec{r}_{F-a} \times \vec{F} \right| \Delta t$$

Torque and final angular velocity in –z direction

$$I\omega_f = \left| RF \right| \Delta t \quad \Rightarrow \quad \Delta t = \frac{I\omega_f}{RF} = \frac{\frac{1}{2}R^2 m\omega_f}{RF} = \frac{Rm\omega_f}{2F} = \frac{Rm\frac{2}{R}\sqrt{\frac{Fl}{m}}}{2F} = \sqrt{\frac{ml}{F}}$$

# Three Fundamental Principles

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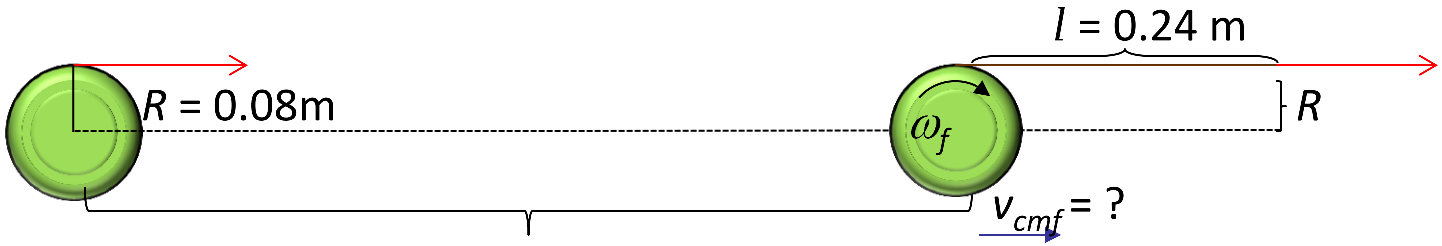



$$I_{puck} = \tfrac{1}{2}mR^2$$

a. What will be its rate of rotation when the string is fully unwound? $\omega_f = \frac{2}{R}\sqrt{\frac{Fl}{m}}$

b. How long was the force applied? $\Delta t = \sqrt{\frac{ml}{F}}$

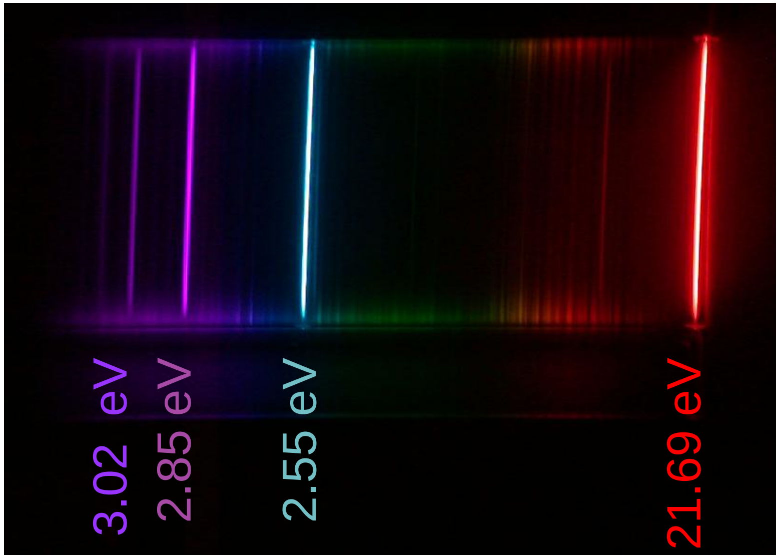
You try:

c. How quickly is the puck finally sliding, $v_{cm\,f}$?

d. How far has the puck moved, d?

# Understanding the Hydrogen Spectrum:

## Bohr's step toward Quantum Mechanics



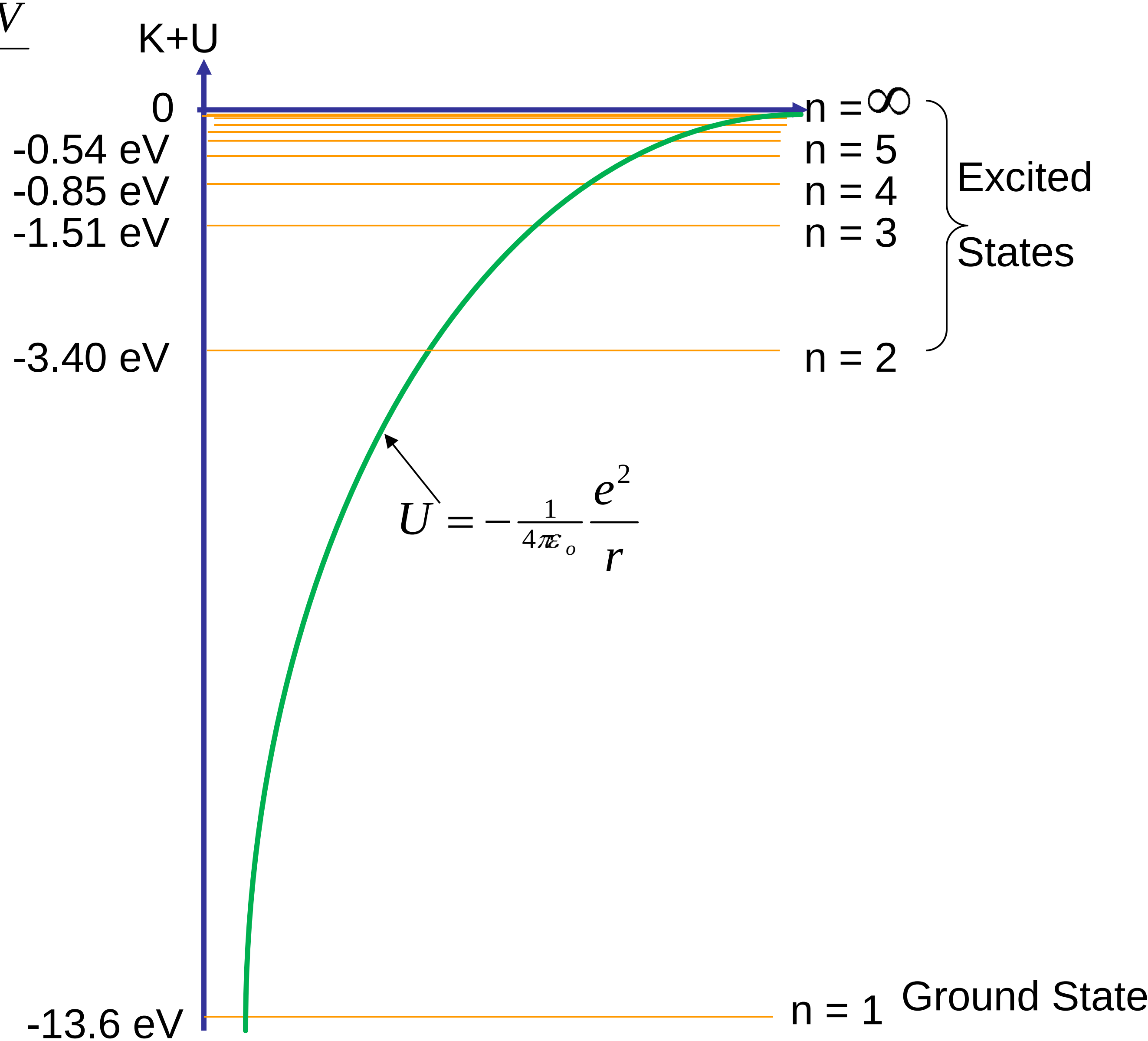
# Hydrogen Energy Levels

$$(K+U)_n = -\frac{13.6eV}{n^2}$$

etc.

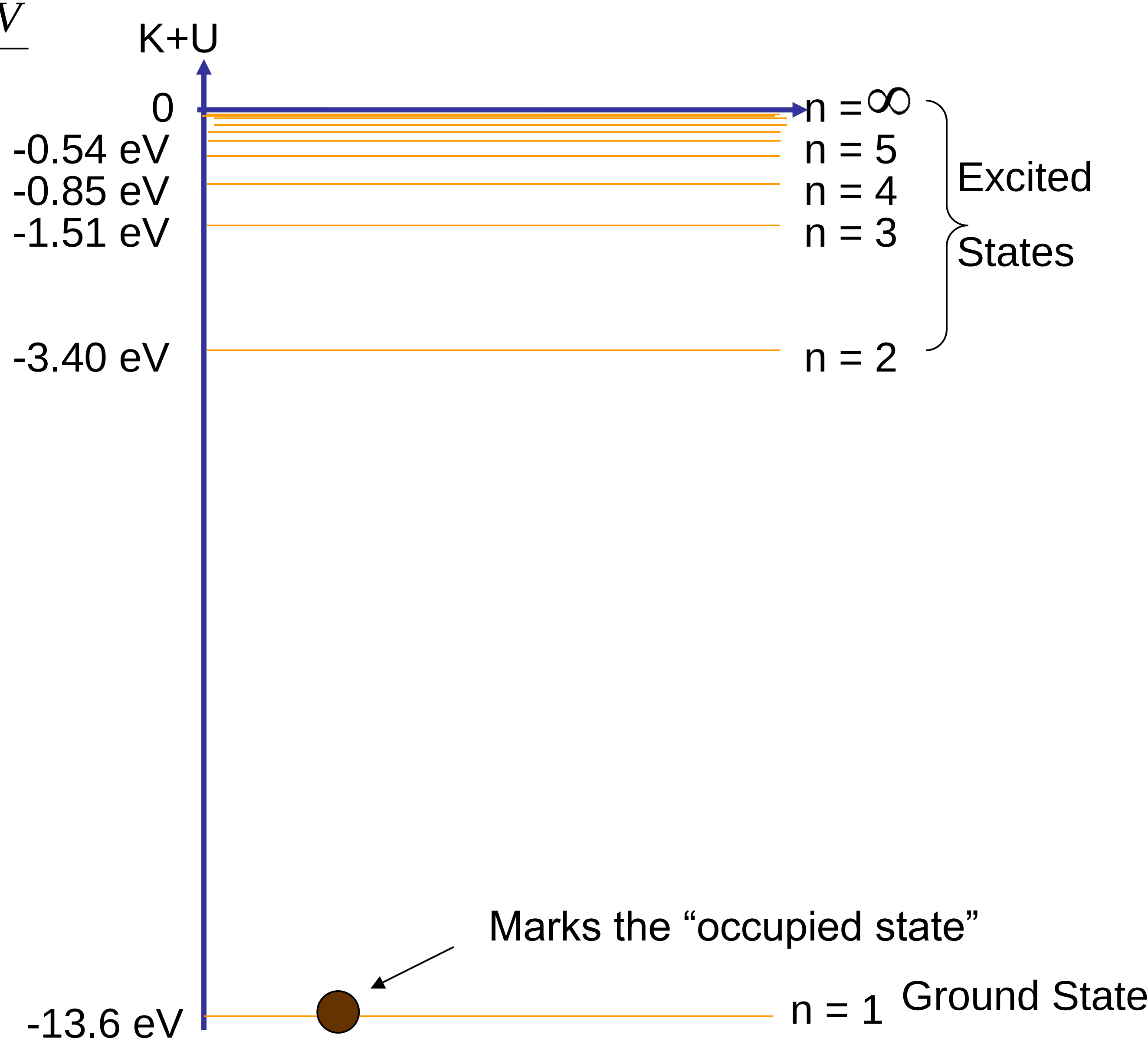
$$(K+U)_2 = -\frac{13.6eV}{2^2}$$

$$(K+U)_1 = -\frac{13.6eV}{1^2}$$



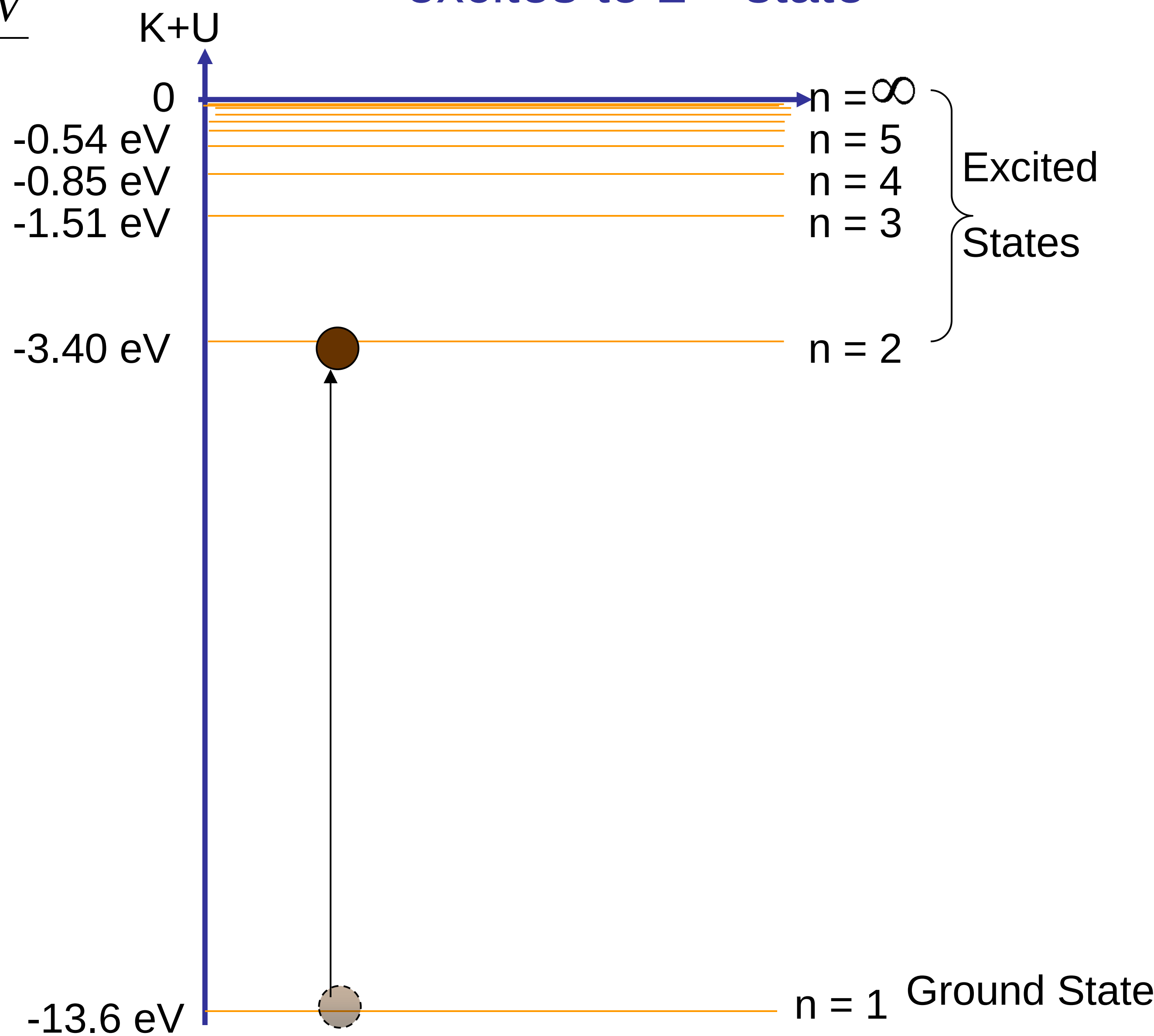
# Hydrogen Excitation: 1st in ground state

$$\left(K+U\right)_n = -\frac{13.6eV}{n^2}$$



# Hydrogen Excitation: 2nd Adsorbs energy from Collision excites to 2nd state

$$(K+U)_n = -\frac{13.6eV}{n^2}$$



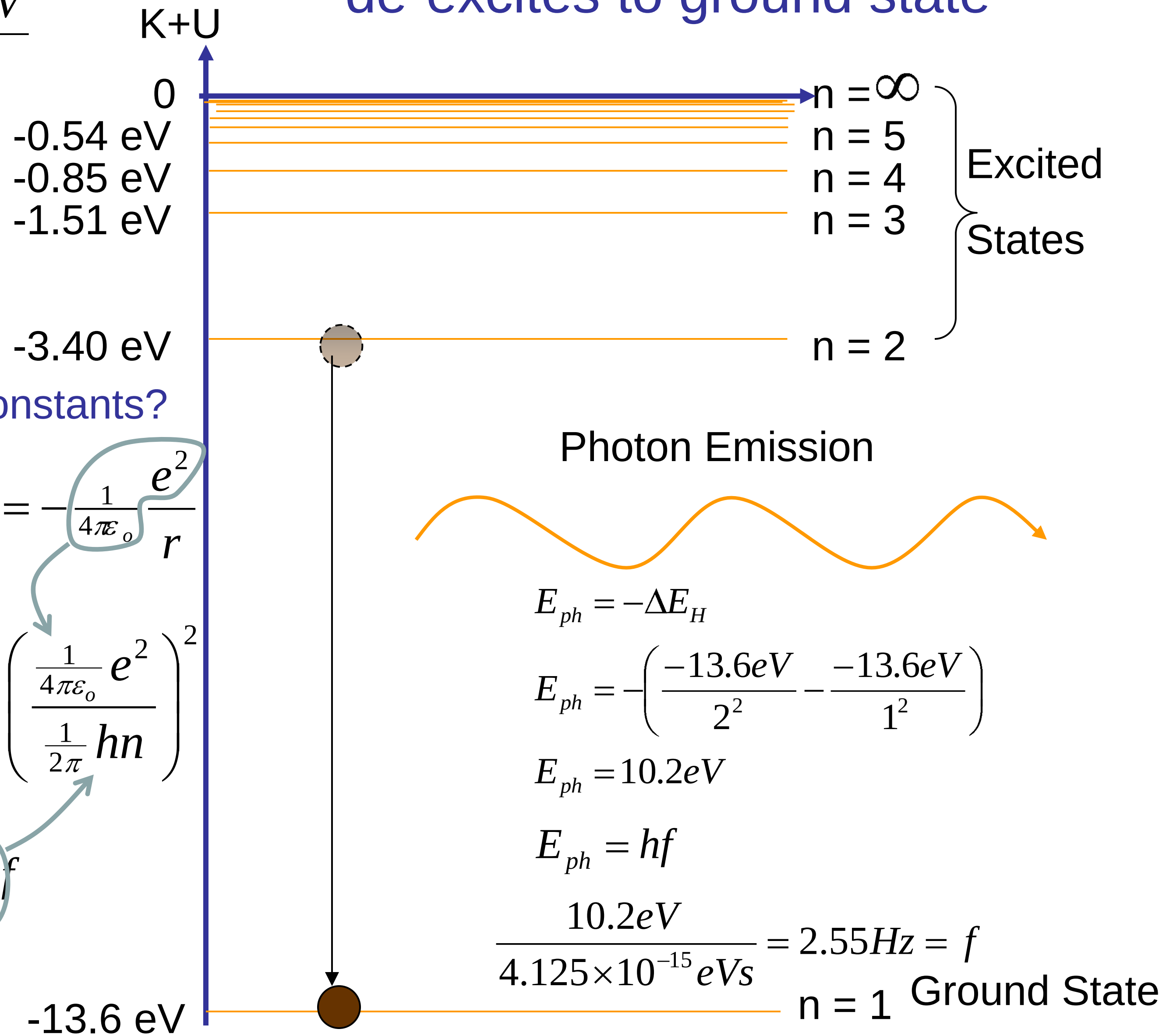
# Hydrogen Excitation: 3rd Looses Energy by photon emission,

de-excites to ground state

Why?

$$\left(K+U\right)_n = -\frac{13.6eV}{n^2}$$

Why?




Some combo of relevant constants?

$$K \approx \frac{1}{2} m_e v^2 \qquad U = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r}$$

$$K+U\Big|_n = -\frac{13.6eV}{n^2} = -\frac{m_e}{2}\left(\frac{\frac{1}{4\pi\varepsilon_o}e^2}{\frac{1}{2\pi}hn}\right)^2$$

$$E_{ph} = hf$$

Why?

$$E_{ph} = -\Delta E_H$$

$$E_{ph} = -\left(\frac{-13.6eV}{2^2} - \frac{-13.6eV}{1^2}\right)$$

$$E_{ph} = 10.2eV$$

$$E_{ph} = hf$$

$$\frac{10.2eV}{4.125\times 10^{-15} eVs} = 2.55Hz = f$$

# Rephrasing Classical Energy Expression for Orbiting Electron

Target expression: $$\left|K+U\right|_n = -\frac{m_e}{2}\left(\frac{\frac{1}{4\pi\varepsilon_o}e^2}{\frac{1}{2\pi}hn}\right)^2$$

$$K+U \approx \tfrac{1}{2}m_e v^2 - \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{r}$$

Specifically for Circular Motion:

$$\left|\frac{d\vec{p}}{dt}\right| = \left|\vec{F}_{net}\right|$$

$$m_e\frac{v^2}{r} \approx \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$$

so

$$m_e v^2 \approx \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{r}$$

$$K+U \approx \tfrac{1}{2}m_e v^2 - m_e v^2 = -\tfrac{m_e}{2}v^2$$

How to eliminate r:

Rephrase orbital kinetic in terms of L:

$$L = m_e v r \quad \text{so} \quad r = \frac{L}{m_e v}$$

Naturally,

$$v^2 \approx \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{m_e r} = \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{m_e\left(\frac{L}{m_e v}\right)} = \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{L}v$$

so

$$v \approx \tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{L}$$

$$K+U \approx -\tfrac{m_e}{2}\left(\tfrac{1}{4\pi\varepsilon_o}\frac{e^2}{L}\right)^2$$

$$K+U \approx -\tfrac{m_e}{2}\left(\frac{\frac{1}{4\pi\varepsilon_o}e^2}{L}\right)^2$$

Works if $L = \dfrac{hn}{2\pi}$ Why?

# Why $L = \frac{hn}{2\pi}$ : De Broglie's Contribution

From Einstein: $E = \sqrt{(pc)^2 + (mc^2)^2}$

For photon (being massless) $E = pc = hf$ From experiments

so $p = \frac{hf}{c}$

frequency – wavelength – wave-speed relation

$$f\lambda = c$$

so $p = \frac{h}{\lambda}$ De Broglie's big idea: what if this is true for particles too – some kind of wave associated with momentum

Then $L = pr$ (for circular motion) means $L = \frac{h}{\lambda} r$

Return to our K+U expression:

$$K + U \approx -\frac{m_e}{2}\left(\frac{\frac{1}{4\pi\varepsilon_o} e^2}{L}\right)^2 = -\frac{m_e}{2}\left(\frac{\frac{1}{4\pi\varepsilon_o} e^2}{h\left(\frac{r}{\lambda}\right)}\right)^2$$

Works if $\frac{r}{\lambda} = \frac{n}{2\pi} \Rightarrow \frac{2\pi r}{\lambda} = n$

Whatever these waves are, they must 'fit' the orbit

Why?

Today we understand the waves to relate to the probability

circular waves demo

# Bohr Radii

$L = m_e v r$ We'd found that $v \approx \frac{1}{4\pi\varepsilon_o}\frac{e^2}{L}$

so $L = m_e\left(\frac{1}{4\pi\varepsilon_o}\frac{e^2}{L}\right)r$

$$r = \frac{L^2}{m_e \frac{1}{4\pi\varepsilon_o} e^2}$$

where $L = \frac{hn}{2\pi} = \hbar n$

$$r = \left(\frac{\hbar^2}{m_e \frac{1}{4\pi\varepsilon_o} e^2}\right)n^2$$

$$\hbar \equiv \frac{h}{2\pi}$$

Along with only specific L values and K+U values, there are only specific radii

$$r = \left( \frac{\hbar^2}{m_e \frac{1}{4\pi\varepsilon_o} e^2} \right) n^2$$

$$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s} \qquad m_e = 9 \times 10^{-31}\ \mathrm{kg}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ \mathrm{N \cdot m^2 / C^2}$$

$$e = 1.6 \times 10^{-19}\ \mathrm{C}$$

| | |
|---|---|
| 1) r = ($8.5 \times 10^{-30}$ meter)$n^2$ | 4) r = ($5.3 \times 10^{-11}$ meter)$n^2$ |
| 2) r = ($5.0 \times 10^{+23}$ meter)$n^2$ | 5) r = ($1.2 \times 10^{-38}$ meter)$n^2$ |
| 3) r = ($4.8 \times 10^{-1}$ meter)$n^2$ | |

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