3	Fri. 9/20	**3.11 –.13** Conservation of Momentum & Multiple Particles	**RE3.c**
4	Mon. 9/23 Tues 9/24	**4.1-.5** Atomic nature of matter / springs	**RE 4.a** **EP 3, HW3:** Ch 3 Pr's 42, 46, 58, 65, 72 & CP

Equipment

- Clickers
- Momentum Principle for multi-Particle system ppt
- Look over WebAssign problems & print out
- Air track with two carts
- L 3 Binary w cm.py, L3 Binary wo cm.py
- Baton

Announcements

Last Time.

- We met the electric force and other fundamental forces

This Time

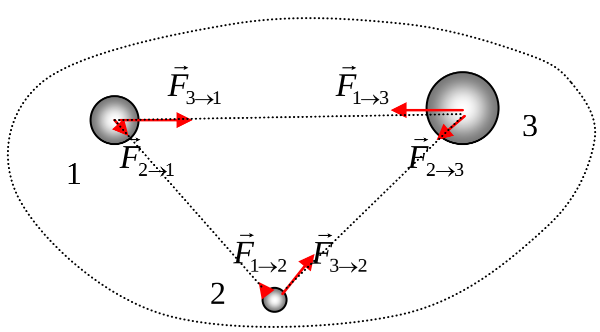
- We'll show that the momentum principle applies for conglomerate systems as well as "point particles."

First: a quick clicker from last time's material - reciprocity.

3.12 The momentum principle for multi-particle systems

- The momentum principle is an extremely powerful tool. It may be surprising that it even holds for whole conglomerates, such as dust clouds, not just individual particles. With the help of the principle of Reciprocity, this can be appreciated by simply considering three interacting particles subject to some external forces (it's easy to imagine generalizing it from there to N particles).
  - **Reciprocity:** Two sides to every story – "I'm not pushing you, *you're* pushing *me!"*

- **3.11 Conservation of Momentum: First – isolated 3-particle system.** Say we have 3, equally-spaced, gravitationally-interacting masses as shown. How should each of the forces compare? The relative masses dictate the relative strengths of the interactions.



The **Reciprocity** principle says that

$$\vec{F}_{3\to1} = -\vec{F}_{1\to3}$$
$$\vec{F}_{2\to1} = -\vec{F}_{1\to2}$$
$$\vec{F}_{3\to2} = -\vec{F}_{2\to3}$$

$$\frac{\Delta\vec{p}_1}{\Delta t} = \vec{F}_{net.1} = \vec{F}_{1\leftarrow2} + \vec{F}_{1\leftarrow3}$$

$$\frac{\Delta\vec{p}_2}{\Delta t} = \vec{F}_{net.2} = \vec{F}_{2\leftarrow1} + \vec{F}_{2\leftarrow3}$$

$$\frac{\Delta\vec{p}_3}{\Delta t} = \vec{F}_{net.3} = \vec{F}_{3\leftarrow1} + \vec{F}_{3\leftarrow2}$$

$$\frac{\Delta\vec{p}_1}{\Delta t} + \frac{\Delta\vec{p}_2}{\Delta t} + \frac{\Delta\vec{p}_3}{\Delta t} = (\vec{F}_{1\leftarrow3} + \vec{F}_{3\leftarrow1}) + (\vec{F}_{3\leftarrow2} + \vec{F}_{2\leftarrow3}) + (\vec{F}_{1\leftarrow2} + \vec{F}_{2\leftarrow1})$$

$$\frac{\Delta}{\Delta t}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) = (0) + (0) + (0)$$

$$\frac{\Delta\vec{p}_{total}}{\Delta t} = 0$$

Conservation of Momentum

- In the case that there *is no* net external force, that is, all interacting objects are part of the system, we have $\frac{d\vec{p}_{total}}{dt} = 0 \Rightarrow \Delta\vec{p}_{total} = 0$. This says that in that case, total momentum is conserved. It can be traded amongst the members of the system, but it never disappears. This is a quantifiable / measureable incarnation of the principle that I've espoused so many times: *motion is neither created nor destroyed, but transferred via interactions.*

Example: You and a friend each hold a lump of wet clay. Each lump has a mass of 20 grams. You each toss your lump of clay into the air, where the lumps collide and stick together. Just before the impact, the velocity of one lump was < 4, 4, -3 > m/s, and the velocity of the other lump was < -4, 0, -6 > m/s.
What was the the total momentum of the lumps just before the impact?
$\vec{p}_{total} = < 0, 0.0800, -0.180 >$ kg·m/s.
What is the momentum of the stuck-together lump just after the collision?
$\vec{p} = < 0, 0.0800, -0.180 >$ kg·m/s.
What is the velocity of the stuck-together lump just after the collision?
$\vec{v}_f = < 0, 2.00, -4.50 >$ m/s.

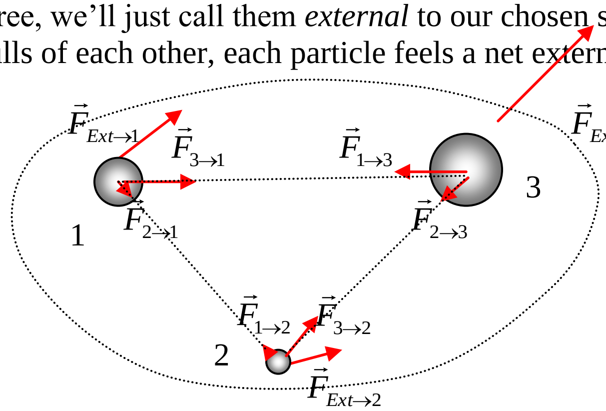
Demo / Example (like WebAssign)

A system consists of a 3 kg block moving with velocity ⟨ 11, 14, 0 ⟩ m/s and a 5 kg block moving with velocity ⟨ −4, 3, 0 ⟩ m/s.

(a) What is the momentum of this two-block system?
$\vec{p}_{total} = \langle 13, 57, 0 \rangle$ kg · m/s

(b) Next, due to interactions between the two blocks, each of their velocities change, but the two-block system is nearly isolated from the surroundings. Now what is the momentum of the two-block system?
$\vec{p}_{total} = \langle 13, 57, 0 \rangle$ kg · m/s

- **Momentum Principle for "open" system (experiencing external forces.)** To make it more interesting, let's imagine there exist other particles aside from our three, we'll just call them *external* to our chosen system. Then, in addition to the pulls of each other, each particle feels a net external force.



- The **Momentum** principle applied to each individual particles the principle yields

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{net.1} = \vec{F}_{ext.1} + \vec{F}_{2\to1} + \vec{F}_{3\to1}$$

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{net.2} = \vec{F}_{ext.2} + \vec{F}_{1\to2} + \vec{F}_{3\to2}$$

$$\frac{d\vec{p}_3}{dt} = \vec{F}_{net.3} = \vec{F}_{ext.3} + \vec{F}_{1\to3} + \vec{F}_{2\to3}$$

- $$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_3}{dt} = \vec{F}_{ext.1} + \vec{F}_{ext.2} + \vec{F}_{ext.3} + (\vec{F}_{1\to3} + \vec{F}_{3\to1}) + (\vec{F}_{3\to2} + \vec{F}_{2\to3}) + (\vec{F}_{1\to2} + \vec{F}_{2\to1})$$

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) = \vec{F}_{ext.1} + \vec{F}_{ext.2} + \vec{F}_{ext.3} + (0) + (0) + (0)$$

$$\boxed{\frac{d\vec{p}_{total}}{dt} = \vec{F}_{ext.net}}$$

  - The terms in brackets went to 0 because of the Reciprocity principle – the force of 3 on 2 is equal and opposite to the force of 2 on 3, so the two cancel. The end result is that the change in the *total* momentum follows from the net *external* force.
  - This has the exact same form as the Momentum principle for just one particle. Thus we can apply it to individual particles or to collections of particles such as dust clouds, planets, or you and me.
  - Thus we will be justified in applying the momentum principle to our planet and our star (in spite of their being large conglomerates of particles.)

    - **The momentum principle for multiparticle systems**

$$\vec{F}_{net.ext} = \frac{d\vec{p}_{total}}{dt}$$

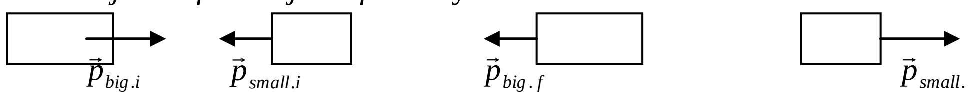
    - **Conservation of momentum:** if no net external force, then

$$\vec{F}_{net.ext} = \frac{d\vec{p}_{total}}{dt} = 0 \Rightarrow d\vec{p}_{total} = 0 \Rightarrow \Delta\vec{p}_{total} = 0$$

Conservation of Momentum & Collisions

- The first tool we'll use is momentum. Recall that $\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$, so, to the extent that there is no net external force, or at least that the interval we're considering is quite brief, $\Delta\vec{p}_{system} = \int_{t_i}^{t_f} \vec{F}_{net.ext} dt \approx 0$. In such a case, we say that momentum is (approximately) conserved during the brief collision.
- The conservation of momentum is a very powerful tool.
- **System = carts.**

  Let's take our system to be the two carts on the air track, and our time interval to be *just before* to *just after* they collide.



  Then

$$\Delta\vec{p}_{system} = \vec{F}_{system\leftarrow net.ext}\Delta t$$

$$\Delta\vec{p}_{big} + \Delta\vec{p}_{small} = (\vec{F}_{big\leftarrow air} + \vec{F}_{big\leftarrow table.friction} + \vec{F}_{big\leftarrow table.normal} + \vec{F}_{big\leftarrow Earth.gravl} + \vec{F}_{small\leftarrow air} + \ldots)\Delta t$$

$$\Delta\vec{p}_{big} + \Delta\vec{p}_{small} \approx 0$$

if Δt is small enough.

  - **Note:** it's true that there is friction, and if you watch a little while, it will have a considerable affect, but *just for the time during the collision,* the affect is quite small.
  - To invoke this relationship, you don't need to know the exact nature of the collision, just that there are no significant *external* interactions, so the total momentum in the system can't change.
  - There are three ways of phrasing this relationship
    - Momentum is conserved, there is not change in momentum:
      - $\Delta\vec{p}_{system} = 0$
        $\Delta\vec{p}_{big} + \Delta\vec{p}_{small} \approx 0$
    - Momentum is conserved, the total amount we had initially equals the total amount we have Finally:
      - $\vec{p}_{sys.i} = \vec{p}_{sys.f}$
        $\vec{p}_{big.i} + \vec{p}_{small.i} = \vec{p}_{big.f} + \vec{p}_{small.f}$
    - Momentum is exchanged, what one member of the system gives up, the other gains:
      - $\Delta\vec{p}_{big} = -\Delta\vec{p}_{small}$
        $(\vec{p}_{big.f} - \vec{p}_{big.i}) = -(\vec{p}_{small.f} - \vec{p}_{small.i})$

  However you phrase it, conservation of momentum gives us *one* of the *two* required equations.

  - **Demo: Maximally (Perfectly) Inelastic.** (collide carts and stick) In the case that the two carts hit and stick together, you also have the equation $\vec{v}_{big.f} = \vec{v}_{small.f}$.

- The carts are clearly going nowhere near the speed of light, so we can approximate the momenta as $\vec{p} = m\vec{v}$.
- We can now predict the final velocities based on the initial conditions and laws of physics:

$$m_{big}\vec{v}_{big.i} + m_{small}\vec{v}_{small.i} = (m_{big} + m_{small})\vec{v}_f$$

- $$\vec{v}_f = \frac{m_{big}\vec{v}_{big.i} + m_{small}\vec{v}_{small.i}}{(m_{big} + m_{small})}$$

Qualitatevly, what does this say?

Q: In the limit that $m_{big} >> m_{small}$, the final velocity is approximately what?

A: $V_{big.i}$

Q: In the limit that $m_{big} = m_{small}$, the final velocity is what?

A: The average of the two velocities: $(\mathbf{V_{big}} + \mathbf{V_{small}})/2$.

Questions about Conservation of Momentum and Choice of System
2nd through last question on P231-GroupProblem-091208.ppt

- **Group Problem.** The following diagrams show hypothetical results for collisions between two identical balls floating in space. The white ball was initially moving to the right along the dotted line before it hit the gray ball, which was initially at rest. The collision is not necessarily head-on. The arrows depict the balls' final velocities. Which outcomes are physically possible? Explain.

- **(2.X.13)** - You hang from a tree branch, then let go and fall. As you fall, the *y* component of your momentum, which was originally zero, becomes large and negative.
  - (a) Choose yourself as the system. Why does the momentum of the system change?
  - (b) Choose yourself and the Earth as the system. Does the total momentum of this system change? Why or why not?

- **2.RQ.21** - A bullet is traveling horizontally when it embeds itself in a wooden block that is on a slippery surface. You want to find the speed of the block just afterwards.
  - (a) What should you choose as the system? (Why?)
    - the bullet
    - the block
    - the bullet and the block
- (b) Which of the following statements is true?
  - After the collision, the speed of the block with the bullet in it is the same as the speed of the bullet before the collision.
  - The momentum of the block with the bullet stuck in it is the same as the momentum of the bullet before the collision.
  - The momentum of the block with the bullet stuck in it is less than the momentum of the bullet before the collision.
- Suppose you have two bullets with equal masses. One is made of metal and the other is made of rubber. If the two bullets can be shot at the same speed, which of the following gives a wooden block a higher speed after the collision?

- (a) The metal bullet gets embedded in the block.
- (b) The rubber bullet bounces off of the block.

- A 45-kg skateboarder on a 3-kg board is holding a 5-kg weight. Beginning from rest, she throws the weight horizontally at 7 m/s *relative to her*. What is her velocity (magnitude and direction) afterwards? Assume that the board rolls without friction.

Center of Mass in Multiparticle Systems

- Consider a system of many particles, perhaps it is a dust cloud in interstellar space and each speck of dust is one of our "particles." On the one hand, if we focus in, we see that each speck is naturally at a different location, has a different mass, and is moving with a different velocity. Yet, if we zoom out, we see a single cloud that behaves somewhat cohesively. If we watch the cloud for a while, as it moves through space, it makes sense to speak of the whole as having some velocity and going from some position to another – in short, we can think of it as a "particle" of its own. For that matter, if we zoom way in, each of our dust "particles" are themselves made of several, much tinier particles. How do we reasonably do this – treat a composite of several particles, each more-or-less doing their own thing, as a single object and yet properly account for the internal workings? That's what this chapter is about.

7.1 The motion of a multparticle system

The chapter opens with two every-day yet rich examples: pulling a block by a string and jumping. In the first case, given what we know about how inter-atomic bonds work, we know that the only way you your "pull" can be transmitted throughout the object is if inter-atomic bonds stretch – so not all the atoms are moving exactly like each other. In the second case, the flexing of the object is quite obvious, and somehow *that* is responsible for you flying off the ground.

7.2 The momentum principle for multiparticle systems.

- We've developed the momentum principle for multiparticle systems. It quite resembled that for the single particle: $\frac{d\vec{p}_{total}}{dt} = \vec{F}_{net,ext}$

  where $\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 ... \approx m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + ...$. Looking at the right hand side, if you recall, all of the internal forces, between the particles of the system, cancel thanks to the reciprocity principle (Newton's 3rd Law: $\vec{F}_{1\to2} = -\vec{F}_{2\to1}$), which leaves only the external forces. This will be our starting point. So, by analogy to the momentum principle for a single particle, $\frac{d\vec{p}_1}{dt} = \vec{F}_{net\to1}$, $\vec{p}_{total}$ plays the role of the momentum of the single, composite system. So then, what's a representative velocity for the whole system? A velocity such that $\vec{p}_{system} \approx m_{system}\vec{v}_{system}$. It seems a no-brainer that $m_{system} = M_{total} = m_1 + m_2 + m_3 + m_4 + ...$ so then

$$\vec{v}_{system} \approx \frac{\vec{p}_{system}}{m_{system}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + \dots}{m_1 + m_2 + m_3 + m_4 + \dots} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + \dots}{M_{total}}$$

- **Weighted Average**: Mathematically, this is the velocity averaged over mass.

Example: A system consists of a 4 kg block moving with velocity ⟨ −4, 6, 0 ⟩ m/s and a 8 kg block moving with velocity ⟨ 3, 7, 0 ⟩ m/s. Calculate the velocity of the center of mass of the two-block system, given that the momentum of the system is $M_{total}\vec{v}_{CM}$.
$\vec{v}_{CM} = \langle 0.667,\ 6.67, 0 \rangle$ m/s

- **Demo: Lab_3 binary wo cm.py** You may remember writing code to simulate two stars gravitationally interacting. They went looping through space weaving around each other. Yet, if we call the two stars together our "system", since there is not net external force on the system, the average velocity of its members must be unchanging – just a constant. What representative point is moving like that?

7.2.1 Center of Mass

- We call the representative point in a complex system the "Center of Mass." You'll see why. So, *where* is it? It's the point that satisfies $\vec{v} = \frac{d\vec{r}}{dt}$.

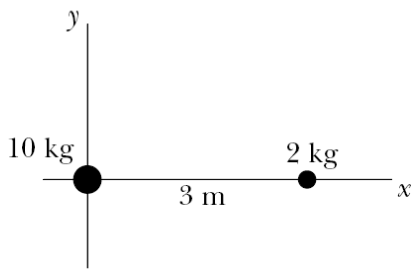
$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4 + ...}{m_1 + m_2 + m_3 + m_4 + ...} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4 + ...}{M_{total}}$$

- for if you take its derivative, you get back our relation for the system's velocity.
- **Demo: Lab_3 binary w cm.py**
  - Note that the center of mass is just a mathematically defined point – it's not a fixed part of either object.

- **Q:** In this simulation, the blue star's mass is 2e30kg and it starts out at the origin; the yellow star's mass is 1e30kg and it starts out at $\vec{r}_y = <1.5e11, 0, 0> m$. Initially, where' the center of mass?

Example: A 17 kg ball is located at ⟨ 6, 0, 0 ⟩ m, and a 3 kg ball is located at ⟨ 13, 3, 0 ⟩ m. Find the center of mass of the two-ball system. You should find that the center of mass is close to the heavier ball.
$\vec{r}_{CM} = \langle 7.05,\ 0.45, 0 \rangle$ m

- **CW**
- Here are two masses located on the *x* axis:



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- What is the x component of the location of the center of mass of the system consisting of both masses?
- $\vec{r}_{cm} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \dfrac{(10kg)(0m)\hat{x} + (2kg)(3m)\hat{x}}{10kg + 2kg} = 0.5m\hat{x}$

- **Center of mass of large objects**
  - Now, in reality, two stars are themselves collections of 'particles' – atoms, electrons, etc. Each has its own center of mass, quite obviously at the geometric center. Let's call the two stars "Yellow" and "Blue", then noting which particles in our system are a part of which star, we have

$$\vec{r}_{CM} = \frac{M_b\vec{r}_b + M_y\vec{r}_y}{M_b + M_y}$$

$$= \frac{(m_{1b}\vec{r}_{1b} + m_{2b}\vec{r}_{2b} + m_{3b}\vec{r}_{3b} + \dots) + (m_{1r}\vec{r}_{1y} + m_{2y}\vec{r}_{2y} + m_{3y}\vec{r}_{3y} + \dots)}{(m_{1b} + m_{2b} + m_{3b} + \dots) + (m_{1y} + m_{2y} + m_{3y} + \dots)}$$

  - So, the relation scales: you can sum over fundamental particles, or over more convenient "objects."

8.X.2. Now for a composite system. Say we make a T of two meter sticks. Taking the origin to be at the base of the T, where's the system's center of mass?

First, where's the center of mass of the vertical stick?

Where's the center of mass of the horizontal stick?

Now apply $\vec{r}_{cm} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$ where both objects have the same mass.