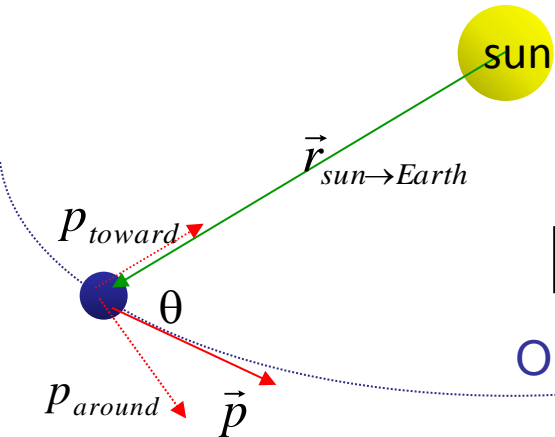


Fri.	11.1 Angular Momentum Quiz 10	RE 11.a; HW10: 13*, 21, 30, 35, "39"
Mon. Tues.	11.2-.3, (.12) Rotational + Translational	RE 11.b EP10
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Mon.	Review for Final (1-11)	HW11: Ch 11 Pr's 39, 57, 64, 74, 78 & Practice Exam

Introducing Angular Momentum

The measure of motion *about* a point

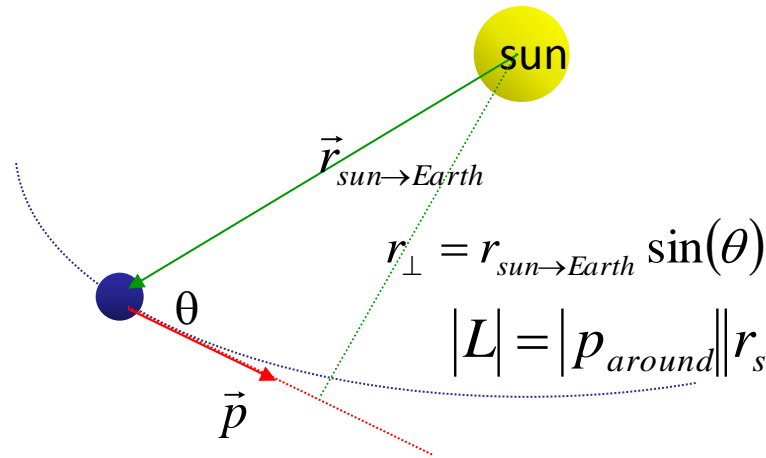
Magnitude



$$|L| = |p_{around}| r_{sun \rightarrow Earth} = |p| r_{sun \rightarrow Earth} \sin(\theta)$$

Only 'around' component of momentum counts

$$p_{around} = p \cos(90^\circ - \theta) = p \sin(\theta)$$



$$r_{\perp} = r_{sun \rightarrow Earth} \sin(\theta)$$

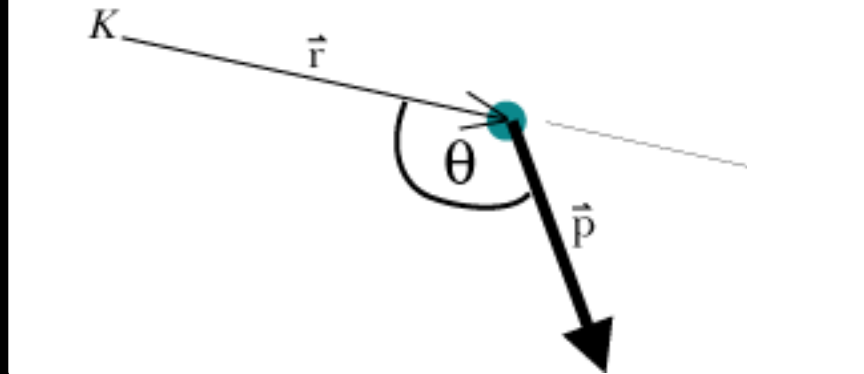
$$|L| = |p_{around}| r_{sun \rightarrow Earth} = |p| r_{sun \rightarrow Earth} \sin(\theta) = |p| r_{\perp}$$

Using Angular Momentum

The measure of motion *about* a point

Magnitude

$$|L| = |p_{\text{around}}||r| = |p||r_{\perp}| = |p||r|\sin(\theta)$$



What is the magnitude of the angular momentum about location K , for the object shown below? The magnitude of the object's momentum $|\vec{p}| = 7 \text{ kg}\cdot\text{m/s}$, the distance $|\vec{r}| = 0.6 \text{ m}$, and the angle $\theta = 150^\circ$

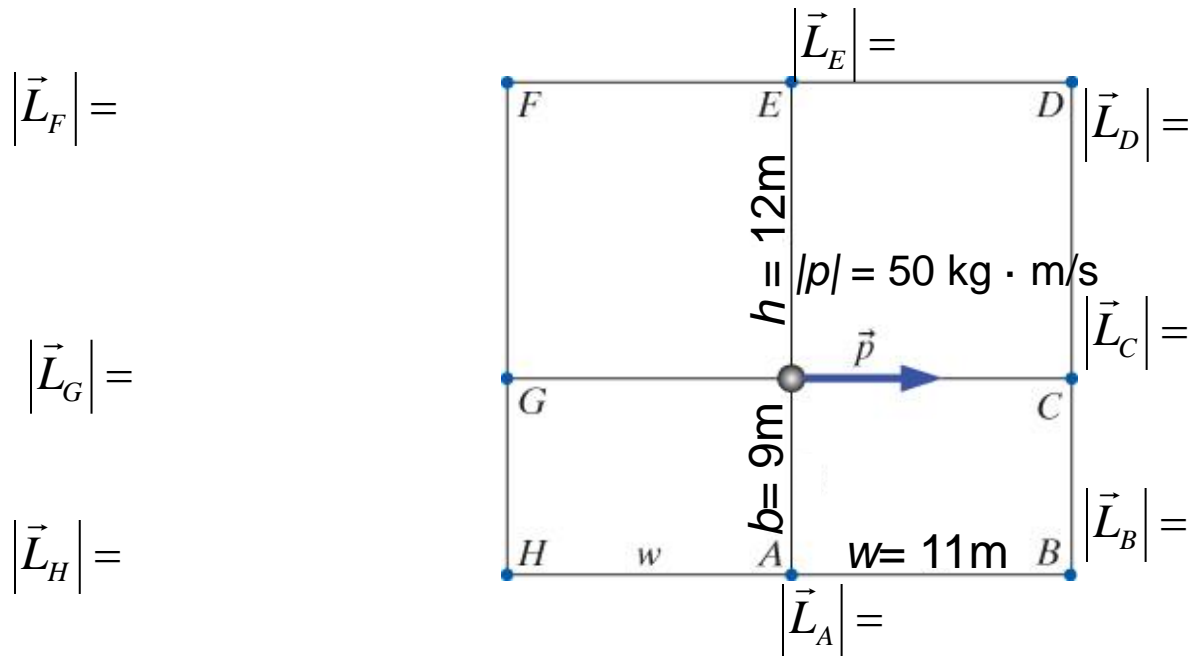
Using Angular Momentum

The measure of motion *about* a point

Magnitude

$$|L| = |p_{around}||r| = |p||r_{\perp}| = |p||r|\sin(\theta)$$

Determine the magnitude of the translational angular momentum of the particle at location O relative to each point: A , B , C , D , E , F , G , and H .



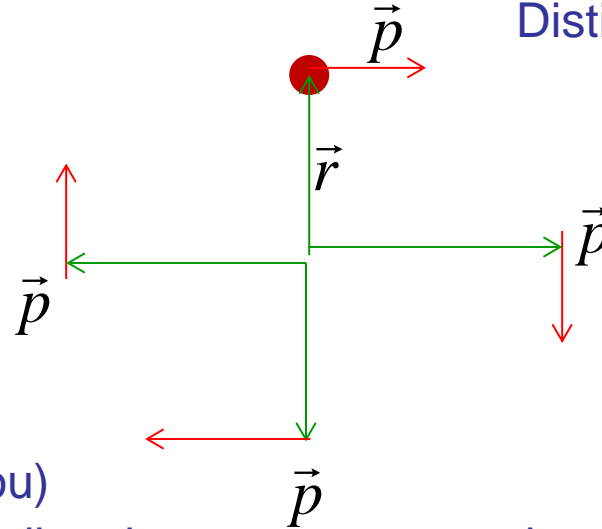
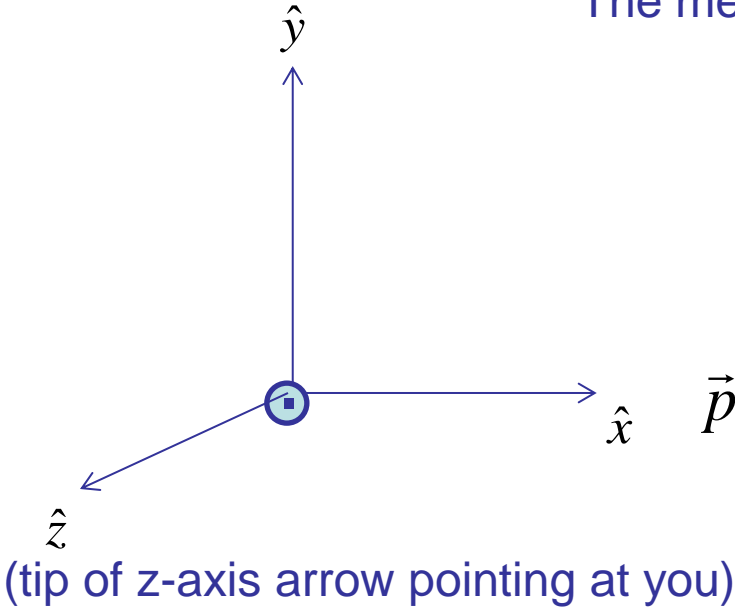
Using Angular Momentum

The measure of motion *about* a point

Direction

Distinguish with Right Hand Rule

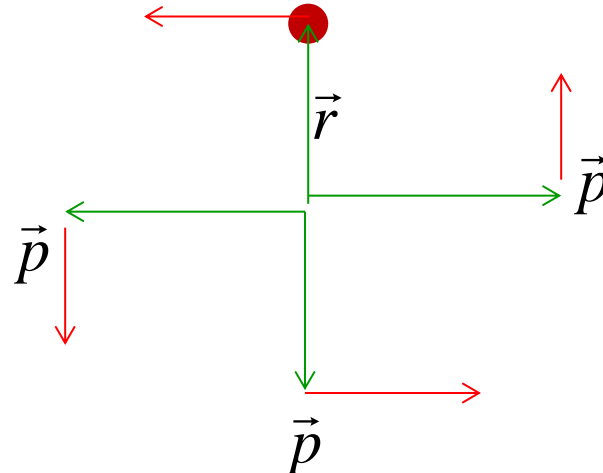
Orient Right hand so fingers curl with motion, then thumb points in conventional direction of angular momentum



(tip of z-axis arrow pointing at you)

The one direction momentum and position vectors *never* point is z – Axis of rotation

But that's also true for \vec{p}



Using Angular Momentum

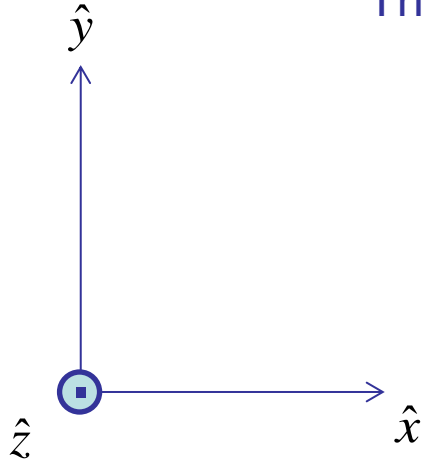
The measure of motion *about* a point

Direction

Example

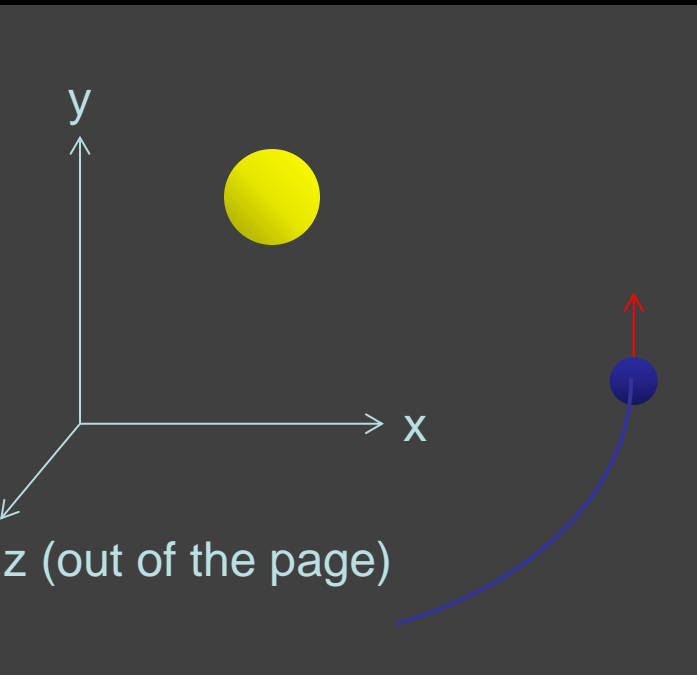
Distinguish with Right Hand Rule

Orient Right hand so fingers curl with motion, then thumb points in conventional direction of angular momentum



(tip of z-axis arrow pointing at you)

A comet orbits the Sun, in the xy plane. Its momentum is shown by the red arrow. What is the direction of the comet's **angular momentum** about the Sun?



- 1) $+x$
- 2) $-x$
- 3) $+y$
- 4) $-y$
- 5) $+z$
- 6) $-z$
- 7) toward the sun
- 8) away from the sun

Using Angular Momentum

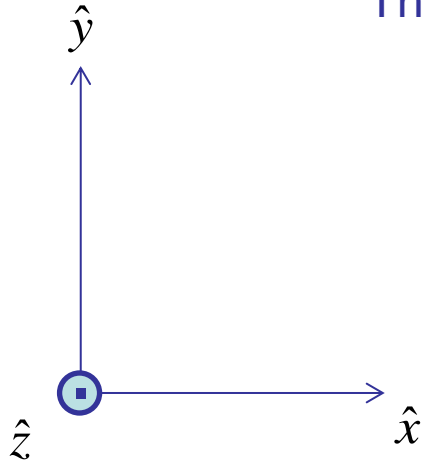
The measure of motion *about* a point

Direction

Example

Distinguish with Right Hand Rule

Orient Right hand so fingers curl with motion, then thumb points in conventional direction of angular momentum



(tip of z-axis arrow pointing at you)

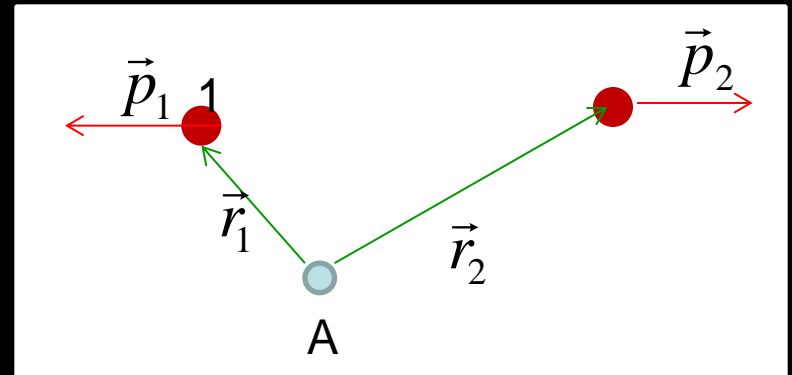
What are the directions of Angular Momentum for particle 1 about point A and particle 2 about point A

a) $\hat{L}_1 = +\hat{z}$ $\hat{L}_2 = +\hat{z}$

b) $\hat{L}_1 = -\hat{z}$ $\hat{L}_2 = +\hat{z}$

c) $\hat{L}_1 = +\hat{z}$ $\hat{L}_2 = -\hat{z}$

d) $\hat{L}_1 = -\hat{z}$ $\hat{L}_2 = -\hat{z}$



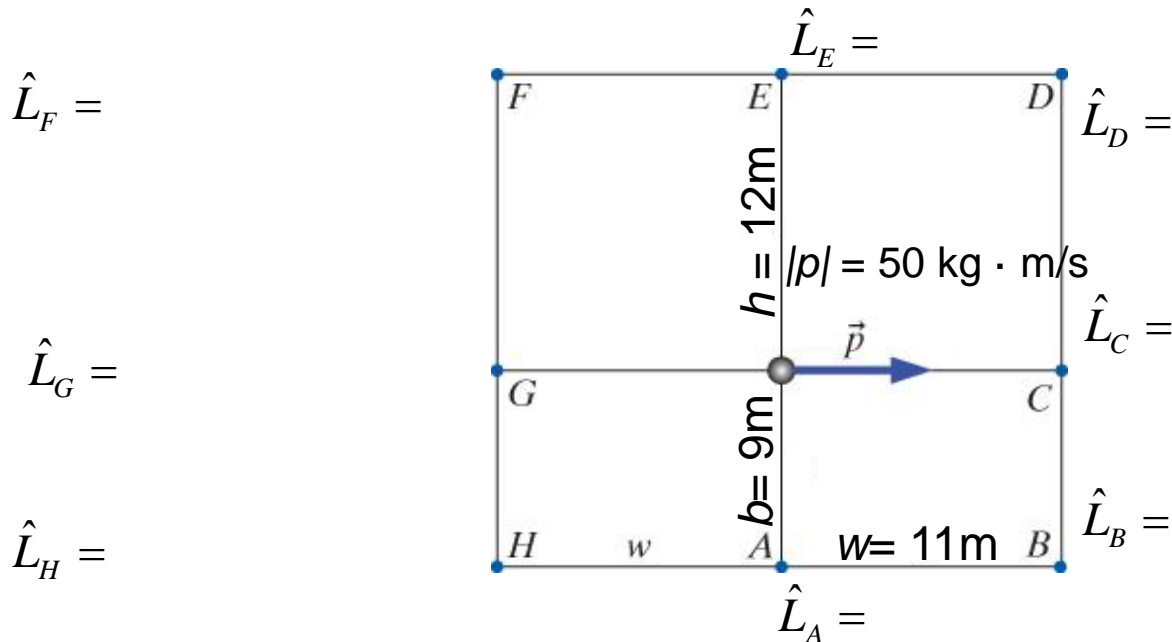
Using Angular Momentum

The measure of motion *about* a point

Direction

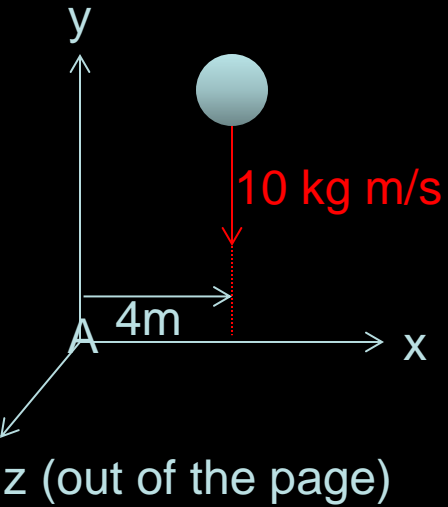
Distinguish with Right Hand Rule

Determine the direction of the translational angular momentum of the particle at location O relative to each point: A , B , C , D , E , F , G , and H .



A ball falls straight down in the xy plane. Its momentum is shown by the red arrow.

What is the direction of the ball's **angular momentum** about location A?



- 1) $+x$
- 2) $-x$
- 3) $+y$
- 4) $-y$
- 5) $+z$
- 6) $-z$
- 7) zero magnitude

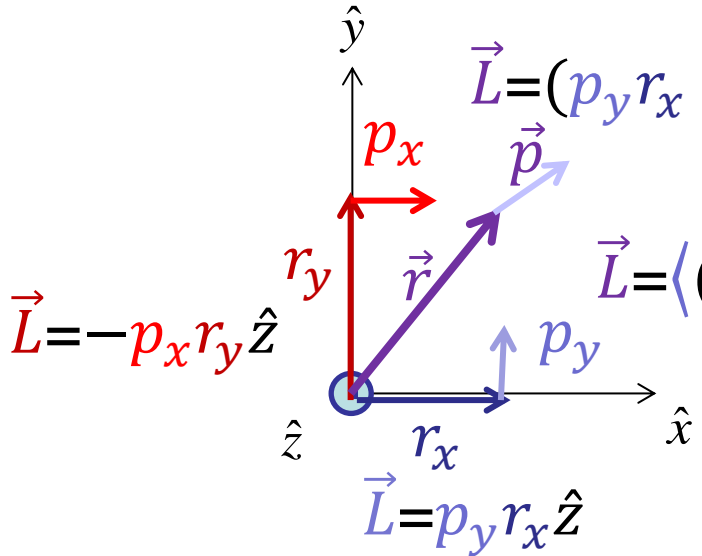
Given these values, what is the magnitude of the ball's angular momentum about A?

- 1) $10\text{ kg m}^2/\text{s}$
- 3) $40\text{ kg m}^2/\text{s}$
- 5) 0

Using Angular Momentum

The measure of motion *about* a point

Magnitude *and* Direction



Most General Expression

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \hat{z} \rangle$$

$$\vec{L} = \vec{r} \times \vec{p}$$

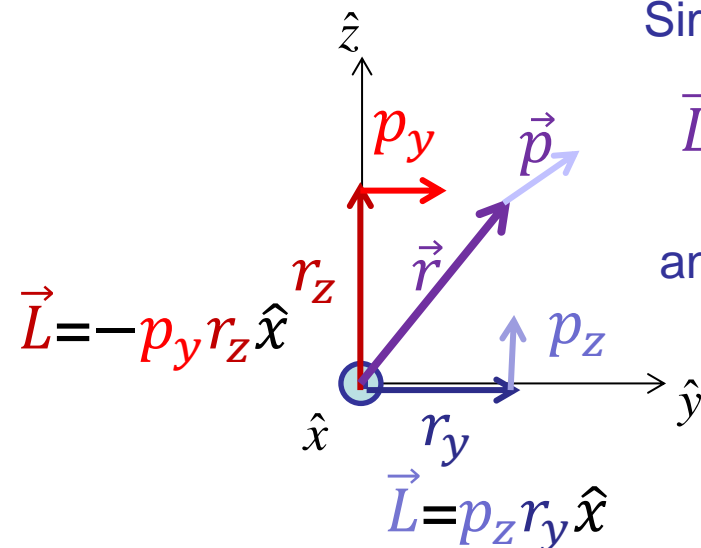
Cross Product

Similarly for position and momentum in the y-z

$$\vec{L} = (p_z r_y - p_y r_z) \hat{x}$$

and for position and momentum in the x-z

$$\vec{L} = (p_x r_z - p_z r_x) \hat{y}$$



Using Angular Momentum

The measure of motion *about* a point

Magnitude *and* Direction

$$\vec{L} = \vec{r} \times \vec{p} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

Example: say you have a mass that, at some instant, has linear momentum $\vec{p} = \langle 4, 2, 0 \rangle \text{ kg} \cdot \text{m/s}$ and is $\vec{r}_A = \langle 5, 3, 0 \rangle \text{ m}$ from some point A. What is its angular momentum about this point?

$$\vec{L} = \vec{r} \times \vec{p} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

What is the direction of

$\langle 0, 0, 3 \rangle$ x $\langle 0, 4, 0 \rangle$?

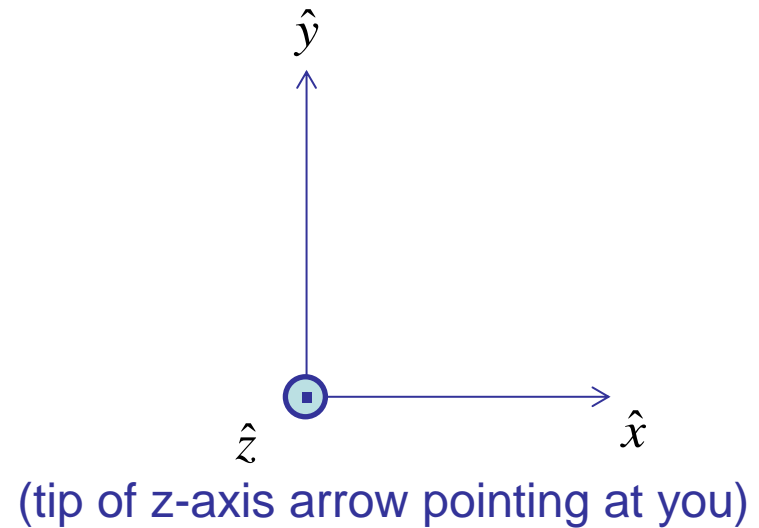
What is the direction of

$\langle 0, 4, 0 \rangle$ x $\langle 0, 0, 3 \rangle$?

What is the direction of

$\langle 0, 0, 6 \rangle$ x $\langle 0, 0, -3 \rangle$?

- 1) +x
- 2) -x
- 3) +y
- 4) -y
- 5) +z
- 6) -z
- 7) zero magnitude



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