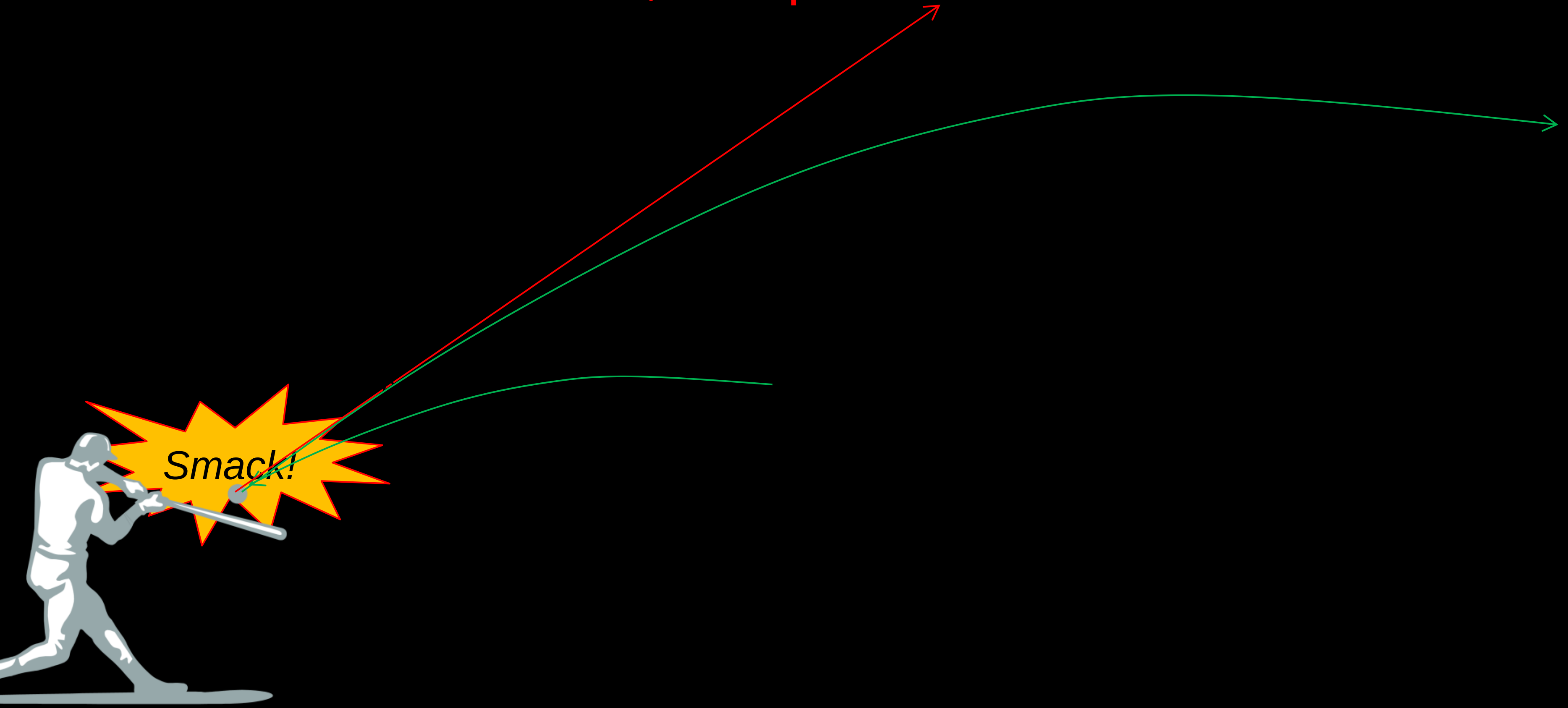
i.	10.6-.8 Scattering	RE 10.b
on., ab ed., i.,	10.9-.10 Collision Complications L10 Collisions 1 10.5, .11 Different Reference Frames 11.1 Translational Angular Momentum Quiz 10	RE 10.c EP9 RE 11.a; HW10: Pr's 13*, 21, 30, "39

Collisions

Short, Sharp Shocks



Which of the following is a property of *all* collisions - both “elastic” and “inelastic” collisions?

(1) The internal energy of the system after the collision is different from what it was before the collision.
(2) The total momentum of the system doesn’t change.
(3) The total kinetic energy of the system doesn’t change.

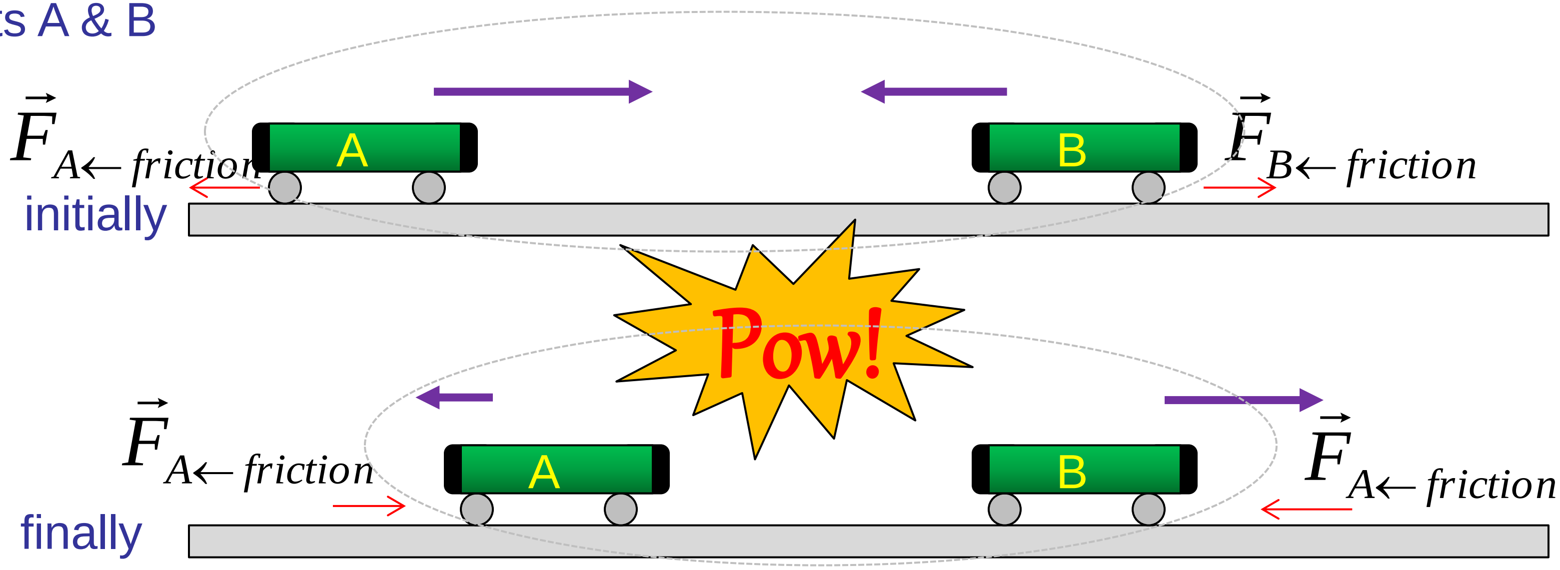
Which of the following is a property of all “elastic” collisions?

(1) The colliding objects interact through springs.
(2) The kinetic energy of one of the objects doesn’t change.
(3) The total kinetic energy is constant at all times -- before, during, and after the collision.
(4) The total kinetic energy after the collision is equal to the total kinetic energy before the collision.
(5) The elastic spring energy after the collision is greater than the elastic spring energy before the collision.

1-D Collision

Example

System = carts A & B



Often know initial motion, want to predict final motion

Two unknowns: $\vec{v}_{A.f}$ and $\vec{v}_{B.f}$

Generally need *two* equations to solve for them

True for *all* collisions

$$\Delta\vec{p}_{A\&B} = \Delta\vec{p}_A + \Delta\vec{p}_B \approx 0$$

$$\Delta E_{A\&B} = \underbrace{\Delta E_A}_{\Delta K_A + \Delta E_{\text{int}.A}} + \underbrace{\Delta E_B}_{\Delta K_B + \Delta E_{\text{int}.B}} + \Delta U_{A\&B} \approx 0$$

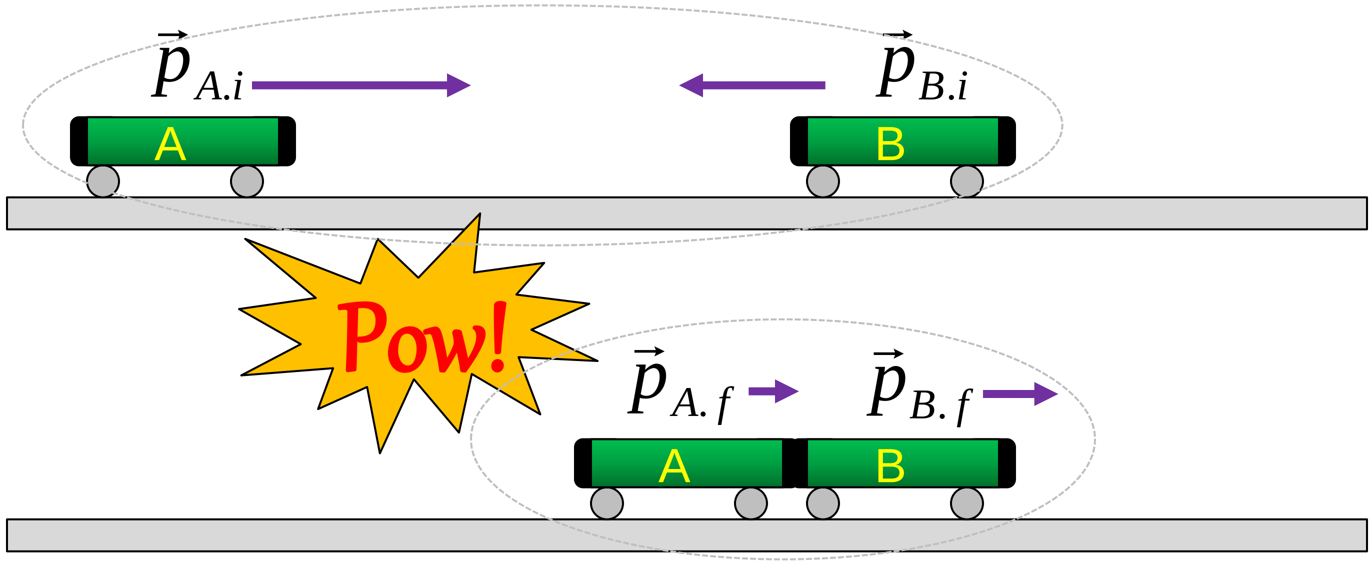
1-D Collision

Special Case: “Maximally Inelastic” – hit & stick

System = carts A & B

initially

finally



$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0 \qquad (v's << c)$$

$$\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right) \approx 0$$

$$\vec{v}_{Af} = \vec{v}_{Bf} \equiv \vec{v}_f$$

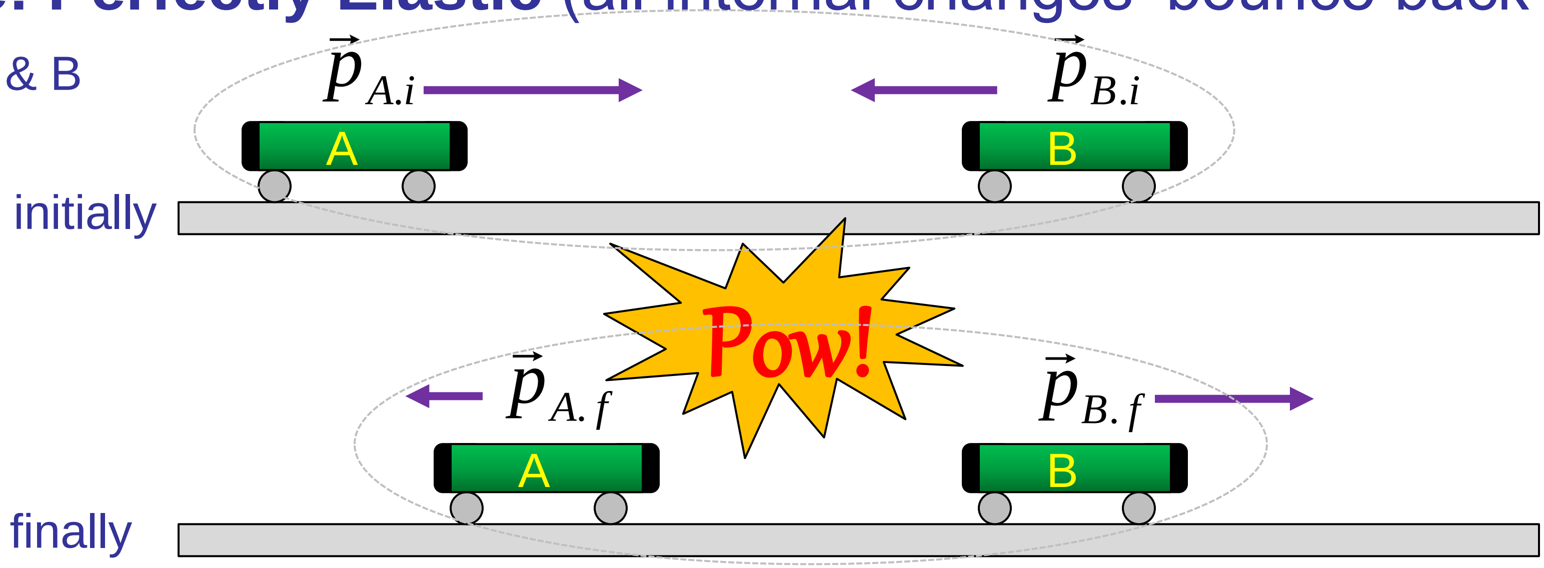
$$\left(m_A\vec{v}_f - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_f - m_B\vec{v}_{B.i}\right) \approx 0$$

$$\vec{v}_f \approx \frac{m_A\vec{v}_{A.i} + m_B\vec{v}_{B.i}}{m_A + m_B}$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

System = carts A & B



$$\Delta E_{A\&B} = \Delta K_A + \cancel{\Delta E_{A.\text{int}}} + \Delta K_B + \cancel{\Delta E_{B.\text{int}}} + \cancel{\Delta U_{A\&B}}$$

$$\left(\tfrac{1}{2} m_A v_{A.f}^2 - \tfrac{1}{2} m_A v_{A.i}^2\right) + \left(\tfrac{1}{2} m_B v_{B.f}^2 - \tfrac{1}{2} m_B v_{B.i}^2\right) \approx 0 \quad (v's << c)$$

Equation 1

$$\left(\frac{p_{A.f}^2}{2m_A} - \frac{p_{A.i}^2}{2m_A}\right) + \left(\frac{p_{B.f}^2}{2m_B} - \frac{p_{B.i}^2}{2m_B}\right) \approx 0$$

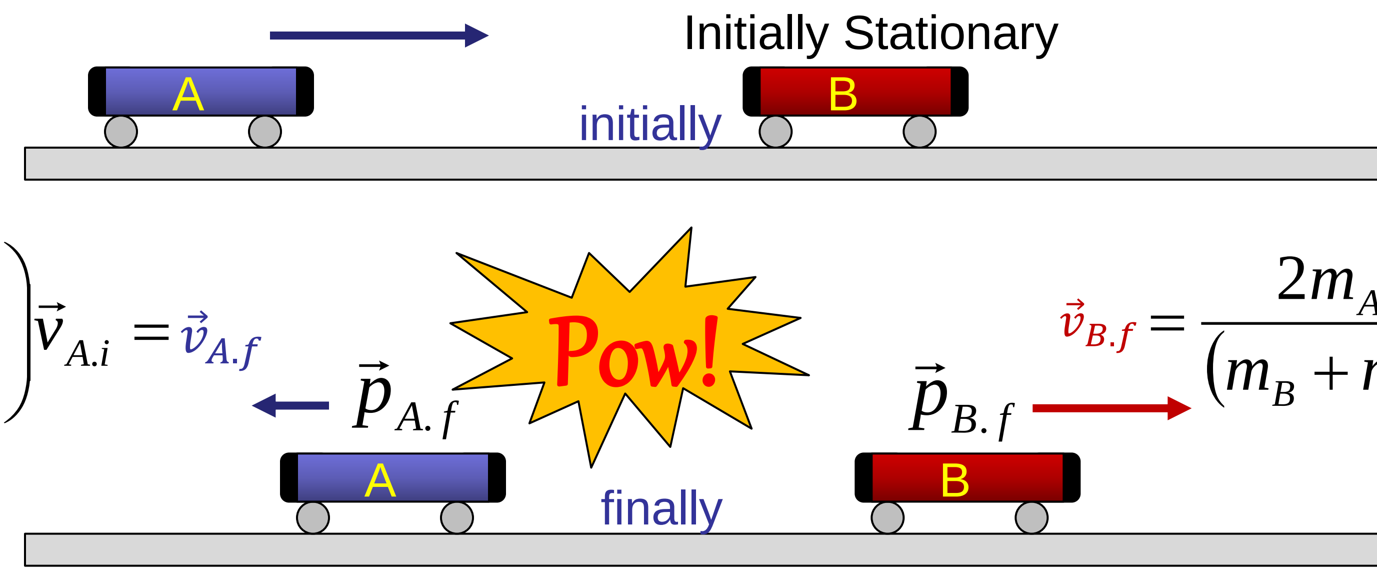
Equation 2

$$\vec{p}_{A.f} + \vec{p}_{B.f} = \vec{p}_{A.i} + \vec{p}_{B.i}$$

1-D Collision

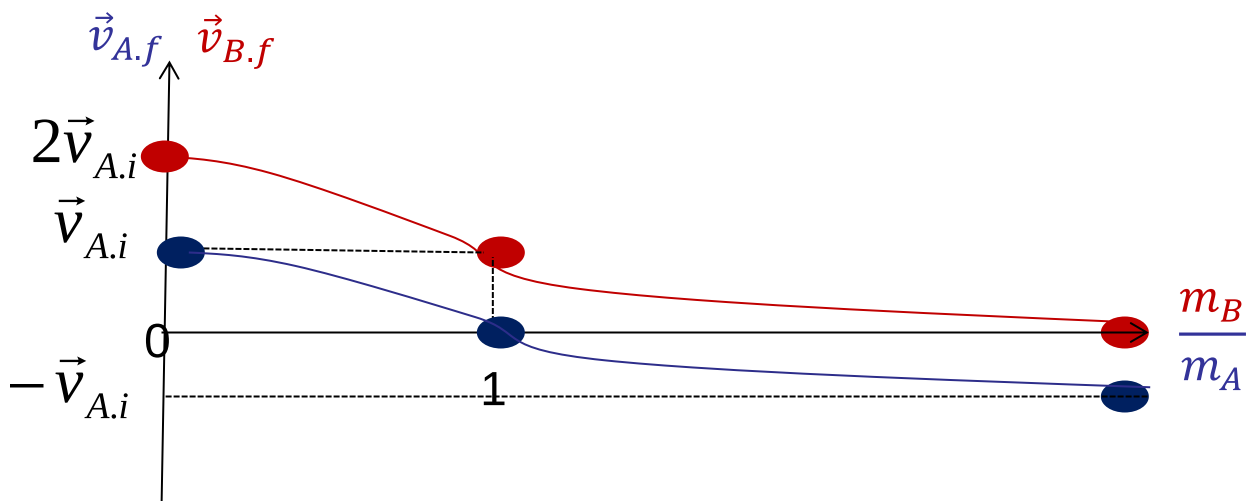
Special Case: Perfectly Elastic (all internal changes 'bounce back')

Extra special case: Say B is initially stationary



$$\left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{v}_{A.i} = \vec{v}_{A.f}$$

$$\vec{v}_{B.f} = \frac{2m_A}{(m_B + m_A)}\vec{v}_{A.i}$$

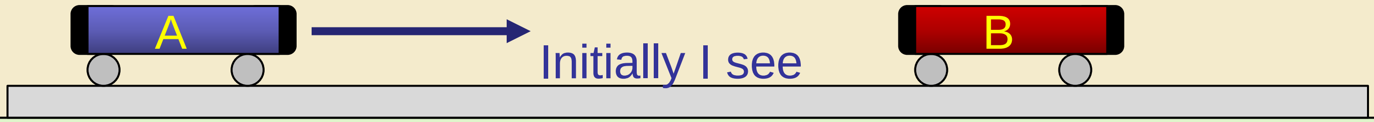


1-D Collision

Special Case: Perfectly Elastic Transform for B initially moving

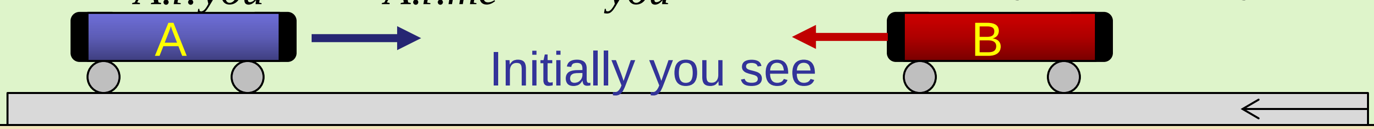
Say I'm watching this collision while I'm just sitting here. According to *me*

$\vec{v}_{A.i.me}$ $\qquad$ $\vec{v}_{B.i.me} = 0$



Say you're watching this collision while walking by with velocity $\vec{v}_{you}$.
Relative to *you* the initial velocities are

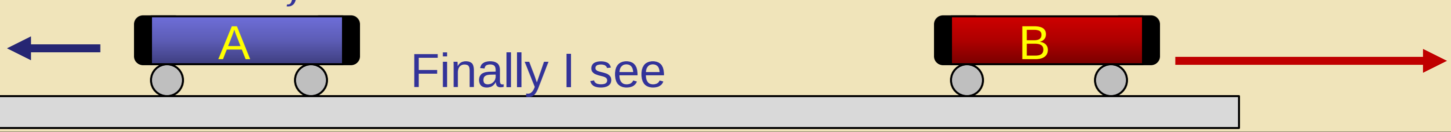
$\vec{v}_{A.i.you} = \vec{v}_{A.i.me} - \vec{v}_{you}$ $\qquad$ $\vec{v}_{B.i.you} = -\vec{v}_{you}$



$-\vec{v}_{you}$

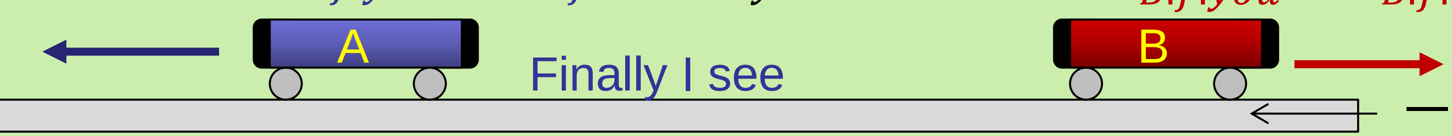
Similarly, after the collision, according to *me*

$\vec{v}_{A.f.me}$ $\qquad$ $\vec{v}_{B.f.me}$



And according to *you*

$\vec{v}_{A.f.you} = \vec{v}_{A.f.me} - \vec{v}_{you}$ $\qquad$ $\vec{v}_{B.f.you} = \vec{v}_{B.f.me} - \vec{v}_{you}$



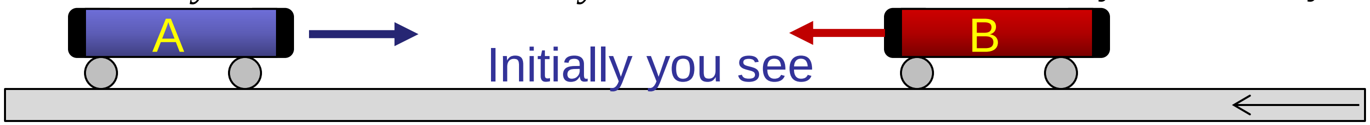
$-\vec{v}_{you}$

1-D Collision

Special Case: Perfectly Elastic Transform for B initially moving

So, completely rephrasing things from your perspective:

$$\vec{v}_{A.i.you} = \vec{v}_{A.i.me} - \vec{v}_{you} \qquad \vec{v}_{B.i.you} = -\vec{v}_{you}$$



$$-\vec{v}_{you}$$

$$\vec{v}_{A.i.you} = \vec{v}_{A.i.me} - \vec{v}_{B.i.you} \Rightarrow \vec{v}_{A.i.me} = \vec{v}_{A.i.you} + \vec{v}_{B.i.you}$$

$$\vec{v}_{A.f.you} = \vec{v}_{A.f.me} - \vec{v}_{you}$$

$$\vec{v}_{B.f.you} = \vec{v}_{B.f.me} - \vec{v}_{you}$$

$$-\vec{v}_{you}$$

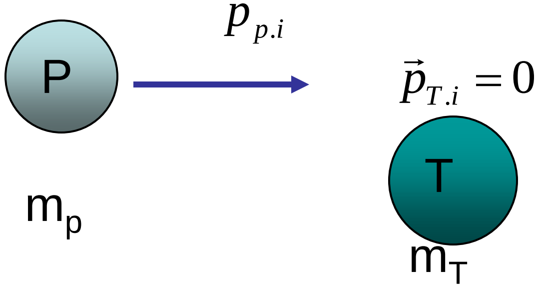
$$\vec{v}_{A.f.you} = \vec{v}_{A.f.me} - \vec{v}_{B.i.you} \qquad \vec{v}_{B.f.you} = \vec{v}_{A.f.me} - \vec{v}_{B.i.you}$$

$$\vec{v}_{A.f.you} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{v}_{A.i.me} - \vec{v}_{B.i.you} \qquad \vec{v}_{B.f.you} = \frac{2m_A}{(m_B + m_A)}\vec{v}_{A.i.me} - \vec{v}_{B.i.you}$$

$$\vec{v}_{A.f} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\left(\vec{v}_{A.i} + \vec{v}_{B.i}\right) - \vec{v}_{B.i} \qquad \vec{v}_{B.f.you} = \frac{2m_A}{(m_B + m_A)}\left(\vec{v}_{A.i} + \vec{v}_{B.i}\right) - \vec{v}_{B.i}$$

2-D Collision: Scattering

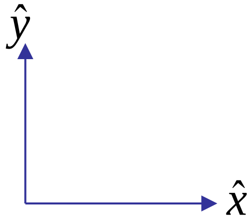
Slow (v<<C)



$\vec{p}_{p.i}$

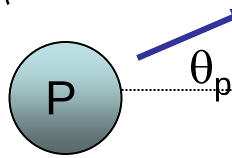
$\vec{p}_{T.i} = 0$

2-D Collision: Scattering



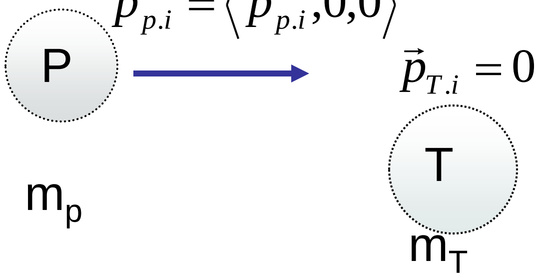
Slow (v<<C)

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$

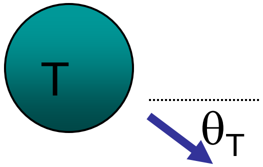
$$\vec{p}_{T.i} = 0$$



Three **Equations / *Four* Unknowns**

Need another equation

Or more information



$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}|\sin\theta_T|, 0\rangle$$

Momentum Principle

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y}: p_{p.f}\sin\theta_p - p_{T.f}|\sin\theta_T| - 0 = 0$$

Energy Principle

$$(E_{p.f} + E_{T.f}) - (E_{p.i} + E_{T.i}) = 0$$

$$K_{p.f} + K_{T.f} - K_{p.i} + \Delta E_{int.p} + \Delta E_{int.t} + \Delta U_{t,p} = 0$$

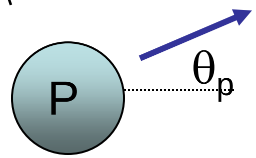
if Elastic

$$\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0$$

2-D Collision: Scattering

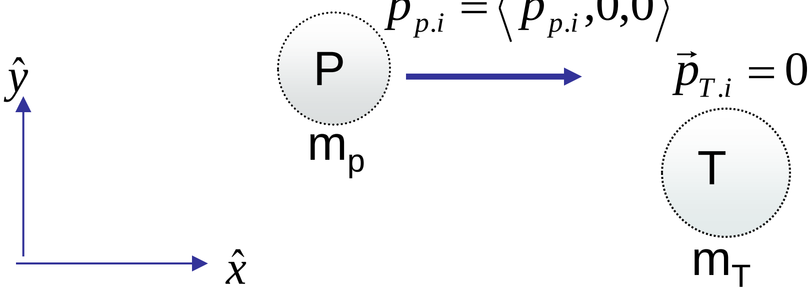
Example. Say a 2kg puck moving at 3m/s collides with a 4kg puck that's just sitting there. Say the 2kg puck travels off at 30° and the 4kg one travels down at 50°. What are the final speeds and is the collision elastic?

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$

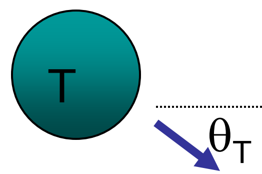
$$\vec{p}_{T.i} = 0$$



Momentum Principle

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y}: p_{p.f}\sin\theta_p - p_{T.f}|\sin\theta_T| - 0 = 0$$

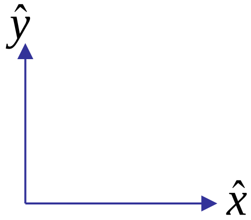


$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}|\sin\theta_T|, 0\rangle$$

Energy Principle – *if Elastic*

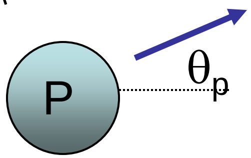
$$\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0$$

2-D Collision: Scattering



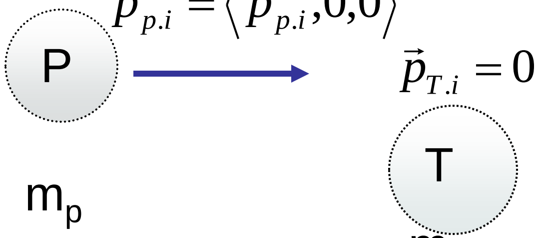
Slow (v<<C), Elastic Collision

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$

$$\vec{p}_{T.i} = 0$$



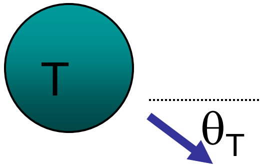
m_p

m_T

Three **Equations / *Four* Unknowns**

Need another equation

Special case: equal masses



$$m_p = m_T$$

$$\frac{p_{p.i}^2}{2m_p} = \frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} \Rightarrow p_{p.i}^2 = p_{p.f}^2 + p_{T.f}^2$$

$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}|\sin\theta_T|, 0\rangle$$

$$\vec{p}_{p.i} = \vec{p}_{p.f} + \vec{p}_{T.f} \Rightarrow p_{pi}^2 = \vec{p}_{pi}\bullet\vec{p}_{pi} = (\vec{p}_{pf} + \vec{p}_{tf})\bullet(\vec{p}_{pf} + \vec{p}_{tf})$$

$$p_{pi}^2 = p_{pf}^2 + p_{Tf}^2 + 2\vec{p}_{pf}\bullet\vec{p}_{Tf}$$

A: projectile missed target

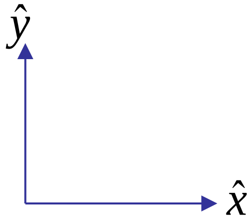
B: $\theta_p + |\theta_T| = 90°$

compare

$$p_{pi}^2 = p_{pf}^2 + p_{Tf}^2 + 2p_{pf}p_{Tf}\cos(\theta_p + |\theta_T|)$$

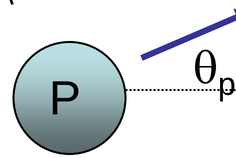
Must be zero

2-D Collision: Scattering



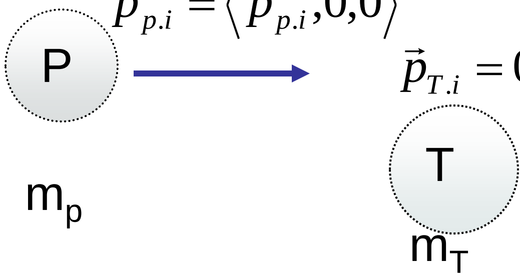
Slow (v<<C), Elastic Collision

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



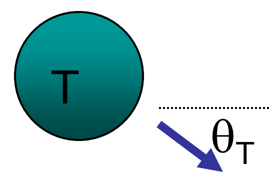
$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$

$$\vec{p}_{T.i} = 0$$



Three **Equations /** ***Four*** **Unknowns**

Need another equation



$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0\rangle$$

Momentum Principle

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

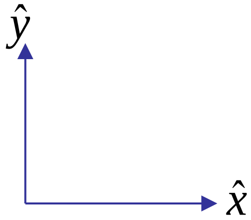
$$\hat{y}: \underline{p_{p.f}}\sin\underline{\theta_p} - \underline{p_{T.f}}\left|\sin\underline{\theta_T}\right| - 0 = 0$$

Energy Principle – *if Elastic*

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

$$K_{p.f} + K_{T.f} - K_{p.i} = 0$$

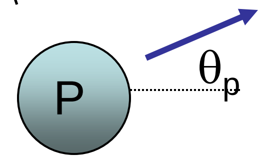
$$\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0$$



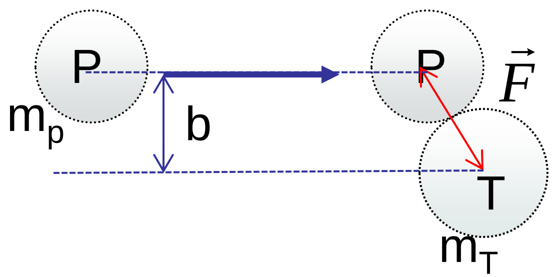
2-D Collision: Scattering

Slow (v<<C), Elastic Collision

$$\vec{p}_{p.f} = \left\langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\right\rangle$$



$$\vec{p}_{p.i} = \left\langle p_{p.i}, 0, 0\right\rangle$$



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p}_{T.i} = 0$$

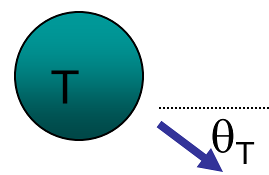
***Three* Equations / *Four* Unknowns**

Need another equation

Collision geometry and Impact Parameter

Component of momentum along line of contact changes, other component remains constant

b = Impact Parameter



$$\vec{p}_{T.f} = \left\langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0\right\rangle$$

Momentum Principle

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

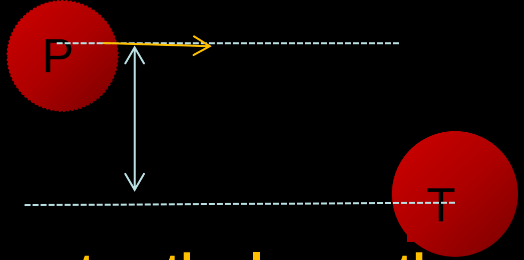
$$\hat{y}: \underline{p_{p.f}}\sin\underline{\theta_p} - \underline{p_{T.f}}\left|\sin\underline{\theta_T}\right| - 0 = 0$$

Energy Principle – *if Elastic*

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

$$K_{p.f} + K_{T.f} - K_{p.i} = 0$$

$$\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0$$

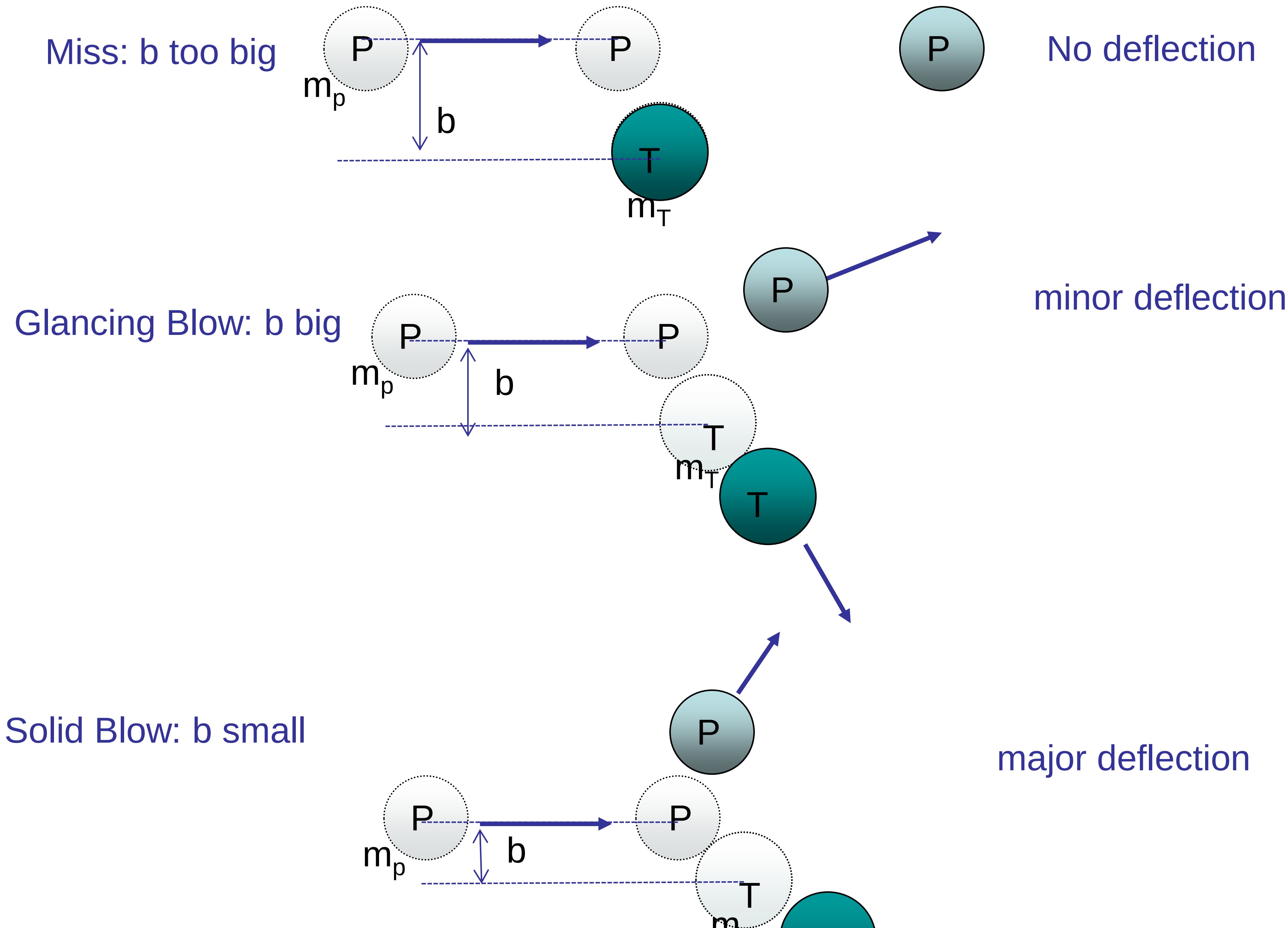


Which of these is true?

1) The larger the impact parameter, the larger the scattering angle (deflection).
2) The larger the impact parameter, the smaller the scattering angle (deflection).

2-D Collision: Scattering

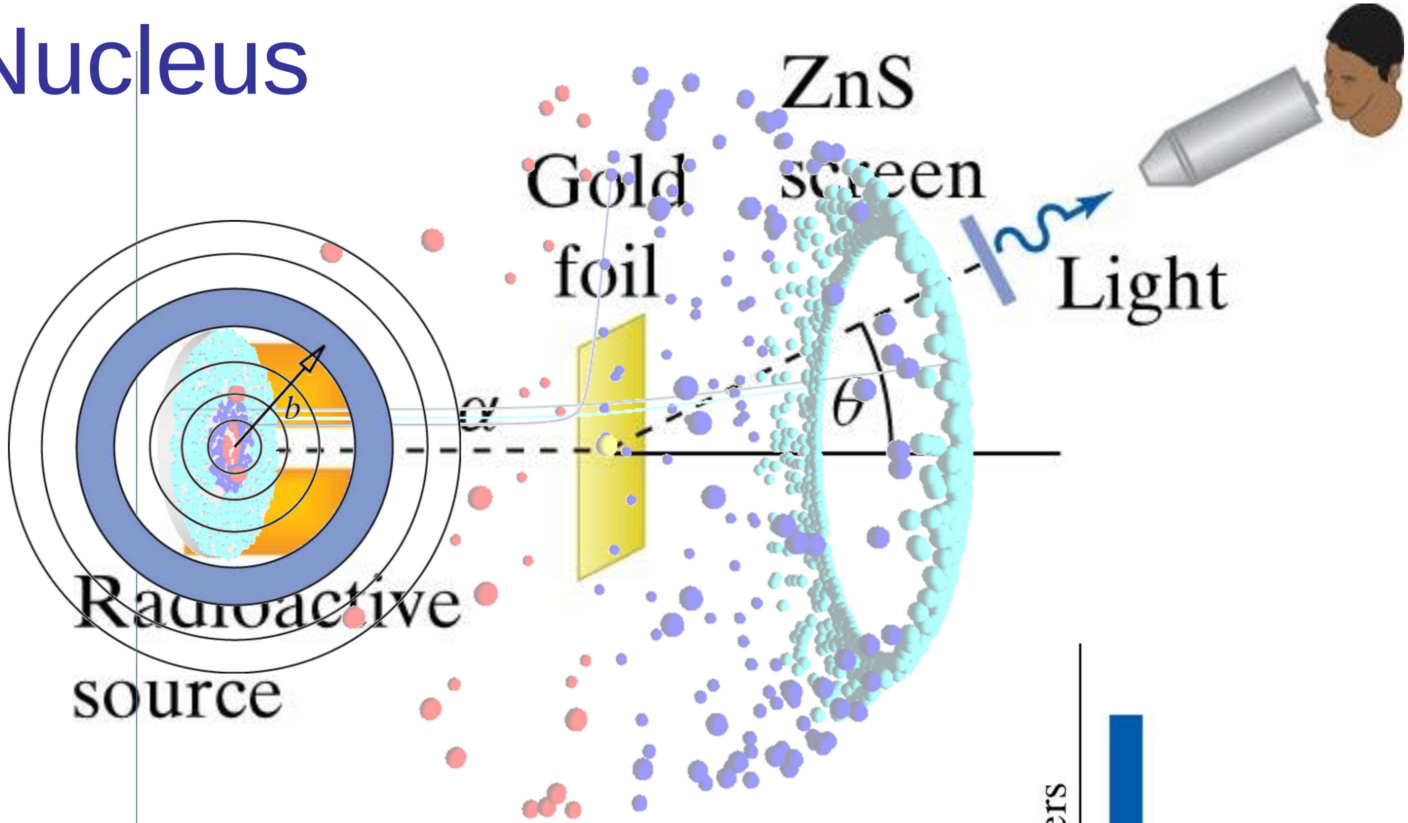
Qualitative effect of Impact Parameter

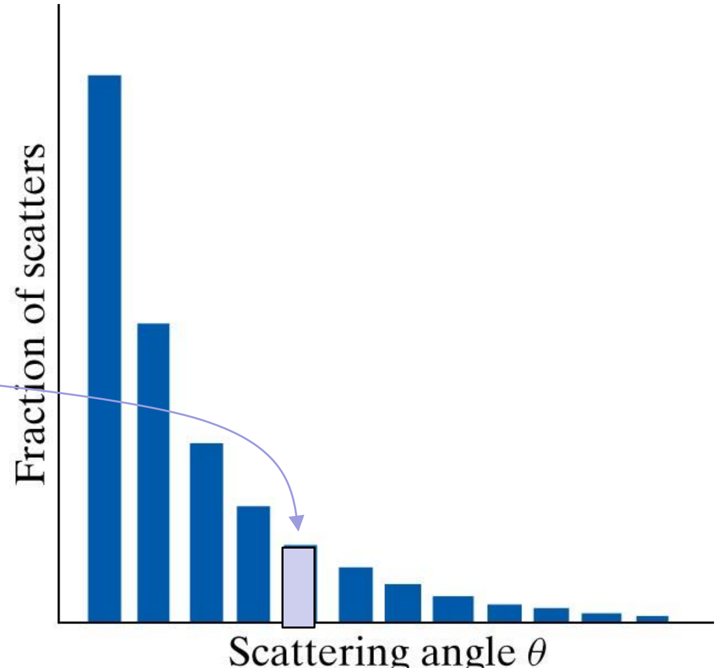


Probability and Cross-sectional Area

Playing Darts Badly

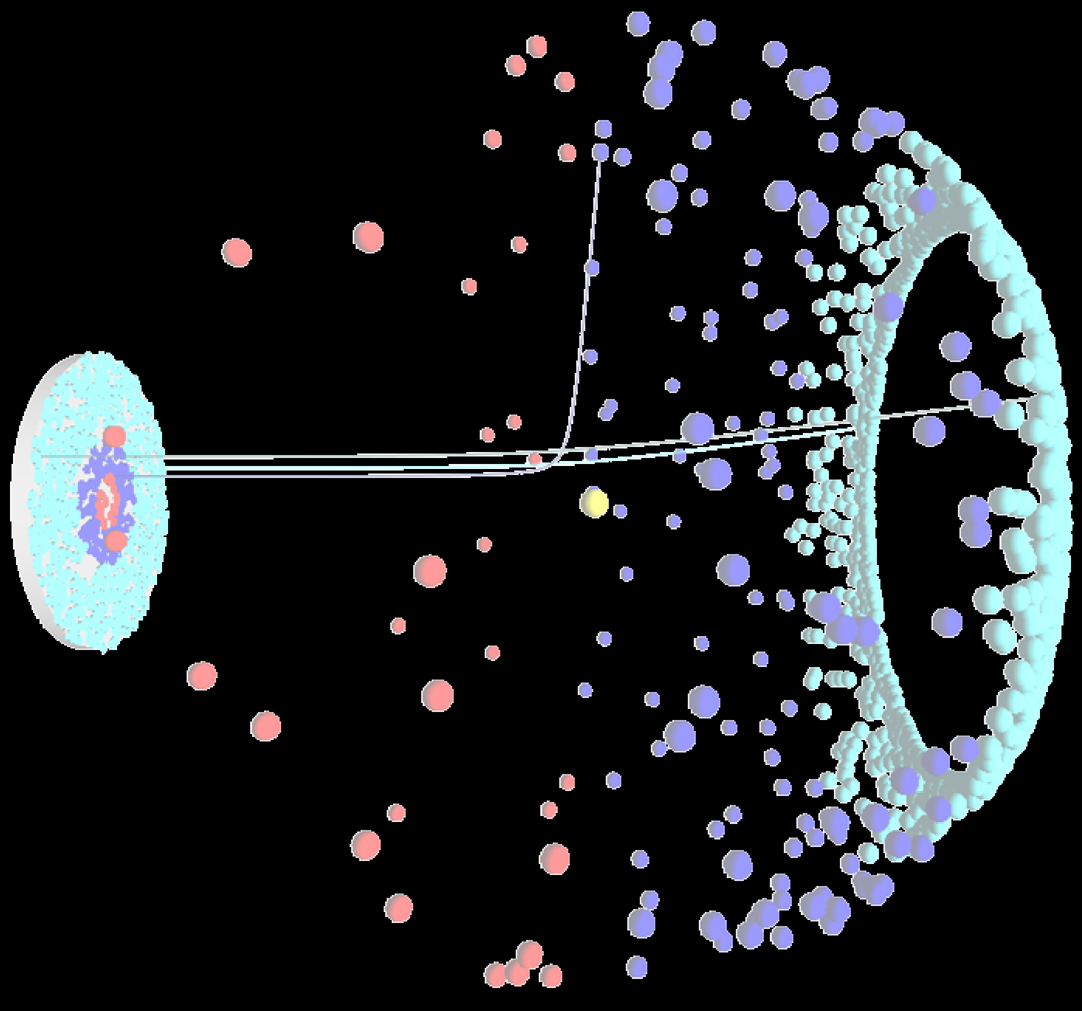
2-D Collision: Scattering – Discovering Nucleus





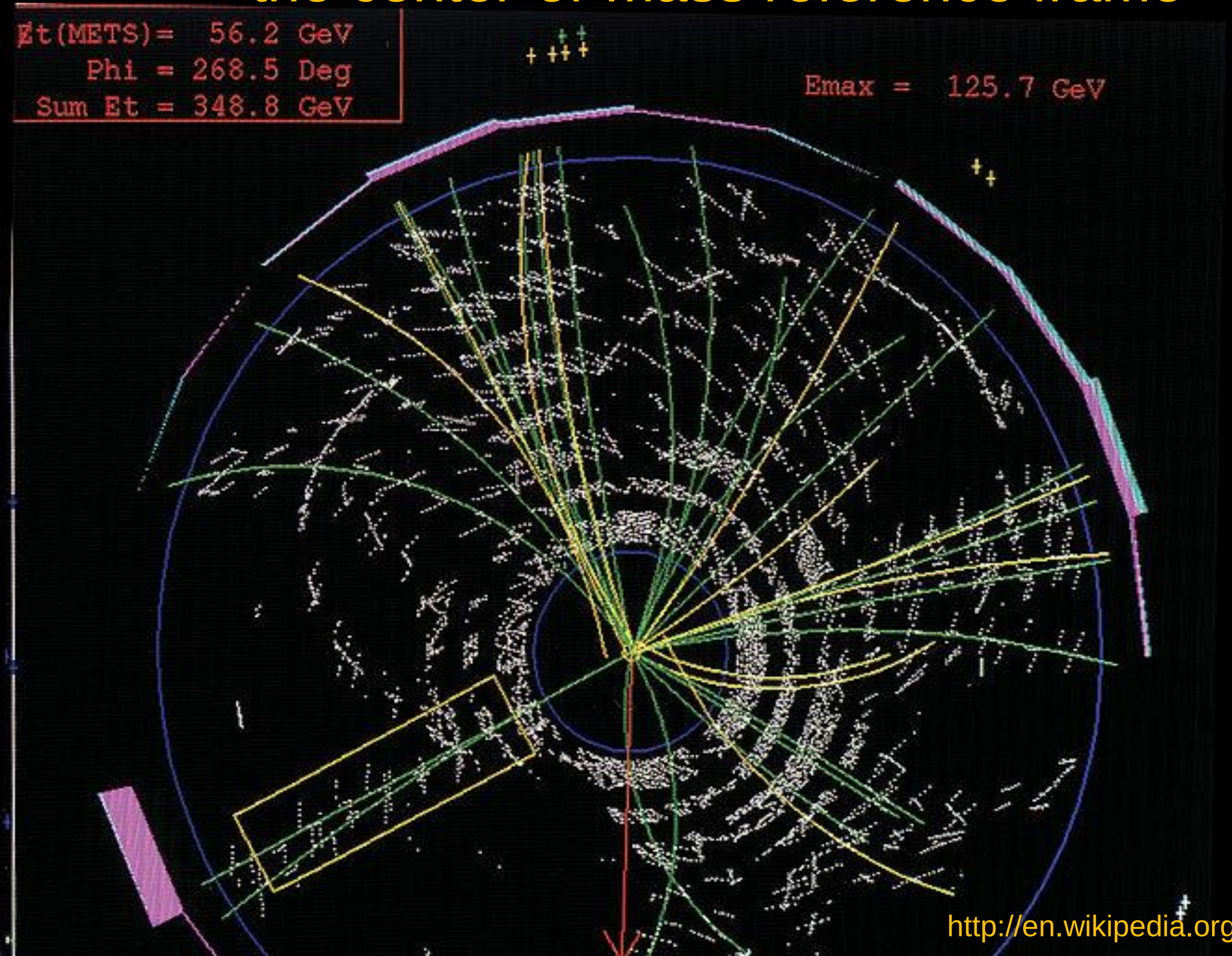
In the Rutherford experiment, what was surprising to the experimenters?

(1) Sometimes the alpha “rays” passed right through the gold foil.
(2) Sometimes the alpha “rays” were deflected slightly when they passed through the gold foil.
(3) Sometimes the alpha “rays” bounced back from the gold foil.



Deeper Collisions

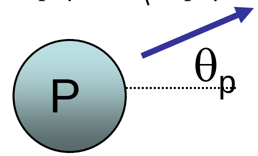
- relativistic speeds
- identifying ‘mystery’ particles
- the mathematical trick of analyzing in the center-of-mass reference frame



http://en.wikipedia.org/wiki/File:CDF_Top_Event.jpg

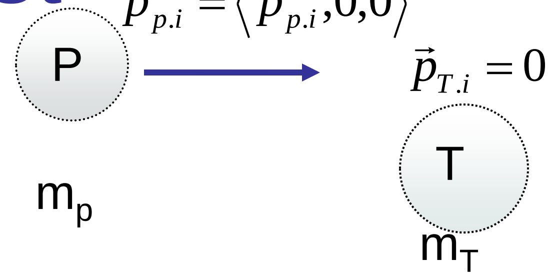
2-D Collision: Scattering Fast

$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$



$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$

$\vec{p}_{T.i} = 0$



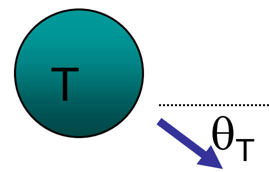
Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y}: p_{p.f}\sin\theta_p - p_{T.f}\left|\sin\theta_T\right| - 0 = 0$$

where $\vec{p} = \dfrac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$



$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0\rangle$

show $E = \sqrt{(pc)^2 + (mc^2)^2}$

By plugging that into it and recovering that.

Conservation of Energy

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

where $E = \dfrac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$$\sqrt{(p_{p.f}c)^2 + (m_{p.f}c^2)^2} + \sqrt{(p_{T.f}c)^2 + (m_{T.f}c^2)^2} - \sqrt{(p_{p.i}c)^2 + (m_{p.i}c^2)^2} - m_{T.i}c^2 = 0$$

Note: initial and final masses differ for inelastic collisions

2-D Collision: Scattering Fast

Note: initial and final masses differ for inelastic collisions

Recall: particle energy

$$E_p = K_p + E_{p.\text{int}}$$

$$E_{p.\text{int}} = m_p c^2$$

Elastic

$$\Delta E_{p.\text{int}} = \Delta m_p c^2 = 0$$

$$\sqrt{(p_{p.f}c)^2 + (m_p c^2)^2} + \sqrt{(p_{T.f}c)^2 + (m_T c^2)^2} - \sqrt{(p_{p.i}c)^2 + (m_p c^2)^2} - m_T c^2 = 0$$

In-Elastic

$$\Delta E_{p.\text{int}} = \Delta m_p c^2 \neq 0$$

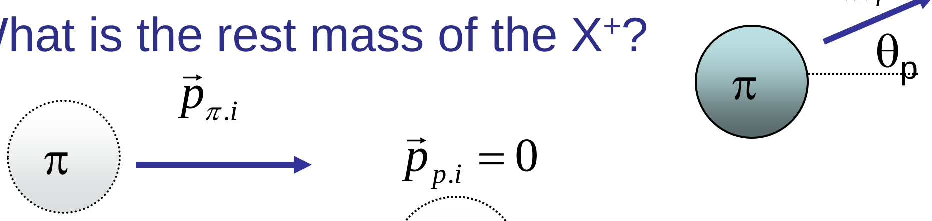
$$\sqrt{(p_{p.f}c)^2 + (m_{p.f} c^2)^2} + \sqrt{(p_{T.f}c)^2 + (m_{T.f} c^2)^2} - \sqrt{(p_{p.i}c)^2 + (m_{p.i} c^2)^2} - m_{T.i} c^2 = 0$$

2-D Collision: Fast

Example: A beam of high energy π^- (negative pions) is shot at a flask of liquid hydrogen, and sometimes a pion interacts through the strong interaction with a proton in the hydrogen, the reaction is $\pi^- + p^+ \to \pi^- + X^+$ where X^+ is a positively charged particle of unknown mass (essentially an excited proton.)

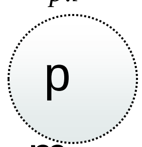
A proton's rest mass is 938MeV, and a pion's rest mass is 140 MeV. The incoming pion has momentum 3GeV/c. It scatters through 40°, and its momentum drops to 1.510 GeV/c.

What is the rest mass of the X^+?

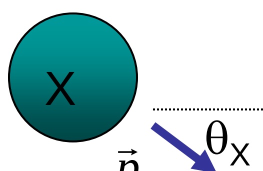


$\vec{p}_{\pi.i}$ $\quad \vec{p}_{\pi.f}$ $\quad \vec{p}_{p.i} = 0$

m_π



m_p



$\vec{p}_{X.f}$

Conservation of Energy

$$\sqrt{(p_{\pi.f}c)^2 + (m_{\pi.f}c^2)^2} + \sqrt{(p_X c)^2 + (m_X c^2)^2} - \sqrt{(p_{\pi.i}c)^2 + (m_\pi c^2)^2} - m_{p.i}c^2 = 0$$

Conservation of Momentum

$$\hat{x} : p_{\pi.f}\cos\theta_\pi + p_{X.f}\cos\theta_X - p_{\pi.i} = 0 \qquad p_{X.f}\cos\theta_X = p_{\pi.i} - p_{\pi.f}\cos\theta_\pi$$

$$\hat{y} : p_{\pi.f}\sin\theta_\pi - p_{X.f}\left|\sin\theta_X\right| = 0 \qquad p_{X.f}\left|\sin\theta_X\right| = p_{\pi.f}\sin\theta_\pi$$

$$\left(p_{X.f}\cos\theta_X\right)^2 + \left(p_{X.f}\left|\sin\theta_X\right|\right)^2 = \left(p_{\pi.i} - p_{\pi.f}\cos\theta_\pi\right)^2 + \left(p_{\pi.f}\sin\theta_\pi\right)^2$$

$$p_{X.f} = \sqrt{\left(p_{\pi.i} - p_{\pi.f}\cos\theta_\pi\right)^2 + \left(p_{\pi.f}\sin\theta_\pi\right)^2} \qquad p_{X.f} = \sqrt{\left(p_{\pi.i}\right)^2 + \left(p_{\pi.f}\right)^2 - 2p_{\pi.i}p_{\pi.f}\cos\theta_\pi}$$

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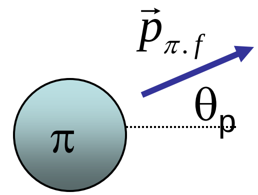

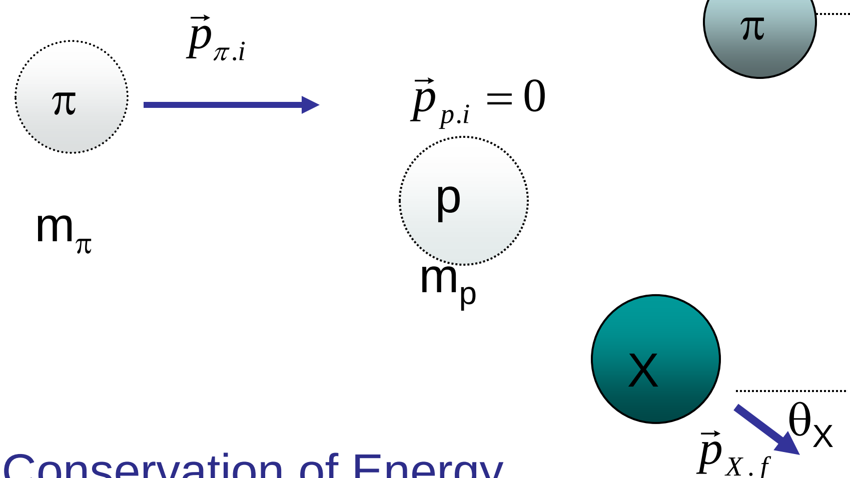

Conservation of Momentum

$$p_{X.f} = \sqrt{(p_{\pi.i})^2 + (p_{\pi.f})^2 - 2p_{\pi.i}p_{\pi.f}\cos\theta_\pi}$$

Conservation of Energy

$$\sqrt{(p_{\pi.f}c)^2 + (m_{\pi.f}c^2)^2} + \sqrt{(p_X c)^2 + (m_X c^2)^2} - \sqrt{(p_{\pi.i}c)^2 + (m_\pi c^2)^2} - m_{p.i}c^2 = 0$$

$$\sqrt{(p_X c)^2 + (m_X c^2)^2} = \sqrt{(p_{\pi.i}c)^2 + (m_\pi c^2)^2} + m_{p.i}c^2 - \sqrt{(p_{\pi.f}c)^2 + (m_{\pi.f}c^2)^2}$$

Could do algebraically further, or just plugin numbers and simplify

i.	10.6-.8 Scattering	RE 10.b
on.,	10.9-.10 Collision Complications	RE 10.c
ab	L10 Collisions 1	EP9
ed.,	10.5, .11 Different Reference Frames	
i.,	11.1 Translational Angular Momentum Quiz 10	RE 11.a; HW10: Pr’s 13*, 21, 30, “39

Collisions

Short, Sharp Shocks