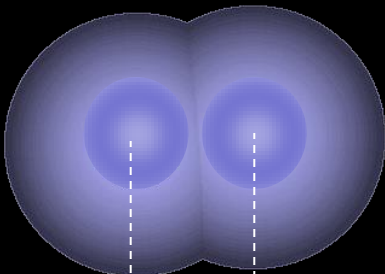


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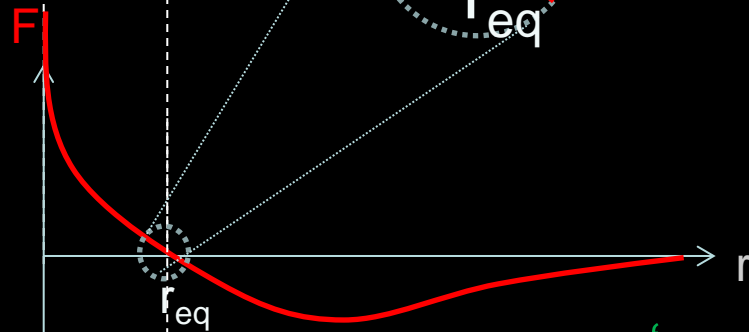
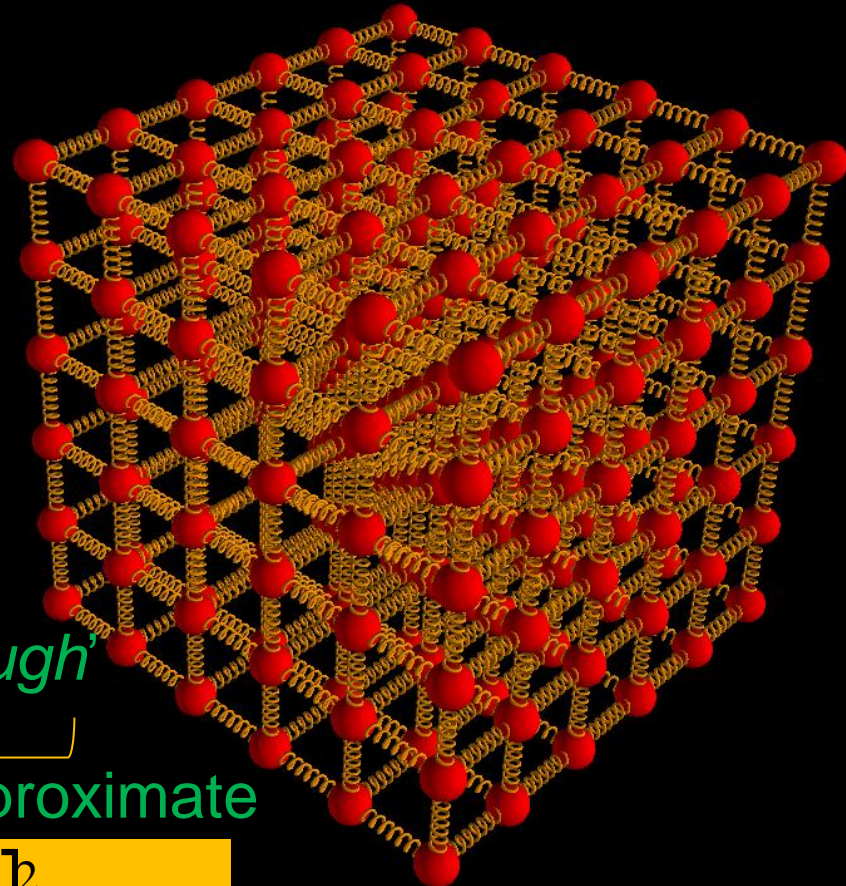
# Ball-Spring Model

Molecule

Solid



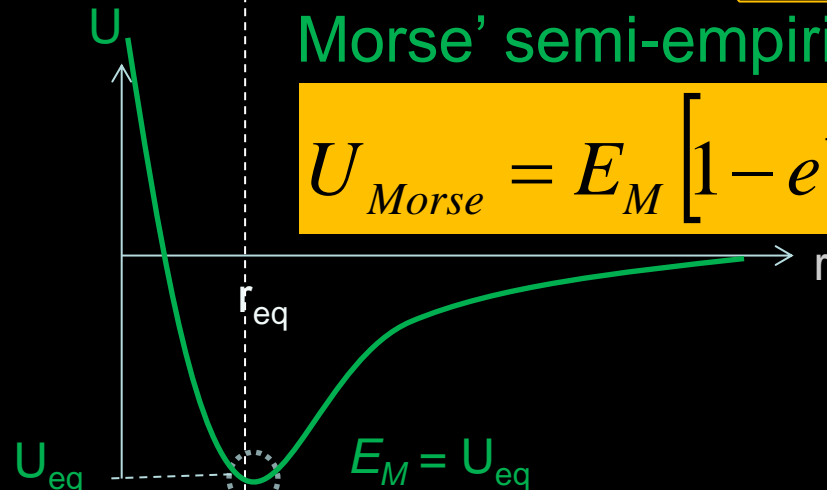
$$F = -k(r - r_{eq})$$



'good enough'

Morse' semi-empirical approximate

$$U_{Morse} = E_M \left[ 1 - e^{-\alpha(r - r_{eq})} \right]^2 - E_M$$



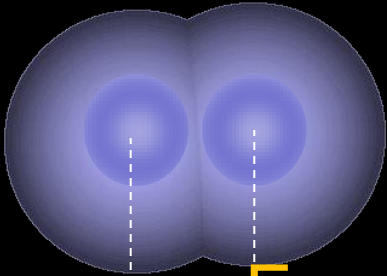
$$E_M = U_{eq}$$

# Ball-Spring Model

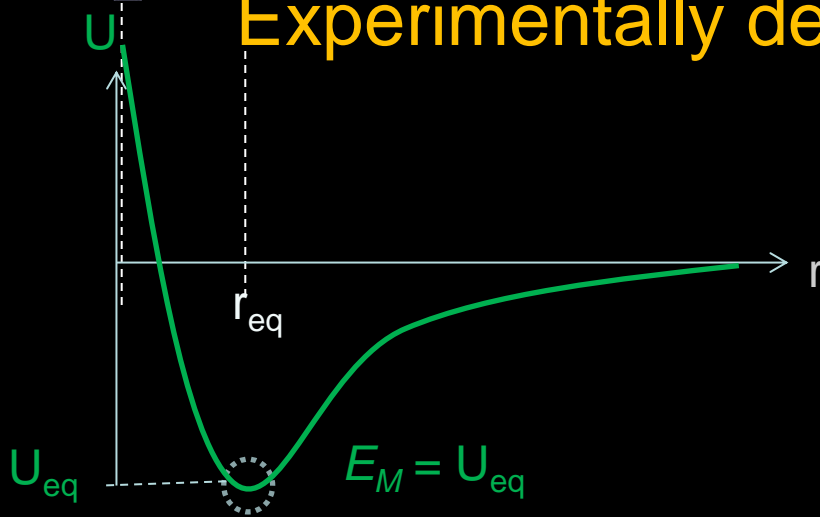
Morse' semi-empirical approximate

$$U_{Morse} = E_M \left[ 1 - e^{-\alpha(r-r_{eq})} \right]^2 - E_M$$

Molecule



Experimentally determining the constants



$E_M = U_{eq}$ : Break molecule

$r_{eq}$ : For solid, from density and atomic mass

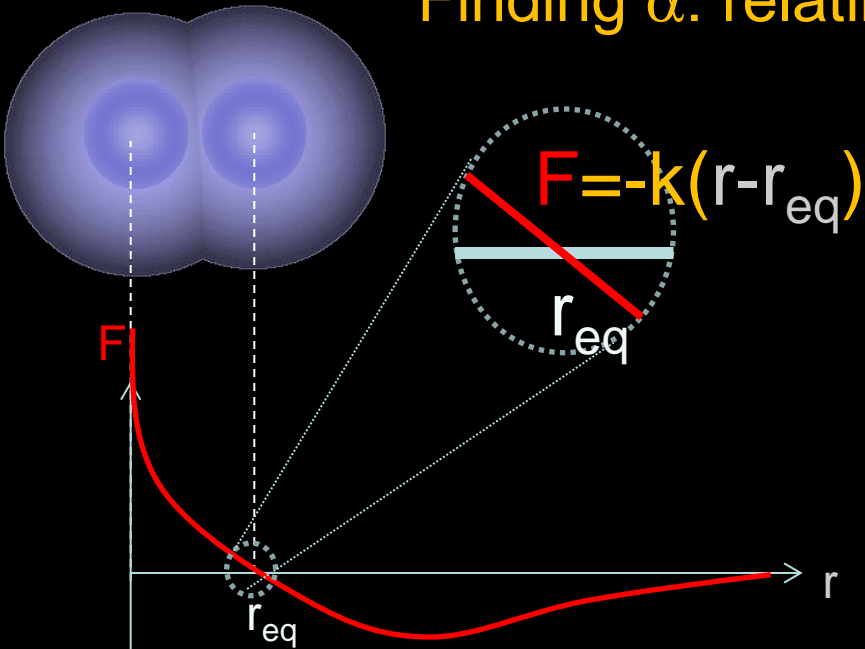
For molecule: rotational spectrum (more later)

$\alpha$ : From vibrational spectrum (more now)

# Ball-Spring Model

Molecule

Finding  $\alpha$ : relating to mass-spring



Taylor Series Approximation

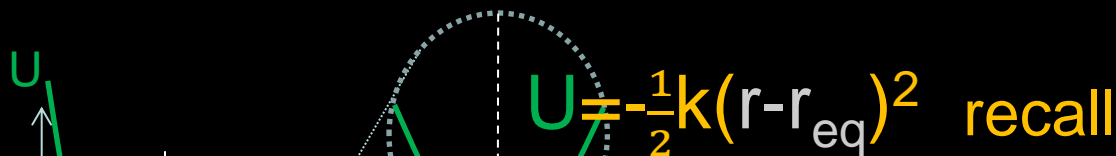
$$U(r) \approx U(r_{eq}) + \left. \frac{dU}{dr} \right|_{r_{eq}} (r - r_{eq}) + \frac{1}{2} \left. \frac{d^2U}{dr^2} \right|_{r_{eq}} (r - r_{eq})^2$$

$$|F| = - \left| \frac{dU}{dr} \right|$$

at equilibrium = 0

$$\frac{d^2U}{dr^2} = \frac{d|F|}{dr} = k$$

$$U(r) \approx U(r_{eq}) + \frac{1}{2} k (r - r_{eq})^2 \quad \frac{d^2U_M}{dr^2} = 2E_M \alpha^2$$



$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Morse' semi-empirical approximate

$U_{eq}$   $E_M = U_{eq}$

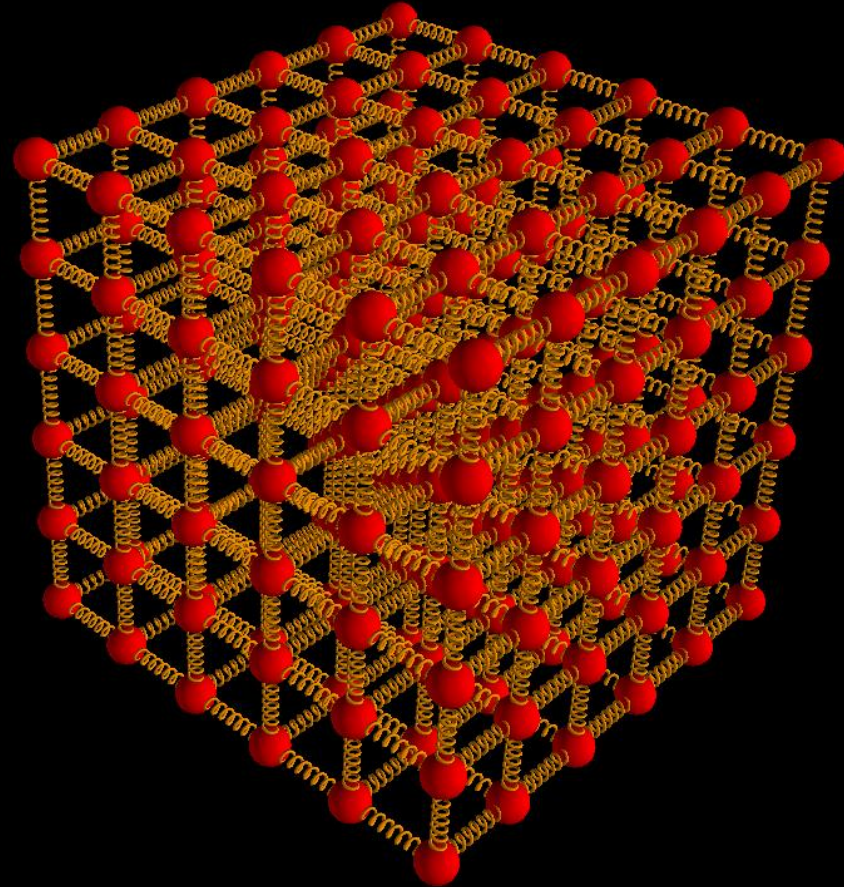
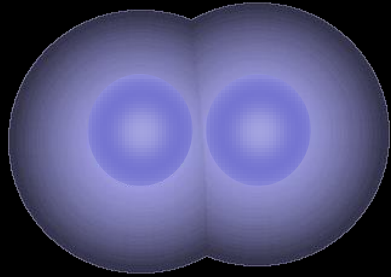
$$U_{Morse} = E_M \left[ 1 - e^{-\alpha(r - r_{eq})} \right]^2 - E_M$$

# Ball-Spring Model

Molecule

Potential Energy in Solid

Solid



$$U_{bond.1}(r) \approx U(r_{eq}) + \frac{1}{2}k(r_1 - r_{eq})^2$$

$$U_{solid}(r) \approx N_{bonds}U(r_{eq}) + \sum_{bonds} \frac{1}{2}k s_{bond}^2$$

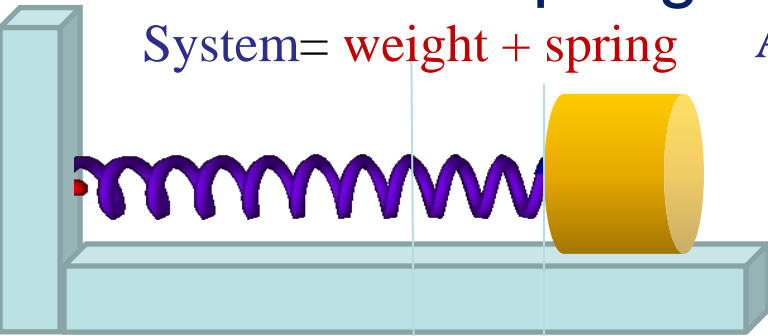
$$U_{solid}(r) \approx N_{bonds}U(r_{eq}) + \frac{1}{2}k_{total}s_{total}^2$$

$$\text{Recall: } k_{total} = \frac{\text{Area}}{\text{Length} \cdot r_{eq}} k_1$$

# Spring force and potential

System = **weight + spring**

Active environment = **none**



$$\Delta E = W_{\text{system} \leftarrow \text{ext}} = 0$$

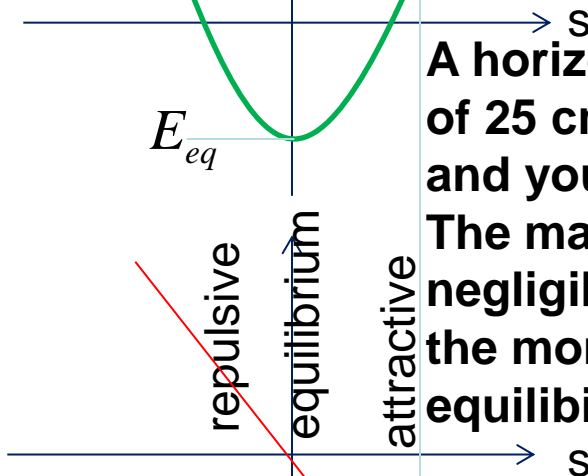
$$\Delta E_{W,S} = \cancel{\Delta E_{\text{rest},W}} + \cancel{\Delta E_{\text{rest},S}} + \Delta K_W + \Delta K_S + \Delta U_{W,S} = 0$$

Negligible mass

$$\Delta E_{W,S} = \Delta K_W + \Delta U_{W,S} = 0$$

$$U_{1,2} = \frac{1}{2} k_s s^2 + E_{eq}$$

$$\frac{1}{2} m_w v_{w,f}^2 - \frac{1}{2} m_w v_{w,i}^2 + \left( \frac{1}{2} k_s s_f^2 \right) - \left( \frac{1}{2} k_s s_i^2 \right) = 0$$



A horizontal spring with stiffness 3 N/m has a relaxed length of 25 cm (0.25 m). A mass of 50 grams (0.050 kg) is attached and you stretch the spring to a total length of 29 cm (0.29 m). The mass is then released from rest and moves with negligible friction. What is the kinetic energy of the mass at the moment when the spring returns through the position its equilibrium length, 25 cm?

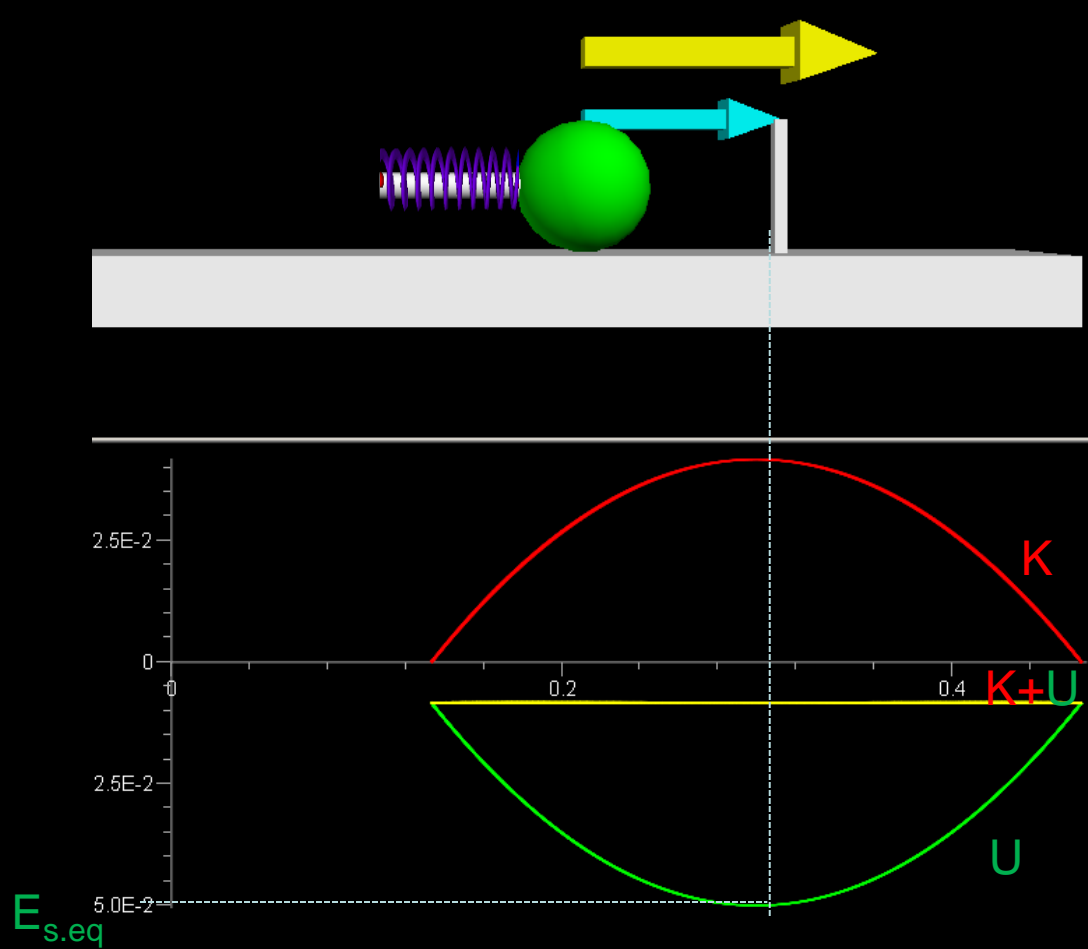
$$-\frac{dU_{1,2}}{ds} = F_{1 \rightarrow 2} = -k_s s$$

- 1) 2.4e-3 J  
2) 4.8e-3 J

- 3) 6e-2 J  
4) 1.26e-1 J

- 5) 1.5 J

# Horizontal Ball on spring



Consider a block falling onto a vertical spring.

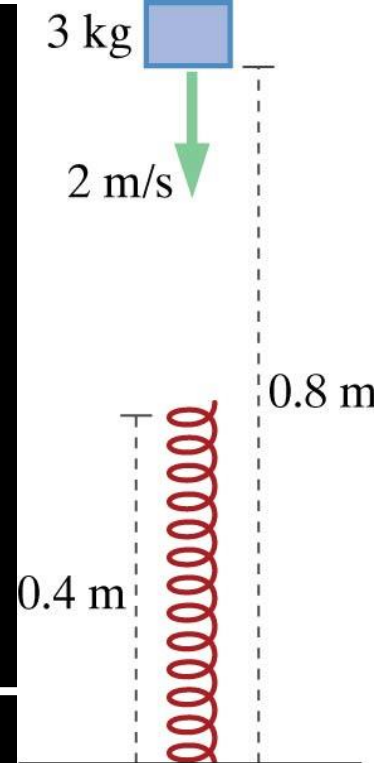
States:

A: Block 0.8 m above floor

B: Block just touching top of spring

C: Block come to rest

To find the speed of the block just before it hits the spring, what should we pick for initial and final states?



	Initial	Final
1)	A	B
2)	B	C
3)	A	C
4)	C	A



Consider a block falling onto a vertical spring.

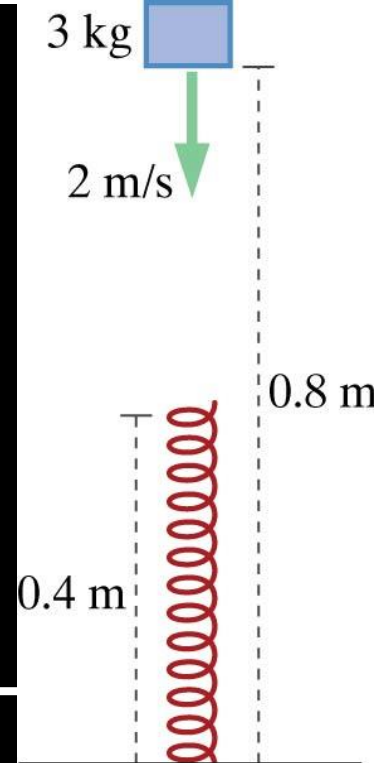
States:

A: Block 0.8 m above floor

B: Block just touching top of spring

C: Block come to rest

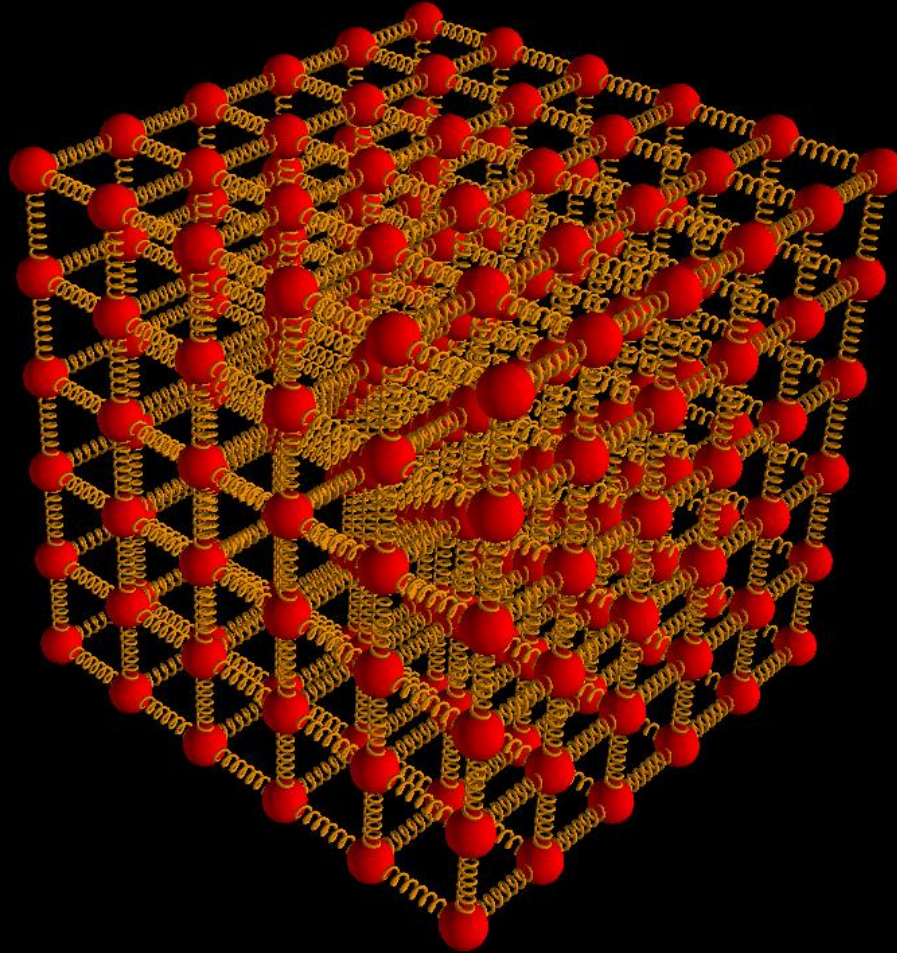
To find the **maximum compression of the spring**, what should we pick for initial and final states?



	Initial	Final
1)	A	B
2)	B	C
3)	A	C
4)	C	A

# Ball-Spring Model

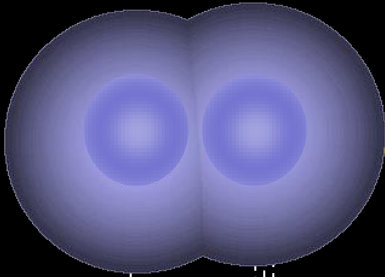
## Kinetic Energy in Solid



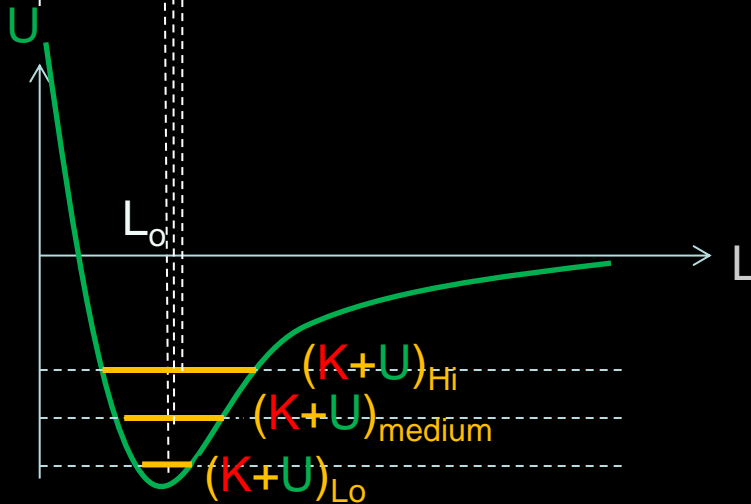
# Ball-Spring Model

## Thermal Expansion

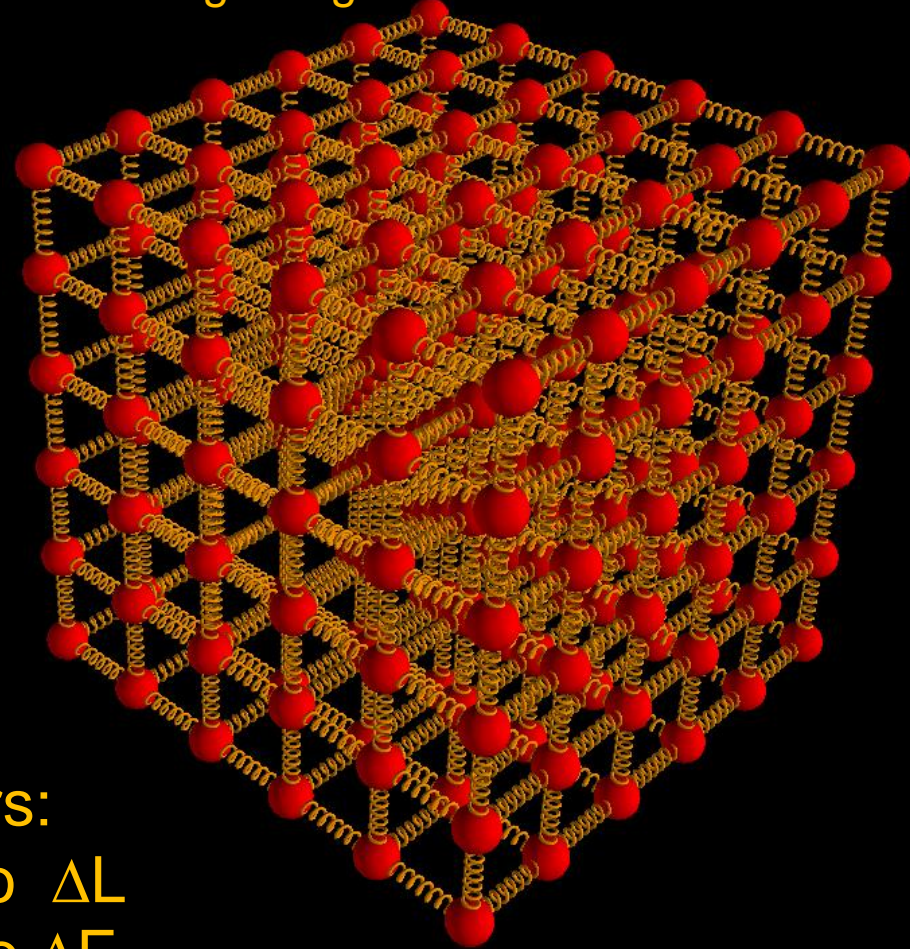
Molecule



or higher energies, average bond length is greater



$\Delta L$  proportional to  $\Delta E$



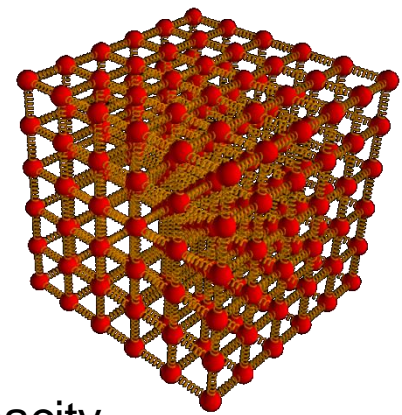
Thermometers:

$\Delta T$  proportional to  $\Delta L$

$\Delta L$  proportional to  $\Delta E$

$\Delta T$  proportional to  $\Delta E$

# Heat Capacity



$\Delta T$  proportional to  $\Delta E_{\text{int}}$

$$\Delta E_{\text{int}} = C\Delta T$$

$C$  = Heat capacity

$C$  should be material specific and scale with amount of material

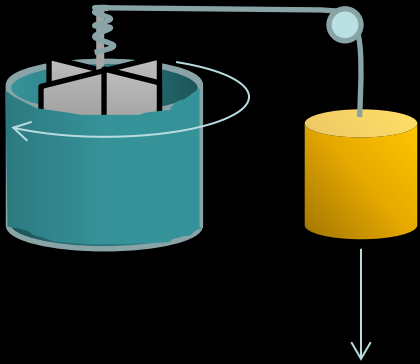
$$C = c_v m \quad c_v = (\text{mass}) \text{ specific Heat capacity}$$

$$\Delta E_{\text{int}} = c_v m \Delta T$$

The thermal energy of the 1000 grams of water increased 7000 J. What was the temperature increase in Kelvins of the water? The heat capacity of water is  $c_v = 4.2 \text{ J/K}$  on a per-gram basis.

- 1) 0.0006 K
- 2) 0.6 K
- 3) 1.7 K
- 4) 1667 K
- 5) Insufficient information

Say you do the historic falling-weight / paddle-wheel in bucket experiment. The mass drops, the wheel spins, and the 10 kg's of water warm up. We find that the difference between the change in gravitational potential and change in kinetic energies is  $40 \times 10^3$  J. A thermometer stuck in the water tells you that its temperature has risen  $\Delta T = 0.96$  Kelvin. What's the mass-specific heat capacity of water?



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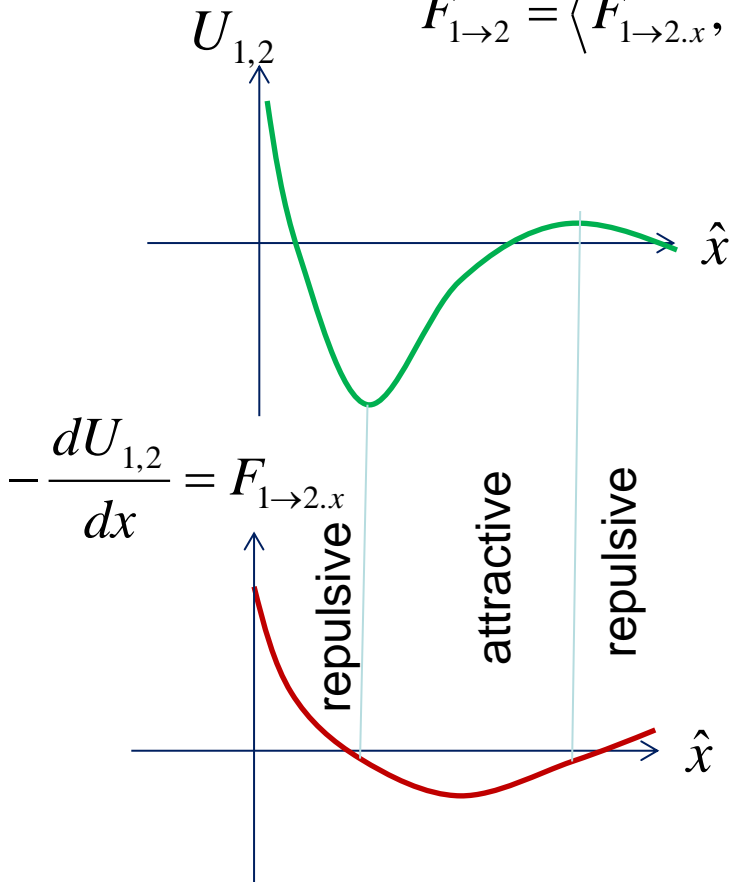
# Force as negative gradient (3-D slope) of Potential Energy

small change in potential

$$dU_{1,2} = -\vec{F}_{1 \rightarrow 2} \cdot d\vec{r}_{1 \rightarrow 2} = -(F_{1 \rightarrow 2.x} dx + F_{1 \rightarrow 2.y} dy + F_{1 \rightarrow 2.z} dz)$$

so

$$\vec{F}_{1 \rightarrow 2} = \langle F_{1 \rightarrow 2.x}, F_{1 \rightarrow 2.y}, F_{1 \rightarrow 2.z} \rangle = - \left\langle \frac{\partial U_{1,2}}{\partial x_{1 \rightarrow 2}}, \frac{dU_{1,2}}{dy_{1 \rightarrow 2}}, \frac{dU_{1,2}}{dz_{1 \rightarrow 2}} \right\rangle$$



Which of the following expressions satisfies the requirement that  $F_x = -dU_s / dx = -k_s x$  where  $x$  is measured from the equilibrium position ( $C$  is a constant)?

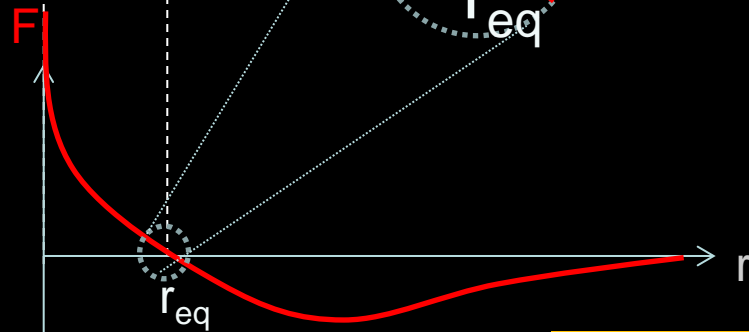
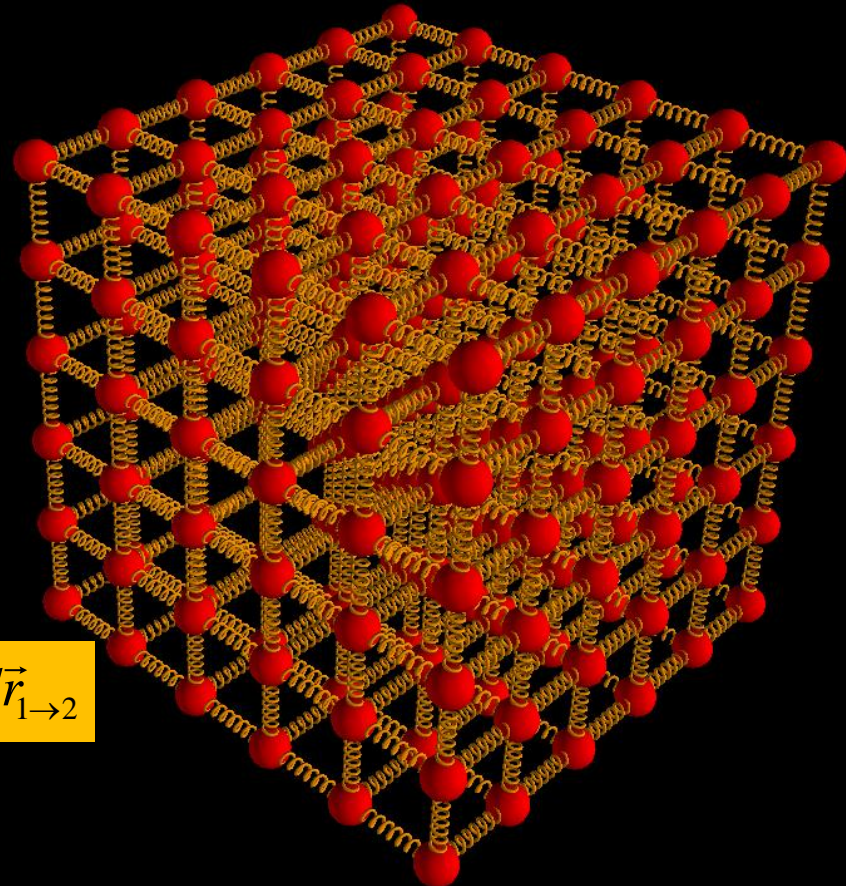
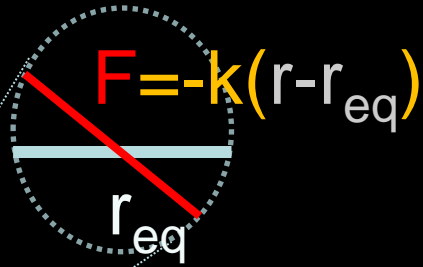
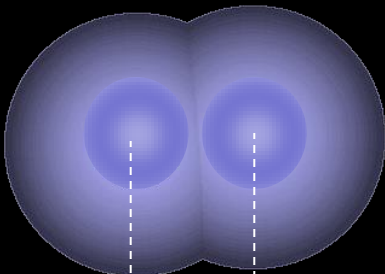
- 1)  $U_s = \frac{1}{2} k_s x^2 + C$
- 2)  $U_s = -\frac{1}{2} k_s x^2 + C$
- 3)  $U_s = k_s x^2 + C$
- 4)  $U_s = k_s + C$
- 5)  $U_s = -k_s + C$



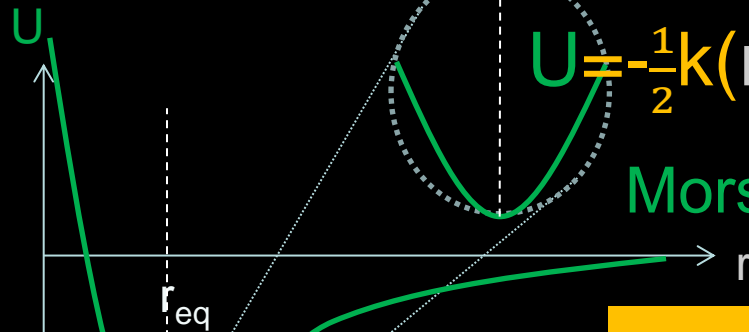
# Ball-Spring Model

Molecule

Solid



$$\Delta U_{1,2} = -\int \vec{F}_{1 \rightarrow 2} \cdot d\vec{r}_{1 \rightarrow 2}$$



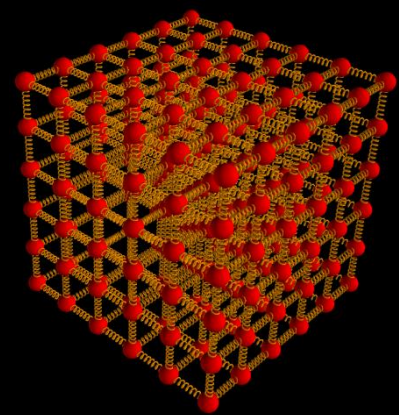
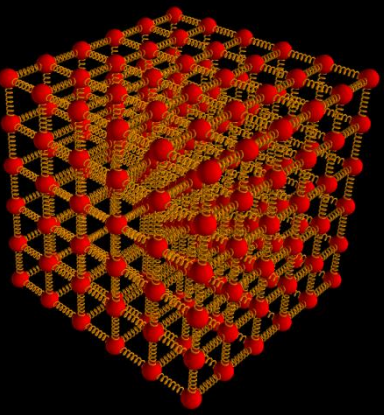
$$U = -\frac{1}{2}k(r - r_{eq})^2$$

Morse' semi-empirical approximate

$$U_{Morse} = E_M \left[ 1 - e^{-\alpha(r - r_{eq})} \right]^2 - E_M$$

$U_{eq}$   $E_M = U_{eq}$





**Two lead bricks moving in the  $+x$  and  $-x$  directions, each with kinetic energy  $K$ , smash into each other and come to a stop. What happened to the energy?**

- 1) The observable kinetic energy changed to thermal energy, a form of rest energy.**
- 2) The total energy of the system decreased by an amount  $2K$ .**
- 3) Since the blocks were moving in opposite directions, the initial kinetic energy of the system was zero, so there was no change in energy.**