Wed. Fri.	6.8-.10, 12,13 Introducing Potential Energy, Gravitational 6.11, 14-17 Visualizing Electric and Rest Energy	RE 6.c RE 6.d,e
Mon. Tues.	Things Engineers and Physicists Do	 EP6, HW6: Ch 6 Pr's 58, 59, 91, 99(a-c), 105(a-c)

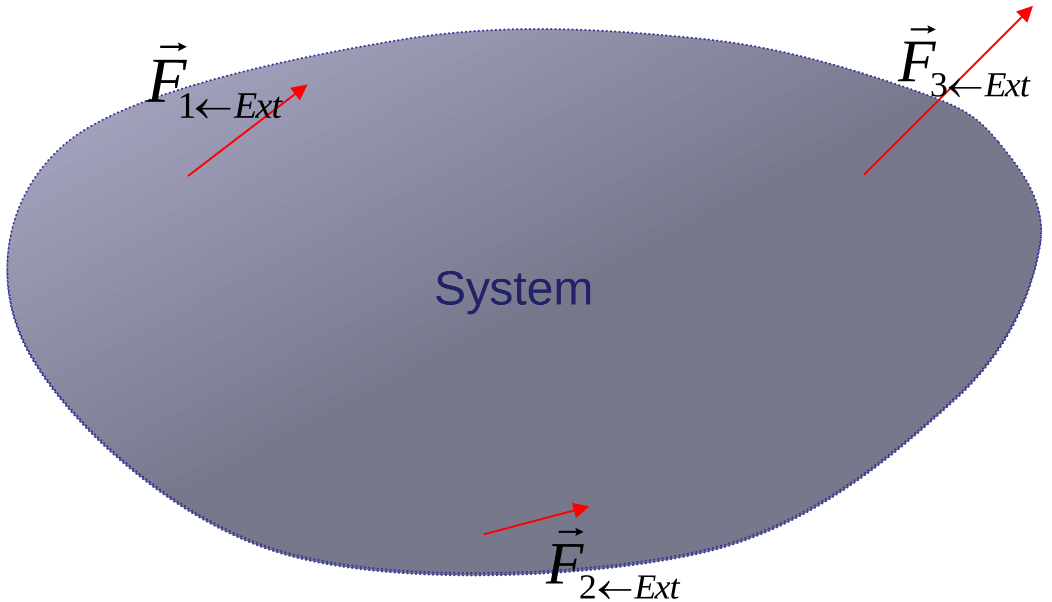
motion is neither created nor destroyed, but transferred via interactions.

Energy → $$\gamma_f mc^2 - \gamma_i mc^2 = \sum_{all}\left(\int_i^f \vec{F}_{\rightarrow sys}\cdot d\vec{r}_{sys}\right)$$ ← *Work*

$$\Delta E = W$$

Accounting for Interactions internal *to the system – change in* potential *energy.*

Energy and internal interactions



$$\Delta E_1 = \vec{F}_{1\leftarrow net}\cdot\Delta\vec{r}_1 = \left(\vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} + \vec{F}_{1\leftarrow ext}\right)\cdot\Delta\vec{r}_1 = W_{1\leftarrow 2} + W_{1\leftarrow 3} + W_{1\leftarrow ext}$$

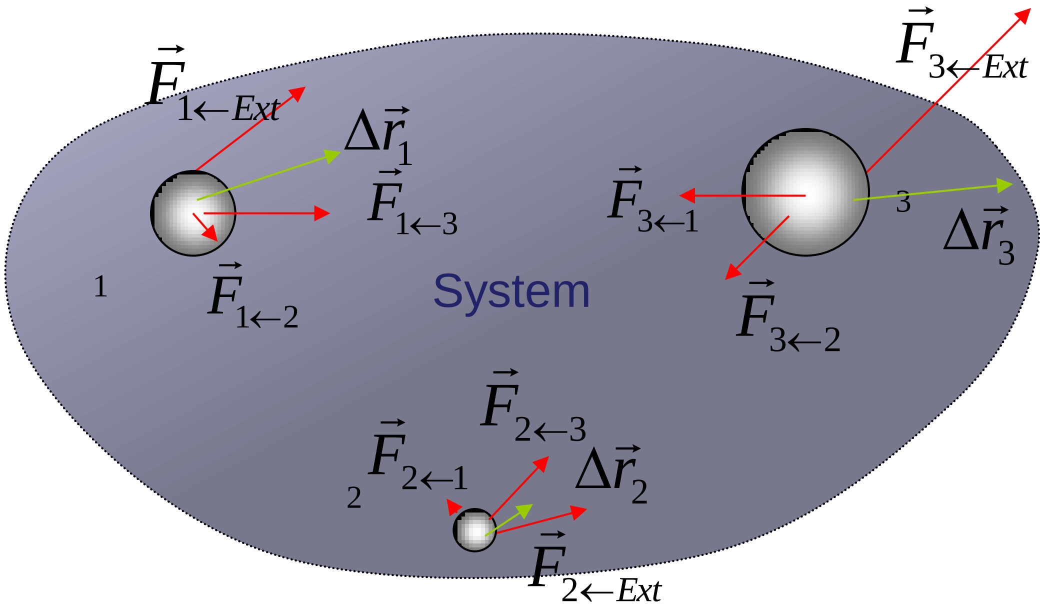
$$\Delta E_2 = \vec{F}_{2\leftarrow net}\cdot\Delta\vec{r}_2 = \left(\vec{F}_{2\leftarrow 1} + \vec{F}_{2\leftarrow 3} + \vec{F}_{2\leftarrow ext}\right)\cdot\Delta\vec{r}_2 = W_{2\leftarrow 1} + W_{2\leftarrow 3} + W_{2\leftarrow ext}$$

$$\Delta E_2 = \vec{F}_{3\leftarrow net}\cdot\Delta\vec{r}_3 = \left(\vec{F}_{3\leftarrow 1} + \vec{F}_{3\leftarrow 2} + \vec{F}_{3\leftarrow ext}\right)\cdot\Delta\vec{r}_3 = W_{3\leftarrow 1} + W_{3\leftarrow 2} + W_{3\leftarrow ext}$$

$$\Delta\left(E_1 + E_2 + E_3\right) = \left(W_{1\leftarrow 2} + W_{2\leftarrow 1}\right) + \left(W_{1\leftarrow 3} + W_{3\leftarrow 1}\right) + \left(W_{2\leftarrow 3} + W_{3\leftarrow 2}\right) + W_{1\leftarrow ext} + W_{2\leftarrow ext} + W_{3\leftarrow ext}$$

$$\Delta\left(E_1 + E_2 + E_3\right) = W_{\text{internal}} + W_{system\leftarrow ext}$$

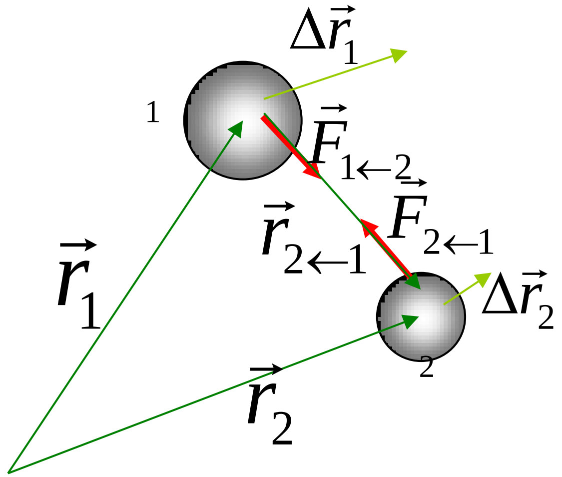
Energy and internal interactions



$$W_{\text{internal}} = \left(W_{1\leftarrow 2} + W_{2\leftarrow 1}\right) + \left(W_{1\leftarrow 3} + W_{3\leftarrow 1}\right) + \left(W_{2\leftarrow 3} + W_{3\leftarrow 2}\right)$$

Unlike Internal Forces, Internal Work's don't necessarily cancel.
For Example…

Work on/by Pair of Particles



$$W_{1,2} = W_{1\leftarrow 2} + W_{2\leftarrow 1}$$

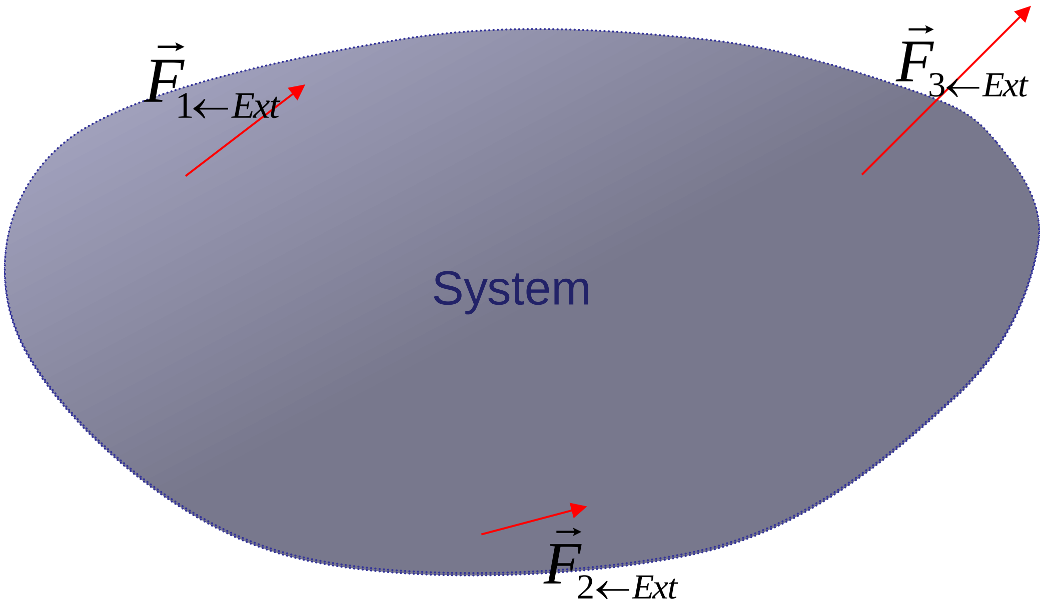
$$W_{1,2} = \left(\vec{F}_{1\leftarrow 2}\cdot\Delta\vec{r}_1 + \vec{F}_{2\leftarrow 1}\cdot\Delta\vec{r}_2\right)$$

By Reciprocity $\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1}$

$$W_{1,2} = \left(\vec{F}_{1\leftarrow 2}\cdot\Delta\vec{r}_1 - \vec{F}_{1\leftarrow 2}\cdot\Delta\vec{r}_2\right) = \vec{F}_{1\leftarrow 2}\cdot\left(\Delta\vec{r}_1 - \Delta\vec{r}_2\right)$$

$$W_{1,2} = \vec{F}_{1\leftarrow 2}\cdot\Delta\vec{r}_{1\leftarrow 2}$$

Energy and internal interactions



$$\Delta\left(E_1+E_2+E_3\right)=W_{\text{internal}}+W_{system\leftarrow ext}$$

$$\Delta\left(E_1+E_2+E_3\right)-W_{\text{internal}}=W_{system\leftarrow ext}$$

Potential Energy

$$\Delta U_{system}\equiv -W_{\text{internal}}$$

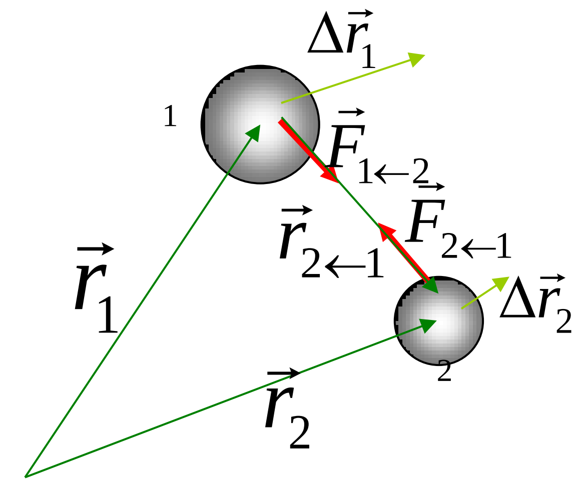
$$\Delta\left(\underbrace{E_1+E_2+E_3+U_{system}}_{E_{system}}\right)=W_{system\leftarrow ext}$$

$$E_{system}=W_{system\leftarrow ext}$$

Say we have an isolated system, so there are no external interactions. If the sum of particle energies increases by 50 J, what must be the change in the system's potential energy?

Potential Energy Change of Pair of Particles

Work by

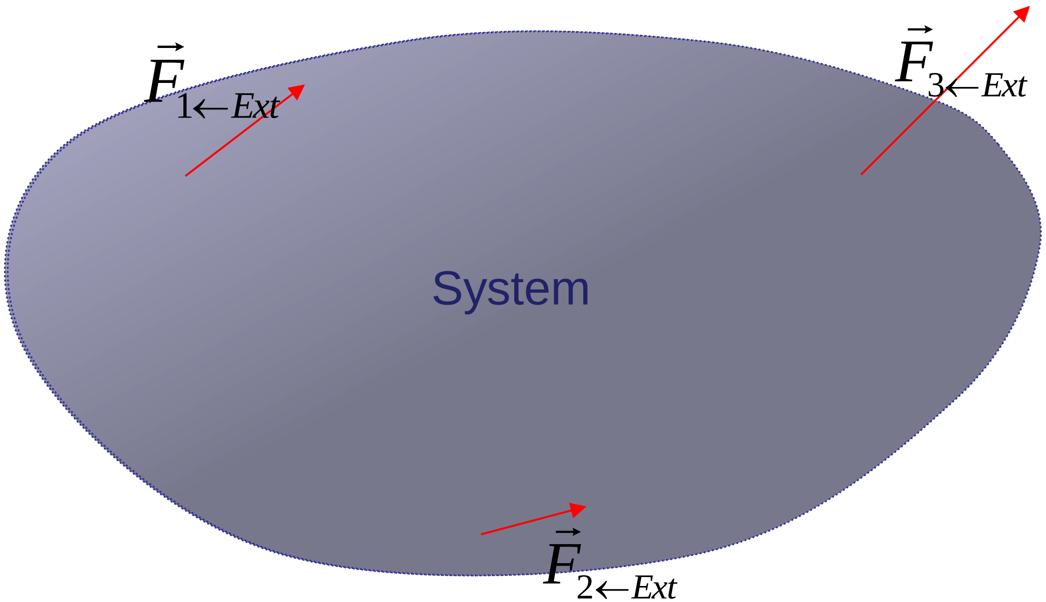


$$W_{1,2} = \vec{F}_{1\leftarrow 2} \cdot \Delta\vec{r}_{1\leftarrow 2}$$

$$\Delta U_{1,2} = -\vec{F}_{1\leftarrow 2} \cdot \Delta\vec{r}_{1\leftarrow 2}$$

Potential Energy is *shared* by members of a system

Energy and internal interactions



$$\Delta\left(E_1 + E_2 + E_3 + U_{system}\right) = W_{system\leftarrow ext}$$

$$\Delta U_{system} = \Delta U_{1,2} + \Delta U_{2,3} + \Delta U_{3,1}$$

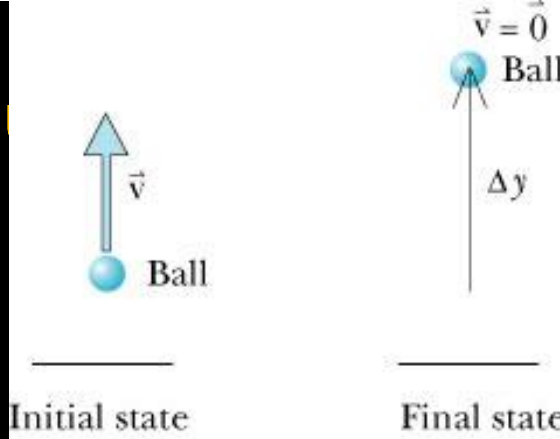
$$E_{system} = E_1 + E_2 + E_3 + U_{1,2} + U_{1,3} + U_{2,3}$$

Say you have a system of 4 particles; in the system's total energy expression, how may pair-wise potential energy terms (like $U_{1,2}$) are there?
(it may help to write them out)

a) 2
b) 4
c) 6
d) 8
e) 10
f) 12
g) 14
h) 16

Write the energy of a system containing four particles, including the relativistic particle energies E_1 etc. plus the potential energy pairs U_{12} etc.

A spacecraft travels from near the Earth toward the Moon. How many gravitational potential energy terms U_g are there in the Energy Principle? System: Earth, Moon, spacecraft	a) 1 b) 2 c) 3 d) 6 e) 0

A thrown ball heads straight SYSTEM: Ball 	What is the *work done by th* *surroundings*? 1) 0 2) mgΔy 3) –mgΔy 4) something else
A thrown ball heads straight up. SYSTEM: Ball + Earth	What is the *work done by th* *surroundings*? 1) 0 2) mgΔy 3) –mgΔy 4) something else
A thrown ball heads straight up. SYSTEM: Ball + Earth	What is the *change in* *potential energy*? 1) 0 2) mgΔy 3) –mgΔy 4) something else

Earth & Ball Revisited

You drop a metal ball from 1 m up. How fast is it going just before it hits the ground?

$$\Delta E = W_{system \leftarrow ext} = 0$$

System= ball+Earth

Active members of environment = none

(neglecting air's resistance)

Not changing

$$\Delta E_{E,b} = \Delta E_{rest.E} + \Delta E_{rest.b} + \Delta K_E + \Delta K_b + \Delta U_{E,b} = 0$$

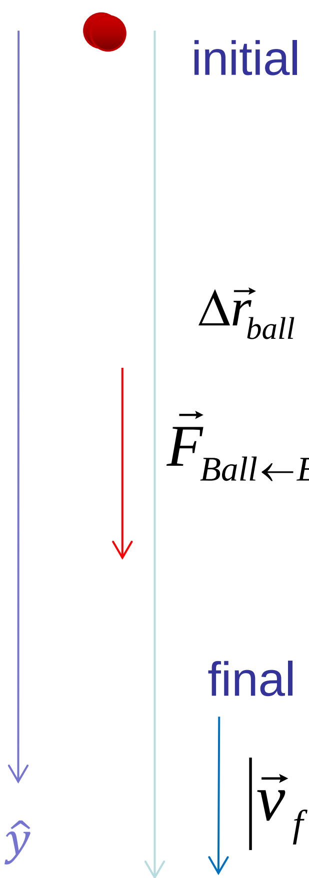
$$\Delta U_{E,b} = -\int \vec{F}_{b \leftarrow E} \cdot d\vec{r}_{b \leftarrow E}$$

Constant force, so…

$$\Delta U_{E,b} = -\vec{F}_{b \leftarrow E} \cdot \Delta \vec{r}_{b \leftarrow E}$$

$$\Delta U_{E,b} = -mg\Delta y$$

Pause and Consider: does sign make sense?



$$\Delta \vec{r}_{ball} = (\Delta y)\hat{y}$$

$$\Delta y = 1\text{m}$$

$$\vec{F}_{Ball \leftarrow Earth} = mg\hat{y}$$

$$|\vec{v}_f| = ?$$

$$\Delta K_b = K_{b.f} - K_{b.i}$$

Pretty sure *v*<<*c*, so $K \approx \frac{1}{2}mv^2$

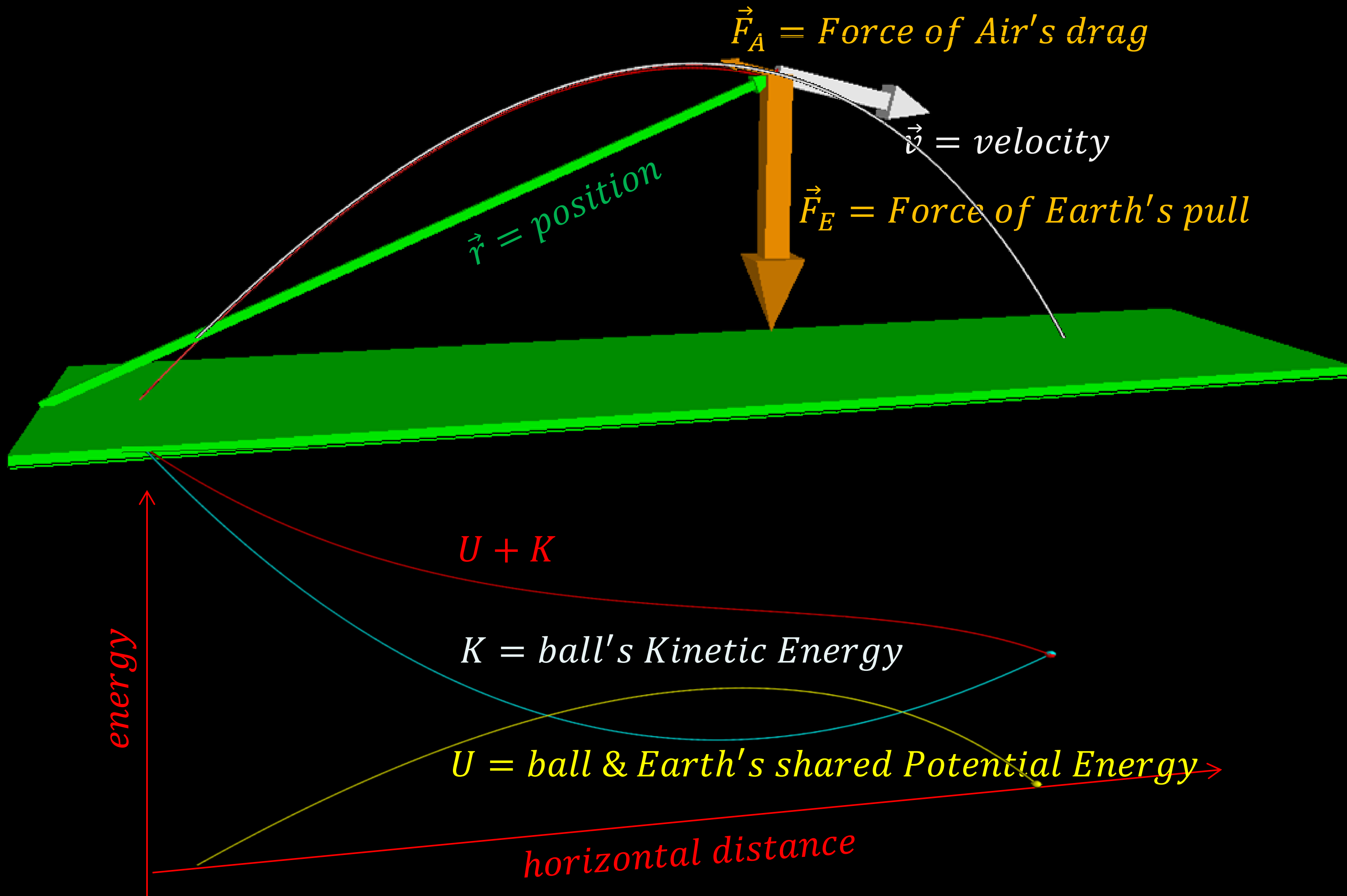
$$\Delta E_{E,b} = \Delta K_b + \Delta U_{E,b} = 0$$

$$\tfrac{1}{2}mv_f^2 - mg\Delta y = 0$$

$$\tfrac{1}{2}mv_f^2 = mg\Delta y$$

$$v_f = \sqrt{2g\Delta y} = \sqrt{2\left(9.8\,{}^{m}\!/\!_{s^2}\right)(1m)} = 4.4\,{}^{m}\!/\!_{s}$$

Visualizing Energy



Thinking about Potential Energy

Have vs. Do

Work is something you *do*

Potential is something you *have*

Example: Work-Study Contract

Energy of Configuration

The Physics is in the *change* in Potential

A ball of mass 0.1 kg is dropped from rest near the Earth.
The ball travels downward 2 m, speeding up.

SYSTEM: Ball

What is the work done by the surroundings?

a) 0
b) + 1.96 J
c) – 1.96 J

A ball of mass 0.1 kg is dropped from rest near the Earth.
The ball travels downward 2 m, speeding up.

SYSTEM: Ball + Earth

What is the work done by the surroundings?

a) 0
b) + 1.96 J
c) – 1.96 J

A cart on a track connected by a string, over a pulley to a weight that hangs off the edge of the table.

SYSTEM: cart + weight + Earth

How many energy terms (rest, kinetic, potential) are in the initial work-energy relation? Include ones that probably aren't changing.

a) 2
b) 3
c) 4
d) 5
e) 6
f) 7
g) 8
h) 9
i) 10

Fun with near-Earth Gravitation

System= Earth + cart + weight

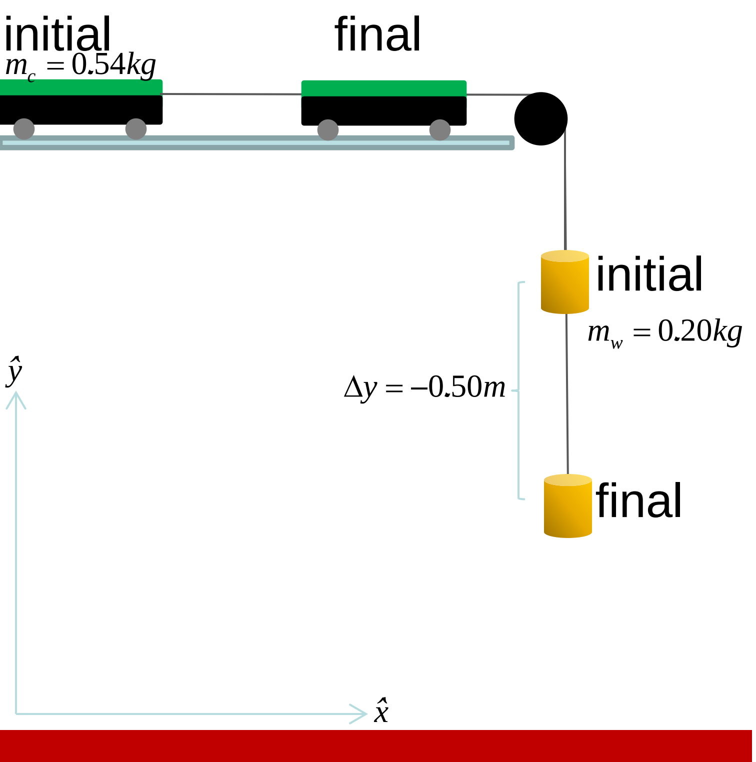
Active surrounds = negligible (ignoring friction and energy invested in spinning up the pulley)

If it starts from rest, how quickly is the 0.54-kg cart moving once the 0.20-kg weight has fallen 0.50m?

$$\Delta E_{E,b} = W_{E,b\leftarrow surroundings} = 0$$

$$\cancel{\Delta E_{rest.E} + \Delta E_{rest.c} + \Delta E_{rest.w}} + \cancel{\Delta K_E} + \Delta K_c + \Delta K_w + \cancel{\Delta U_{E,c}} + \Delta U_{E,w} + \cancel{\Delta U_{c,w}} = 0$$

$$\left(\tfrac{1}{2} m_c v_f^2 - \tfrac{1}{2} m_c v_i^2\right) + \left(\tfrac{1}{2} m_w v_f^2 - \tfrac{1}{2} m_w v_i^2\right) + m_w g(\Delta y) = 0$$



$$\left(\tfrac{1}{2}(m_c + m_w)v_f^2 - \tfrac{1}{2}(m_c + m_w)\cancel{v_i^2}\right) + m_w g(\Delta y) = 0$$

$$v_f^2 + \frac{2m_w}{m_c + m_w} g(\Delta y) = 0$$

Pause:
reasonable dependences?

$$v_f = \sqrt{\frac{2m_w}{m_c + m_w} g(-\Delta y)}$$

check:
units work?

$$v_f = \sqrt{\frac{2(0.2kg)}{0.54kg + 0.2kg}(9.8\,{}^m\!/\!_{s^2})(0.5m)} = 1.6\,{}^m\!/\!_s$$

Fun with near-Earth Gravitation

System= Earth + ball

Active surrounds = negligible

$$\Delta E_{E,b} = W_{E,b \leftarrow surroundings} = 0$$

$$\Delta E_{rest.E} + \Delta E_{rest.b} + \Delta K_E + \Delta K_b + \Delta U_{E,b} = 0$$

$$\tfrac{1}{2} m_b v_{b.f}^2 - \tfrac{1}{2} m_b v_{b.i}^2 + m_b g\left(y_f - y_i\right) = 0$$

$$v_{b.f}^2 - v_{b.i}^2 + 2g\left(y_f - y_i\right) = 0$$

a) If it is thrown and caught at the same elevation, how are the initial and final speeds related?

$$v_{b.f}^2 - v_{b.i}^2 = 0$$ They're the same.

b) Say at a certain instant, the ball is moving 6 m/s and at an elevation of 10 m. How fast is it going when it is at 5 m?

$$v_{b.f} = \sqrt{v_{b.i}^2 - 2g\left(y_f - y_i\right)}$$

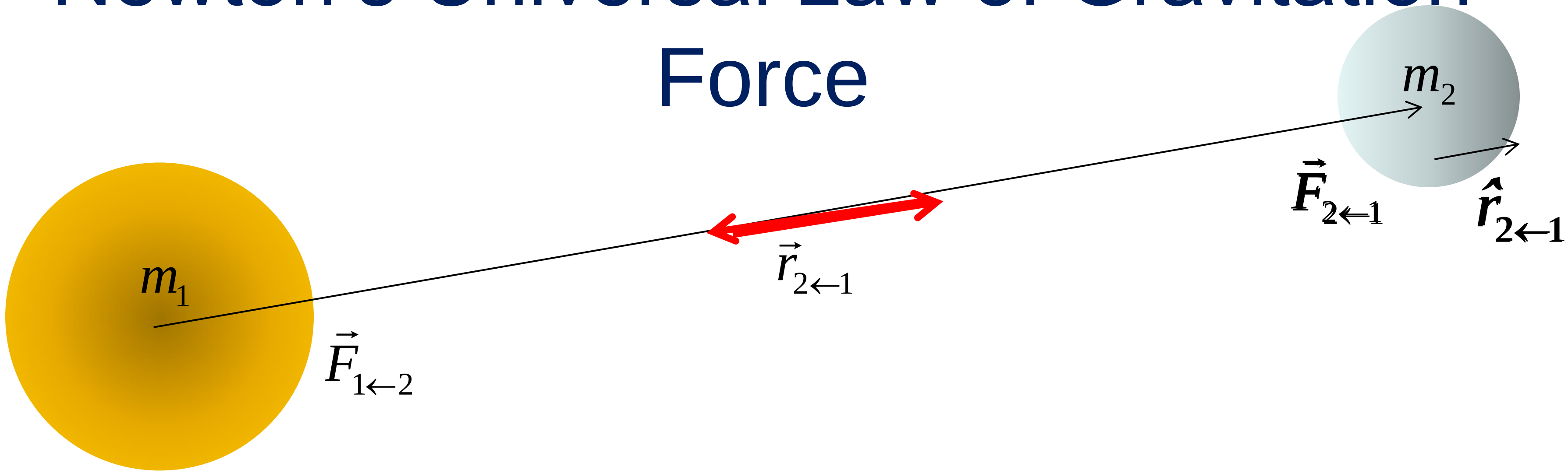
$$v_{b.f} = \sqrt{(6m/s)^2 - 2\left(9.8m/s^2\right)\left(5m - 10m\right)} = 18.2m/s$$

c) If it were thrown straight up and it peaked at around 5.1m above the launch height (in the ball park of what happened in lab), what must have been it's launch / initial speed?

$$v_{b.f}^2 - v_{b.i}^2 + 2g\left(y_f - y_i\right) = 0$$

$$v_{b.i} = \sqrt{2g\left(y_f - y_i\right)} = \sqrt{2\left(9.8m/s^2\right)\left(5.1m\right)} = 10\,{}^{m}\!/\!{}_{s}$$

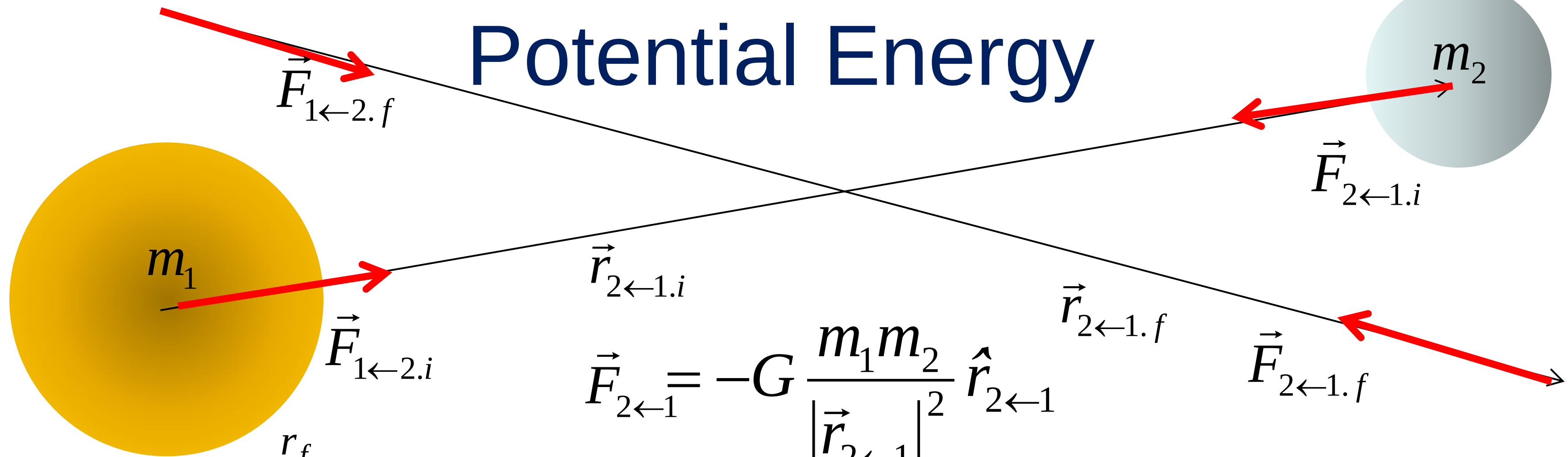
Newton's Universal Law of Gravitation Force



$$= - \quad \cdot \hat{r}_{2\leftarrow 1}$$

$$G = 6.67\times 10^{-11}\,\frac{N\cdot m^2}{(kg)^2} \qquad = \frac{\vec{r}_{2\leftarrow 1}}{\left|\vec{r}_{2\leftarrow 1}\right|}$$

Newton's Universal Law of Gravitation Potential Energy



$$\vec{F}_{2\leftarrow 1} = -G\frac{m_1 m_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}$$

$$\Delta U_{1,2} = -\int_{r_{1\leftarrow 2}=r_i}^{r_f} \vec{F}_{1\leftarrow 2}\cdot d\vec{r}_{1\leftarrow 2}$$

$$\Delta U_{1,2} = -\int_{r_{1\leftarrow 2}=r_i}^{r_f} \left(-G\frac{m_1 m_2}{r_{1\leftarrow 2}^2}\hat{r}_{1\leftarrow 2}\right)\cdot d\vec{r}_{1\leftarrow 2} = G m_1 m_2 \int_{r_{1\leftarrow 2}=r_i}^{r_f} \frac{dr_{1\leftarrow 2}}{r_{1\leftarrow 2}^2}$$

$$\Delta U_{1,2} = \left(-G\frac{m_1 m_2}{\left|r_{1\leftarrow 2.f}\right|}\right) - \left(-G\frac{m_1 m_2}{\left|r_{1\leftarrow 2.i}\right|}\right)$$

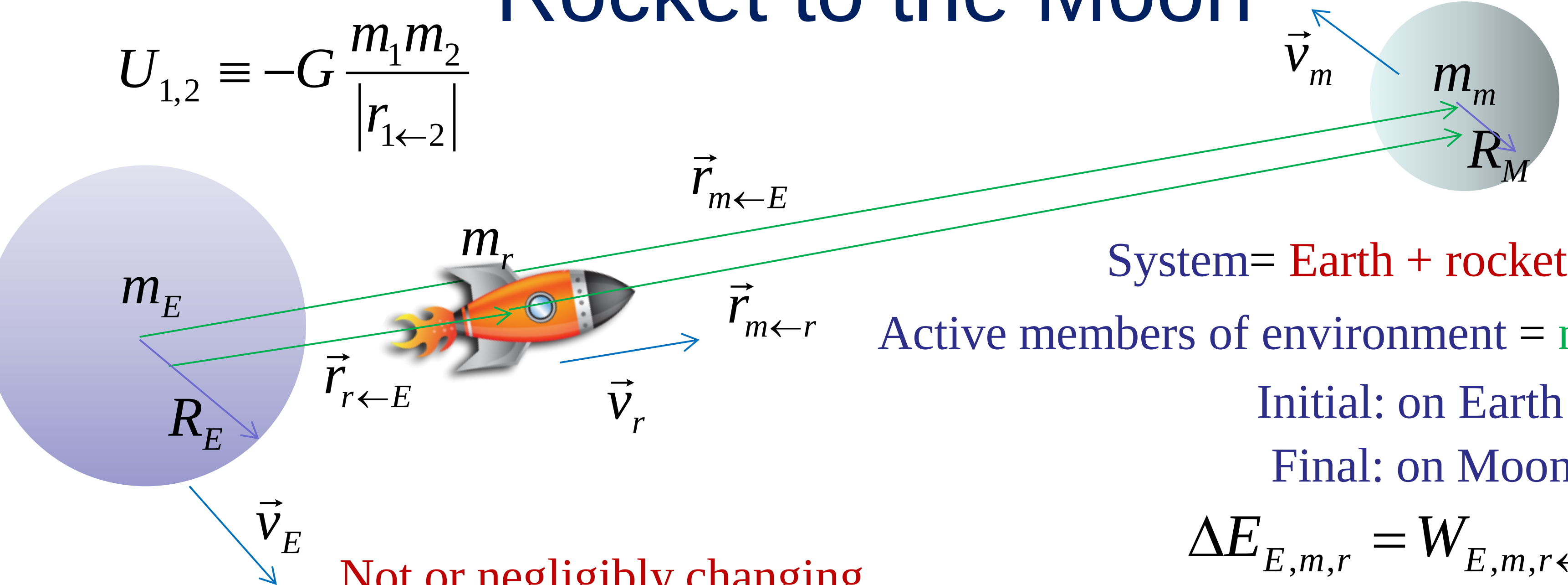
At any particular separation

$$U_{1,2} \equiv -G\frac{m_1 m_2}{\left|r_{1\leftarrow 2}\right|}$$

Warning: the physics is in the *change* not the *value*

Rocket to the Moon

$$U_{1,2} \equiv -G\frac{m_1 m_2}{\left|r_{1\leftarrow 2}\right|}$$



System= Earth + rocket + Moon

Active members of environment = negligible

Initial: on Earth Surface

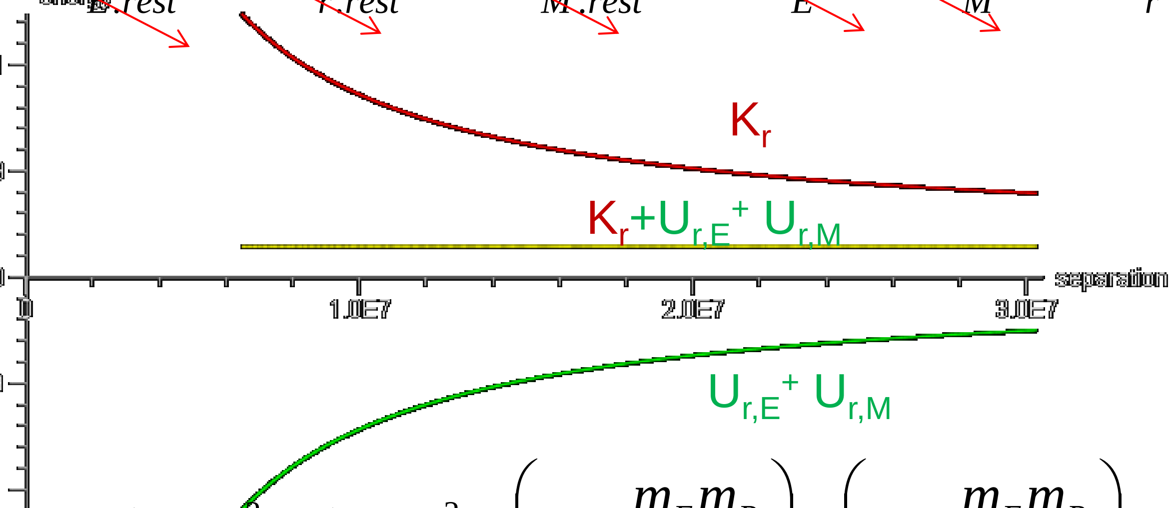
Final: on Moon Surface

$$\Delta E_{E,m,r} = W_{E,m,r\leftarrow ext} = 0$$

Not or negligibly changing

$$\Delta E_{E,rest} + \Delta E_{r,rest} + \Delta E_{M,rest} + \Delta K_E + \Delta K_M + \Delta K_r + \Delta U_{E,r} + \Delta U_{E,M} + \Delta U_{M,r} = 0$$

$$\Delta K_r + \Delta U_{E,r} + \Delta U_{M,r} = 0$$

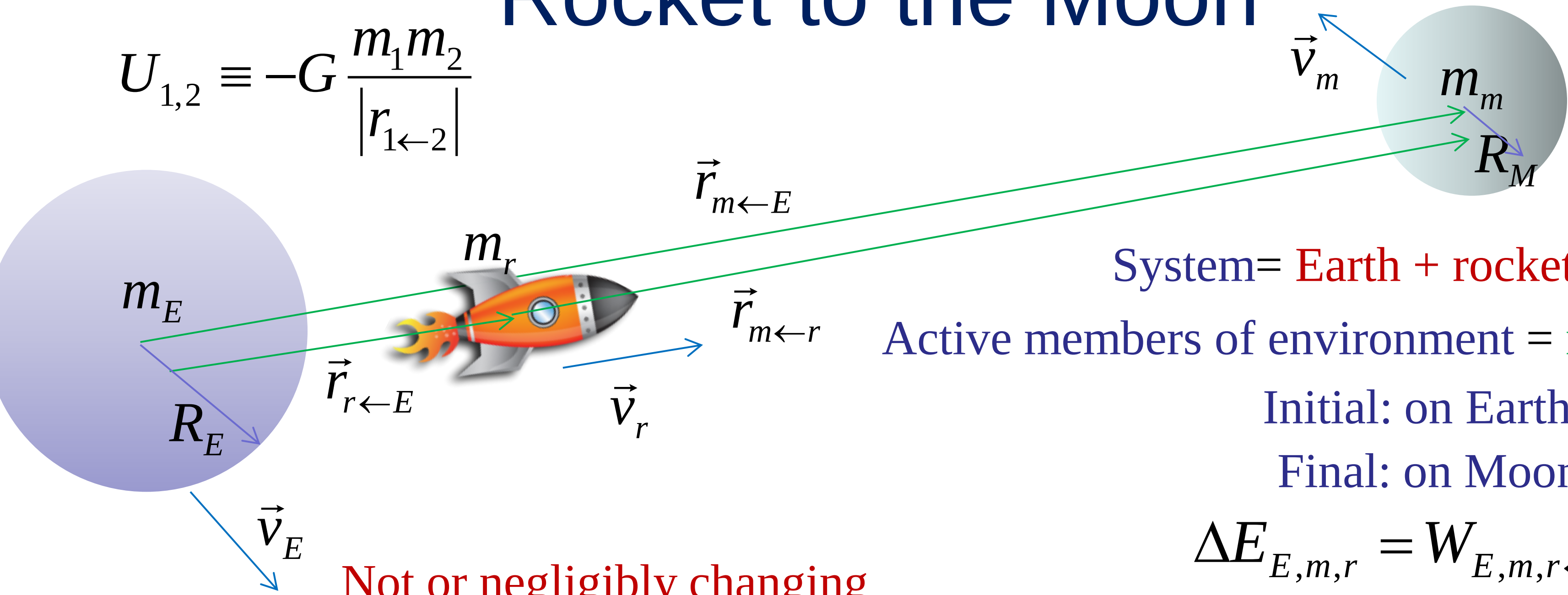


04_02.py

$$\tfrac{1}{2}m_R v_f^2 - \tfrac{1}{2}m_R v_i^2 + \left(-G\frac{m_E m_R}{R_{E-M}}\right) - \left(-G\frac{m_E m_R}{R_E}\right) + \left(-G\frac{m_M m_R}{R_M}\right) - \left(-G\frac{m_M m_R}{R_{M-E}}\right) = 0$$

Rocket to the Moon

$$U_{1,2} \equiv -G\frac{m_1 m_2}{\left|r_{1\leftarrow 2}\right|}$$



System= Earth + rocket + Moon

Active members of environment = negligible

Initial: on Earth Surface

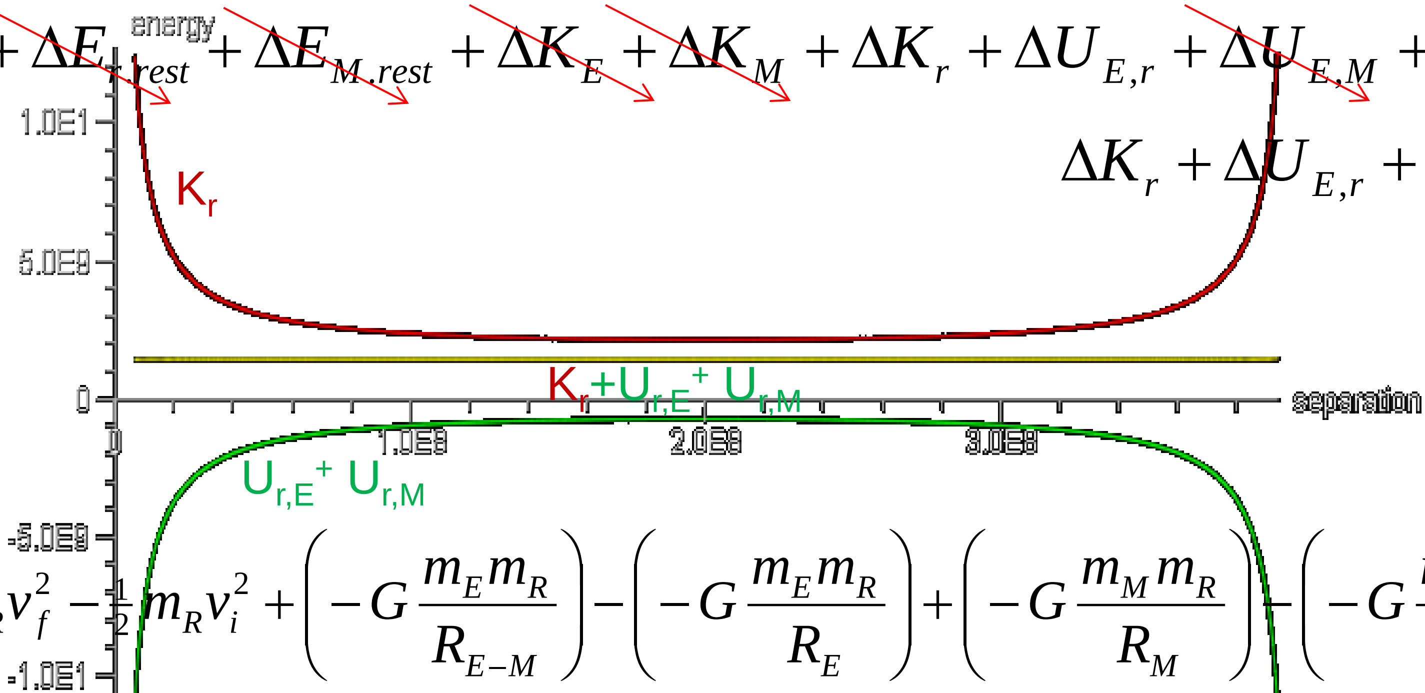
Final: on Moon Surface

$$\Delta E_{E,m,r} = W_{E,m,r\leftarrow ext} = 0$$

Not or negligibly changing

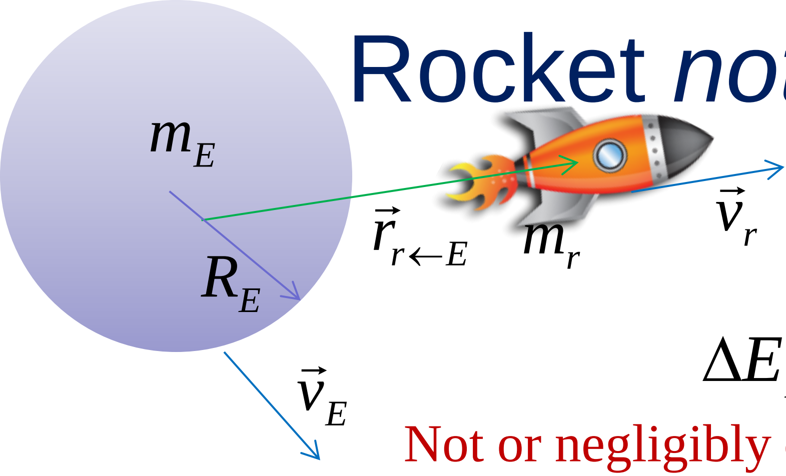
$$\Delta E_{E,rest} + \Delta E_{r,rest} + \Delta E_{M,rest} + \Delta K_E + \Delta K_M + \Delta K_r + \Delta U_{E,r} + \Delta U_{E,M} + \Delta U_{M,r} = 0$$

$$\Delta K_r + \Delta U_{E,r} + \Delta U_{M,r} = 0$$



$$\tfrac{1}{2} m_R v_f^2 - \tfrac{1}{2} m_R v_i^2 + \left(-G\frac{m_E m_R}{R_{E-M}}\right) - \left(-G\frac{m_E m_R}{R_E}\right) + \left(-G\frac{m_M m_R}{R_M}\right) - \left(-G\frac{m_M m_R}{R_{M-E}}\right) = 0$$

Rocket *not quite* to the Moon



System= Earth + rocket

Active members of environment = negligible

Initial: on Earth Surface

Final: …Stalled

$$\Delta E_{E,r} = W_{E,r\leftarrow ext} = 0$$

Not or negligibly changing

$$\Delta E_{E.rest} + \Delta E_{r.rest} + \Delta K_E + \Delta K_r + \Delta U_{E,r} = 0$$

$$\Delta K_r + \Delta U_{E,r} = 0$$

$$\left(K_{r.f} - K_{r.i}\right) + \left(U_{E,r.f} - U_{E,r.i}\right) = 0$$

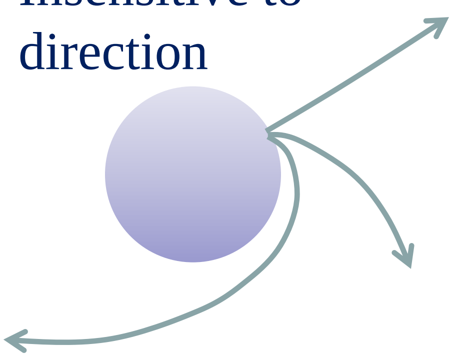
$K_r + U_{r,E}$ remains constant

$$\left(K_{r.i} + U_{E,r.i}\right) = \left(K_{r.f} + U_{E,r.f}\right)$$

$$\left(\tfrac{1}{2} m_R v_i^2 - G\frac{m_E m_R}{R_E}\right) = \left(\tfrac{1}{2} m_R v_f^2 - G\frac{m_E m_R}{r_f}\right)$$

$$\left(\tfrac{1}{2} m_R v_i^2 - G\frac{m_E m_R}{R_E}\right) = \left(-G\frac{m_E m_R}{r_f}\right)$$

Insensitive to direction



Escape Speed

$$v_i = \sqrt{\frac{2Gm_E}{R_E} - \frac{2Gm_E}{r_f}} \qquad \lim_{r_f \to \infty} \quad v_{escape} \equiv \sqrt{\frac{2Gm_E}{R_E}}$$

A huge asteroid smacks into the leading edge of the Earth – stopping the Earth's orbit. Subsequently, the Earth falls straight into the sun! With what speed would the Earth hit the Sun's surface?

m_E

$\vec{v}_{E.i} = 0$

$r_{E\leftarrow S.i} = 1.5\times 10^{11} m$

$\vec{v}_{E.f} = ?$

$m_S = 1.99\times 10^{30} kg$

$r_{E\leftarrow S.f} = R_E + R_S$

$= 7.02\times 10^{8} m$

System= Earth + Sun

Active environment = none

$$\Delta E = W_{system\leftarrow ext} = 0$$

Not changing

$$\Delta E_{E,S} = \Delta E_{rest.E} + \Delta E_{rest.S} + \Delta K_E + \Delta K_S + \Delta U_{E,S} = 0$$

$$\Delta E_{E,S} = \Delta K_E + \Delta U_{E,S} = 0$$

$$\Delta K_E = K_{E.f} - K_{E.i} = \tfrac{1}{2} m_E v_{E.f}^2 \quad \Delta U_{ES} = \Delta\left(-G\frac{m_E m_s}{r_{ES}}\right) = \left(-G\frac{m_E m_s}{r_{ES.f}}\right) - \left(-G\frac{m_E m_s}{r_{ES.i}}\right)$$

$$\Delta E_{E,S} = \tfrac{1}{2} m_E v_{E.f}^2 - G\frac{m_E m_s}{r_{ESf}} + G\frac{m_E m_s}{r_{ESi}} = 0$$

$$v_{E.f} = \sqrt{2Gm_s\left(\frac{1}{r_{ESf}} - \frac{1}{r_{ESi}}\right)}$$

$$= \sqrt{2\left(6.67\times 10^{-11} Nm^2/kg^2\right)\left(1.99\times 10^{30} kg\right)\left(\frac{1}{7.02\times 10^{8} m} - \frac{1}{1.5\times 10^{11} m}\right)} = 6.14\times 10^{5} m/s$$

System: comet + star

As a comet travels away from a star, how does the kinetic energy and potential energy of the system change?

K	U
a) increase	decrease
b) increase	increase
c) decrease	increase
d) decrease	decrease
e) no change	no change

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Mon.	Things Engineers and Physicists Do	
Tues.		EP6, HW6: Ch 6 Pr's 58, 59, 91, 99(a-c), 105(a-c)

motion is neither created nor destroyed, but transferred via interactions.

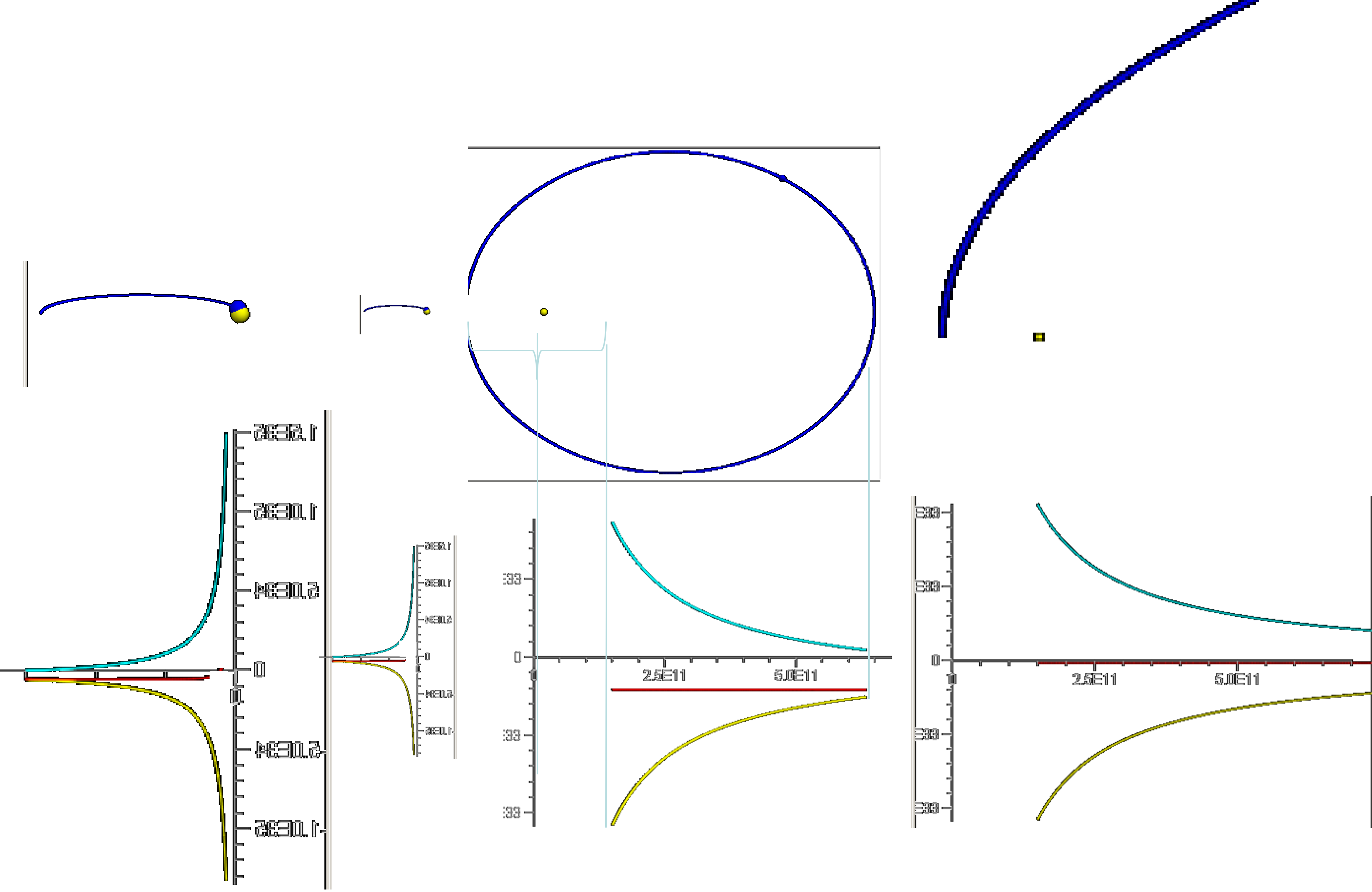
Energy → $$\gamma_f mc^2 - \gamma_i mc^2 = \sum_{all}\left(\int_i^f \vec{F}_{\to sys}\cdot d\vec{r}_{sys}\right)$$ ← *Work*

$$\Delta E = W$$

Accounting for Interactions internal *to the system – change in* potential *energy.*

Different Initial Speeds / kinetic Energies, Different Paths

(orbit noncircular, with energy vs position.py)



04_potential_energy_well.py