Wed.	6.1-.4 (.21) Introducing Energy & Work Quiz 5	RE 6.a
Lab	L5: Buoyancy, Circles & Pendulums	laptop
Fri.,	6.5-.7 (.22) Rest Mass,Work by Changing Forces Columbia Rep 3pm, here	RE 6.b (*last day to drop*)
Wed.	6.8-.9(.18, .19) Introducing Potential Energy …	RE 6.c
Tues.		HW6: Ch 6 Pr's 58,59, 99(a-c), 105(a-c)

motion is neither created nor destroyed, but transferred via interactions.

Momentum

$$\vec{p} \equiv \frac{m\vec{v}}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

Impulse

$$d\vec{p}_{sys} = \sum_{all} \vec{F}_{\rightarrow sys} dt$$

$$\frac{d\vec{p}_{sys}}{dt} = \sum_{all} \vec{F}_{\rightarrow sys}$$

Energy

$$\int_i^f \frac{d\vec{p}_{sys}}{dt} \cdot d\vec{r}_{sys} = \int_i^f \sum_{all} \vec{F}_{\rightarrow sys} \cdot d\vec{r}_{sys}$$

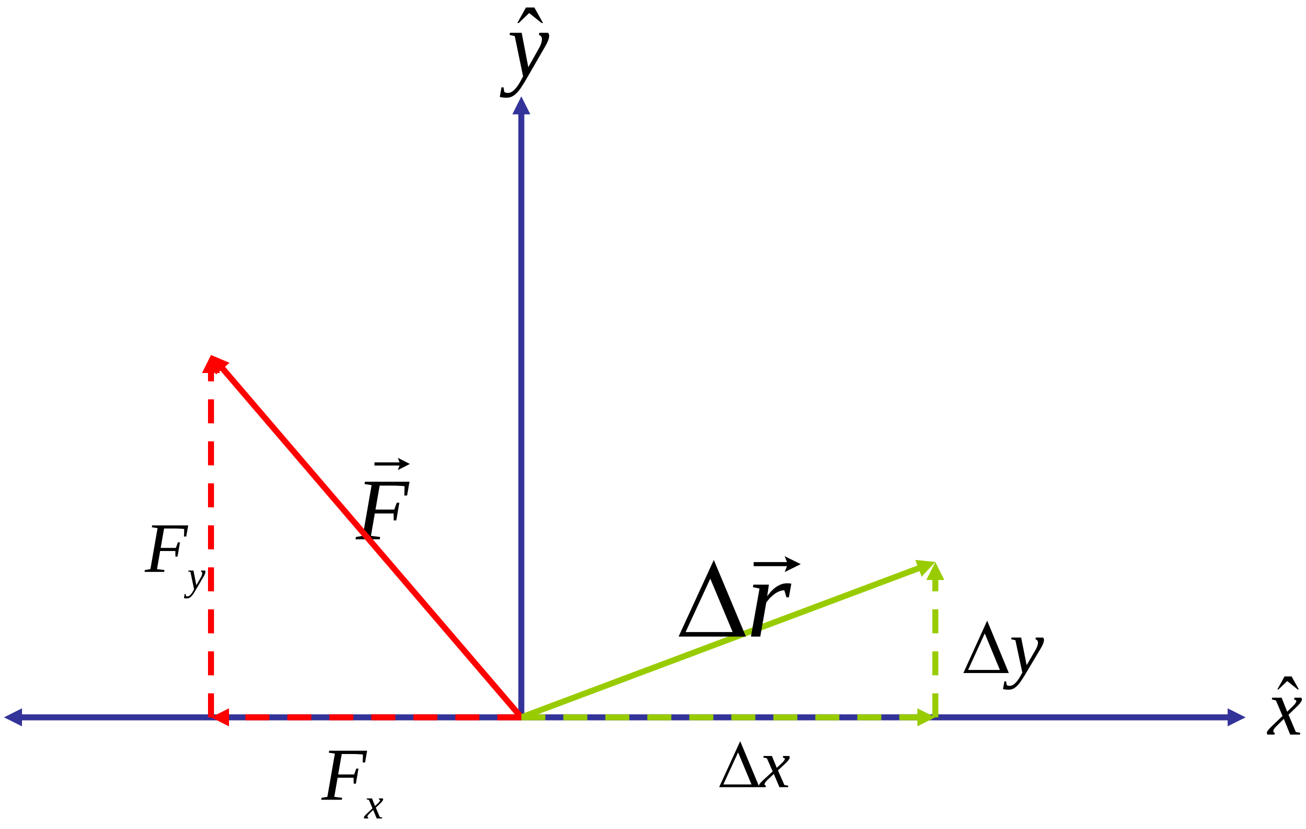
Work

$$\Delta E = W$$

Understanding Work:

Scalar or Dot Product as sum of products of components

$$W = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$



Understanding Work:

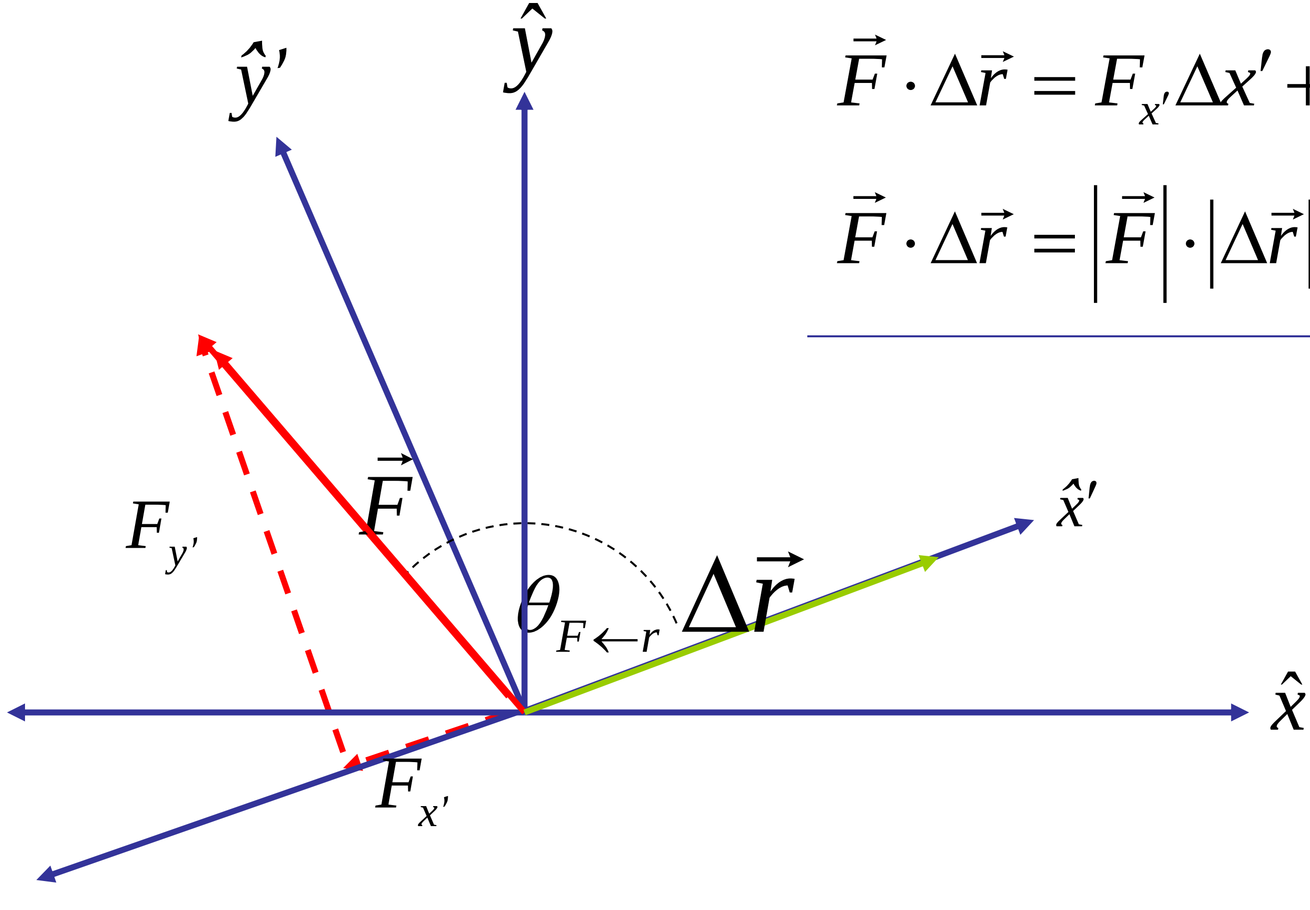
Scalar or Dot Product as projection of one vector onto another – product of magnitudes and cosine

$$W = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

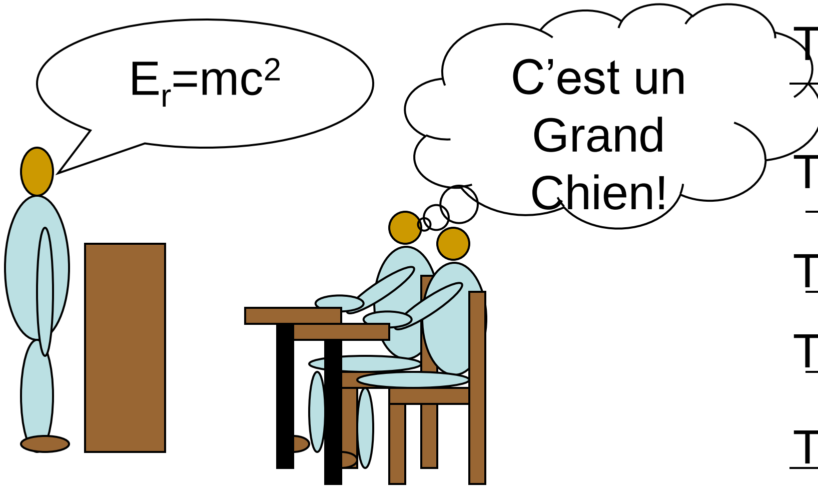
$$\vec{F} \cdot \Delta\vec{r} = F_{x'} \Delta x' + F_{y'} \Delta y' + F_{z'} \Delta z'$$

$$\vec{F} \cdot \Delta\vec{r} = F_{x'} \Delta x' + 0$$

$$\vec{F} \cdot \Delta\vec{r} = \left|\vec{F}\right| \cdot \left|\Delta\vec{r}\right| \cos\theta_{F \leftarrow r}$$



Understanding Work:
conceptual – non-physicist def



Teacher	Students	Work $_{S \leftarrow T}$
Teaches Physics	Learn Physics	
Teaches Physics	Learn Nothing	
Teaches Nothing	Learn Physics	
Teaches Physics	*Un*learn Physics	
Teaches Physics	Learn French	

Work is the product of *effort* and *achievement*

Both in the same direction, it's good (+) work

Both opposite directions, it's bad (-) work

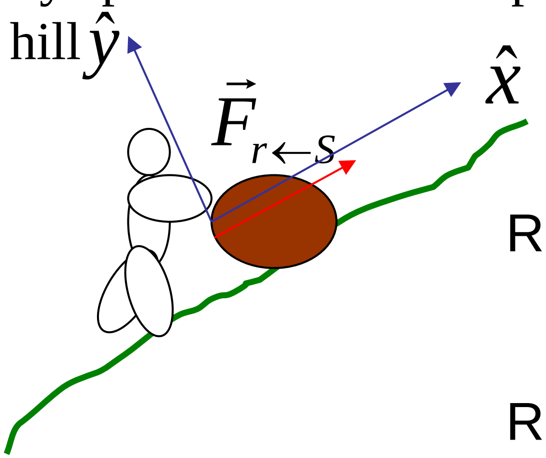
Both unrelated directions, it's no (0) work

One non-existent, it's no (0) work

Understanding Work:

conceptual –physicist def

Sysiphus Pushes rock up-hill



$$W_{r\leftarrow S} = \vec{F}_{r\leftarrow S} \bullet \Delta\vec{r}_r$$

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\Delta x_r + F_{y.r\leftarrow s}\Delta y_r + F_{z.r\leftarrow S}\Delta z_r$$

Rock goes up hill

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\Delta x_r + 0\cdot 0 + 0\cdot 0 > 0$$ + work

Rock stays put

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot 0 + 0\cdot 0 + 0\cdot 0 = 0$$ 0 work

Rock rolls down hill

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot\left(-\left|\Delta x\right|\right) + 0\cdot 0 + 0\cdot 0 < 0$$ - work

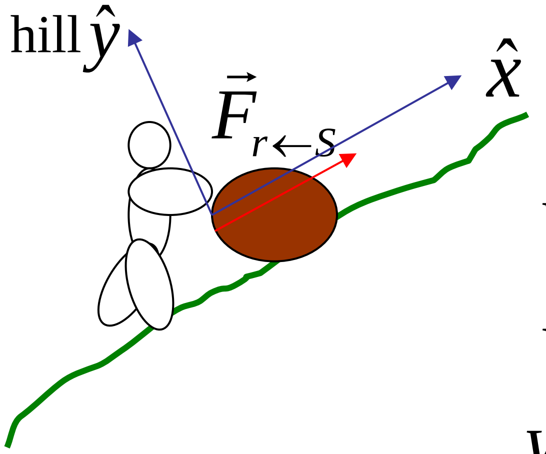
Rock slides sideways

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot 0 + 0\cdot 0 + 0\cdot \Delta z = 0$$ 0 work

Understanding Work:

conceptual –physicist def

Sysiphus Pushes rock up-hill



$$W_{r\leftarrow S} = \vec{F}_{r\leftarrow S} \bullet \Delta\vec{r}_r$$

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\Delta x_r + F_{y.r\leftarrow s}\Delta y_r + F_{z.r\leftarrow S}\Delta z_r$$

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\Delta x_r + 0\cdot 0 + 0\cdot 0 > 0$$ + work

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot 0 + 0\cdot 0 + 0\cdot 0 = 0$$ 0 work

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot\left(-\left|\Delta x\right|\right) + 0\cdot 0 + 0\cdot 0 < 0$$ - work

$$W_{r\leftarrow S} = F_{x.r\leftarrow S}\cdot 0 + 0\cdot 0 + 0\cdot\Delta z = 0$$ 0 work

Work is the product of *Force* and *Displacement*

Both in the same direction, it's good (+) work

Both opposite directions, it's bad (-) work

Both unrelated directions, it's no (0) work

One non-existent, it's no (0) work

Work by Constant Force: $W_{r\leftarrow S} = \vec{F}_{r\leftarrow S} \bullet \Delta\vec{r}_r = F_{x.r\leftarrow S}\Delta x_r + F_{y.r\leftarrow s}\Delta y_r + F_{z.r\leftarrow S}\Delta z_r$

On a space station, you pushed a box that was at rest at location < 0, 0, 10 > m to location < 0,0,14 > m, applying a force < 0, 0, 5 > N. How much work did you do on the box?

1) W = 20 J
2) W = 50 J
3) W = 70 J
4) W = 140 J
5) Not enough information

Assuming no other forces applied to the box What happened?

1) The box slowed down.
2) The box sped up.
3) The box moved at constant speed.

Next your partner pushed back on the moving box with a force < 0, 0, -3 > N. After the box had moved from < 0, 0, 14 > m to < 0, 0, 16 > m. What happened?

1) The box slowed down.
2) The box sped up.
3) The box moved at constant speed.

Work by Constant Force: $W_{r\leftarrow S} = \vec{F}_{r\leftarrow S} \bullet \Delta\vec{r}_r = F_{x.r\leftarrow S}\Delta x_r + F_{y.r\leftarrow s}\Delta y_r + F_{z.r\leftarrow S}\Delta z_r$

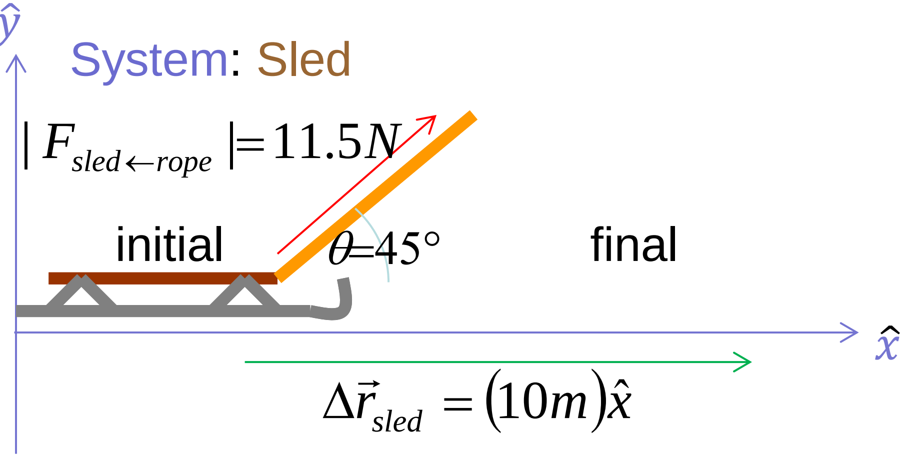
You move an object from < 3, 7, 4 > m to < 2, 10, 12 > m, applying a force < 10, -20, 30 > N. How much work do you do?

1) 10 J
2) 170 J
3) < -10, -60, 240 > J
4) < 30, -140, 120 > J
5) $\sqrt{(-10)^2 + (-20)^2 + (30)^2}$ J

Work Example: Work with Angle

A kid pulls a sled at a constant speed by a rope at a 45° angle across ice for 10 m. If she pulls with an 11.5 N force, How much work does she do on the sled?

System: Sled



$$W_{sled\leftarrow rope} = \int_i^f \vec{F}_{sled\leftarrow rope}\cdot d\vec{r}_{sled}$$

$$W_{sled\leftarrow rope} = \vec{F}_{sled\leftarrow rope}\cdot \Delta\vec{r}_{sled}$$

Recall

$$W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z$$

or

$$\vec{F}\cdot\Delta\vec{r} = |\vec{F}|\cdot|\Delta\vec{r}|\cos\theta_{F\leftarrow r}$$

$$W_{sled\leftarrow rope} = |\vec{F}_{sled\leftarrow rope}||\Delta r_{sled}|\cos\theta$$

$$W_{sled\leftarrow rope} = (11.5N)(10m)\cos 45°$$

$$W_{sled\leftarrow rope} = 81.3Nm = 81.3J$$

Work Energy Relation

Energy ← $$\int_i^f \frac{d\vec{p}_{sys}}{dt}\cdot d\vec{r}_{sys} = \int_i^f \sum_{all} \vec{F}_{\rightarrow sys}\cdot d\vec{r}_{sys}$$ → Work

$$\Delta E \equiv \int_i^f \frac{d\vec{p}_{sys}}{dt}\cdot d\vec{r}_{sys} = \int_i^f d\vec{p}\cdot\frac{d\vec{r}}{dt} = \int_i^f d\vec{p}\cdot\vec{v}$$

You won't have to do anything like this for a few years

where $\vec{p} \equiv \frac{m\vec{v}}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$ so $\vec{v} = \frac{\vec{p}/m}{\sqrt{1+\left(\frac{\vec{p}}{mc}\right)^2}}$

$$\Delta E = \int_i^f d\vec{p}\cdot\vec{v} = \int_i^f d\vec{p}\cdot\frac{\vec{p}/m}{\sqrt{1+\left(\frac{p}{mc}\right)^2}} = mc^2\int_i^f d\left(\frac{\vec{p}}{mc}\right)\cdot\frac{(\vec{p}/mc)}{\sqrt{1+\left(\frac{p}{mc}\right)^2}}$$

Form of

$$mc^2\int_i^f \frac{x\,dx}{\sqrt{1+x^2}} = mc^2\int_i^f \frac{\frac{1}{2}dx^2}{\sqrt{1+x^2}} = mc^2\sqrt{1+x^2}\Big|_i^f$$

sub.-ing back in …

$$\Delta E = m_f c^2\sqrt{1-\left(\frac{p_f}{c}\right)^2} - m_i c^2\sqrt{1-\left(\frac{p_i}{c}\right)^2}$$

Work Energy Relation

Energy ← $$\int_i^f \frac{d\vec{p}_{sys}}{dt}\cdot d\vec{r}_{sys} = \int_i^f \sum_{all} \vec{F}_{\to sys}\cdot d\vec{r}_{sys}$$ → Work

$$\Delta E \equiv \int_i^f \frac{d\vec{p}_{sys}}{dt}\cdot d\vec{r}_{sys} = \int_i^f d\vec{p}\cdot\frac{d\vec{r}}{dt} = \int_i^f d\vec{p}\cdot\vec{v}$$

You won't have to do anything like this for a few years

where $\vec{p} \equiv \dfrac{m\vec{v}}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$

$$E = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2} = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$E = \gamma mc^2$$

$$E = \gamma mc^2$$

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Consider an electron (mass 9e-31 kg) moving with speed v = 0.9c. What is its total (particle) energy?	1) 7.3e-31 J 2) 8.1e-14 J 3) 1.05e-13 J 4) 1.86e-13 J 5) 2.7e8 m/s

Kinetic and Rest Energy

(convenient to define now, will use more later)

Particle's total energy: $E = \gamma mc^2$

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

When at rest: $v = 0 \Rightarrow \gamma = 1$

$$E_{rest} \equiv mc^2$$

Energy associated just with motion: Kinetic Energy

$$K = E_{(total)} - E_{rest}$$

$$K = \gamma mc^2 - mc^2$$

$$K = (\gamma - 1)mc^2$$

Consider an electron (mass 9e-31 kg) moving with speed v = 0.9c.

a) What is its rest energy?
b) What is its (total) particle energy?
c) What is its Kinetic energy?

High-speed Example: An electron (mass 9e-31 kg) is traveling at a speed of 0.95*c* in an electron accelerator. An electric force of $1.6e^{-13}$ N is applied in the direction of motion while the electron travels a distance of 2 m. We want to determine the new speed (as a multiple of c) for the electron, using the energy principle.

Work Energy Relation

v<<c approximation

Energy ← $$\int_i^f \frac{d\vec{p}_{sys}}{dt} \cdot d\vec{r}_{sys} = \int_i^f \sum_{all} \vec{F}_{\to sys} \cdot d\vec{r}_{sys}$$ → Work

$$\Delta E \equiv \int_i^f \frac{d\vec{p}_{sys}}{dt} \cdot d\vec{r}_{sys} = \int_i^f d\vec{p} \cdot \frac{d\vec{r}}{dt} = \int_i^f d\vec{p} \cdot \vec{v}$$

If v<<c $\quad \vec{p} \approx m\vec{v}$

$$\Delta E = \int_i^f d\vec{p} \cdot \vec{v} \approx m \int_i^f d\vec{v} \cdot \vec{v} = \tfrac{1}{2} m v^2 \Big|_i^f = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

(approximate) Kinetic Energy

$$K \approx \tfrac{1}{2} m v^2$$

$$K = (\gamma - 1)mc^2 \qquad K \approx \tfrac{1}{2}mv^2$$

A ball whose mass is 2 kg travels at a velocity of < 0, –3, 4> m/s. What is the kinetic energy of the ball?	**1) < 0, –6, 8 > J** **2) < 0, –3, 4 > J** **3) 2 J** **4) 10 J** **5) 25 J**

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motion is neither created nor destroyed, but transferred via interactions.

Energy $\Delta E = W$ *Work*

$$\Delta\left(\gamma m c^2\right) = \int_i^f \sum_{all} \vec{F}_{\to sys} \cdot d\vec{r}_{sys}$$