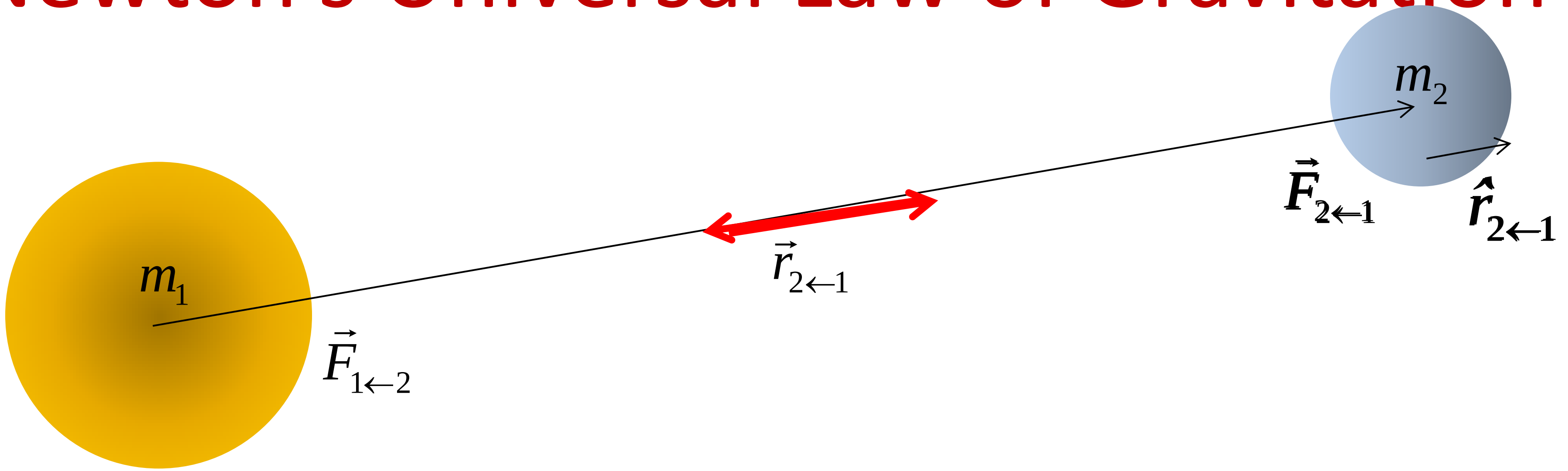
Mon. Tues Wed. Lab Fri.	3.1 – .5, .14-.15 Fundamental F's, Gravitation 3.6-.10 Elect & Strong Force; Quiz 2 L3: Predicting Motion under Non-Constant F 3.11 –.13 Conservation of P & Multiple Particles	RE 3.a EP 2, HW2: Ch 2 Pr's 40, 57, 63, 67 & CP RE 3.b bring laptop, smartphone, pad,… bring headphones if you want RE3.c
Mon. Tues	4.1-.5 Atomic nature of matter / springs	RE 4.a EP 3, HW3: Ch 3 Pr's 42, 46, 58, 65, 72 & CP

- **Fundamental Forces**
  - Gravitation
    - The Law
    - Scenarios
    - Reciprocity
    - Modeling

Newton's Universal Law of Gravitation



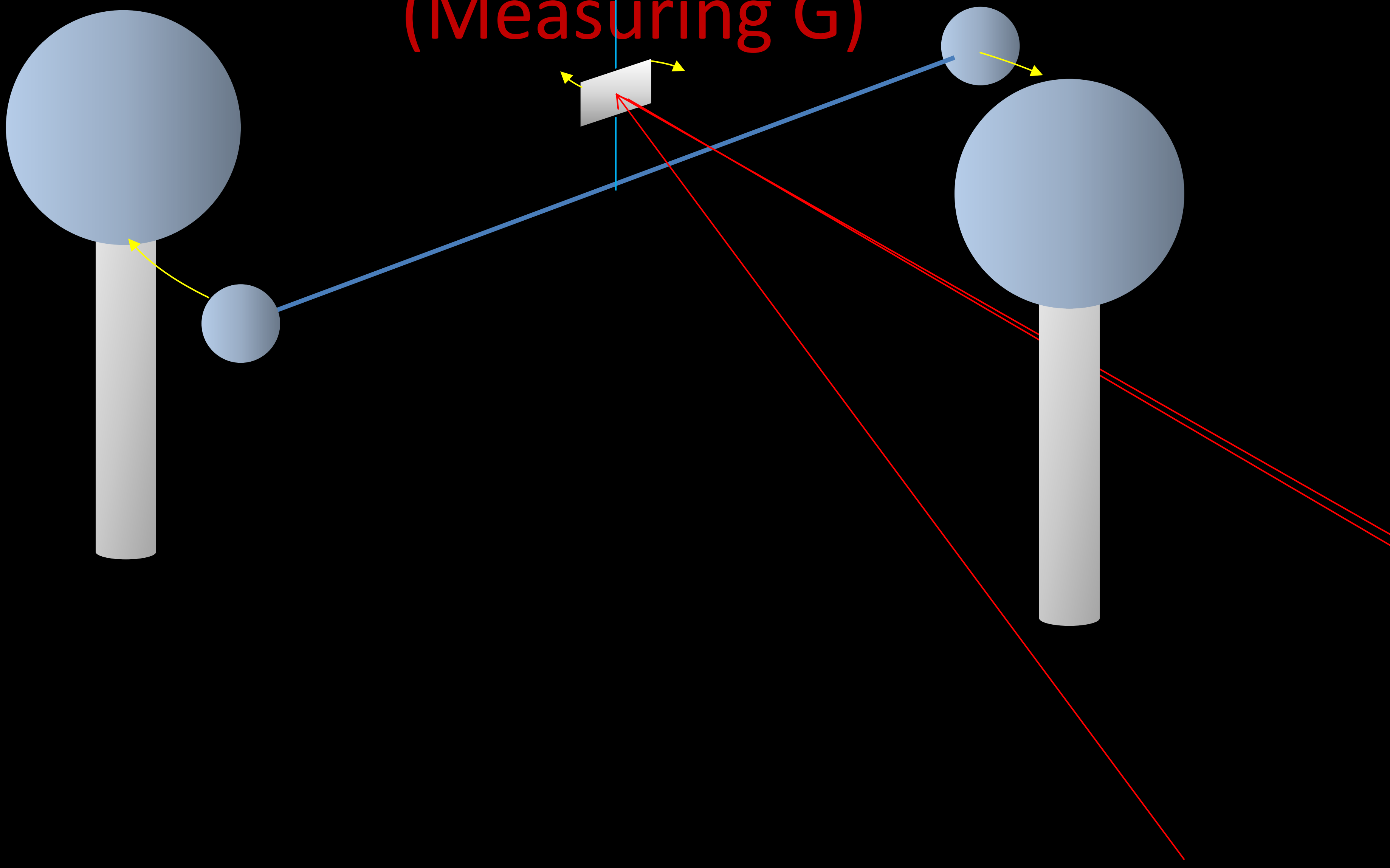
Ex. 2.1a A 3 kg ball and a 5 kg ball are 2 m apart, center to center. What is the magnitude of the gravitational force that the 3 kg ball exerts on the 5 kg ball?

$$= - \qquad \hat{r}_{2\leftarrow 1}$$

$$G = 6.67\times 10^{-11}\,\frac{N\cdot m^2}{(kg)^2}$$

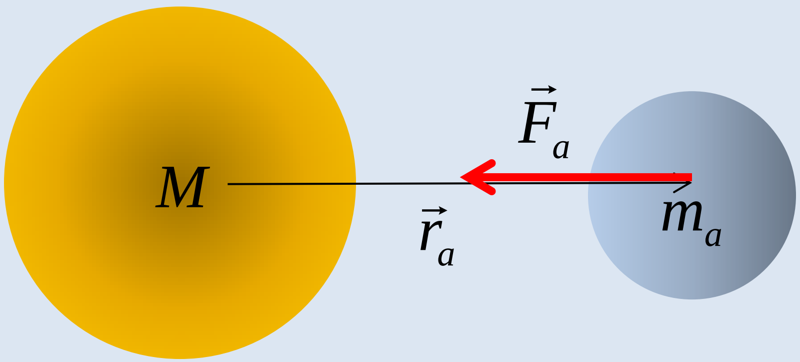
$$= \frac{\vec{r}_{2\leftarrow 1}}{\left|\vec{r}_{2\leftarrow 1}\right|}$$

Newton's Universal Law of Gravitation (Measuring G)

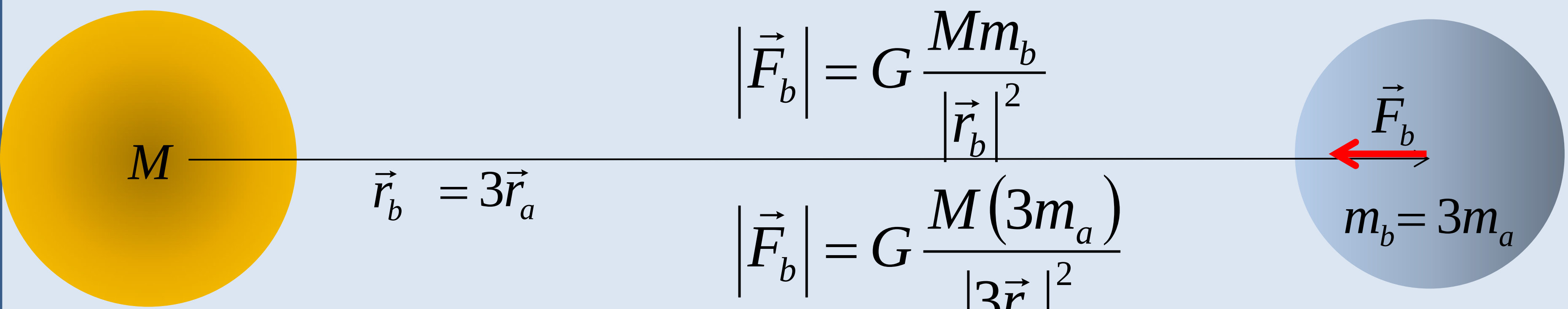


Example: Scenarios & Dependencies

2.2 Masses M and m_a attract each other with a gravitational force $|F_a|$. If mass m_a were replaced with a mass $m_b=3m_a$, and it were moved three times farther away, what is the new force, $|F_b|$, in terms of $|F_a|$?



$$\left|\vec{F}_a\right| = G\frac{Mm_a}{\left|\vec{r}_a\right|^2}$$



$$\left|\vec{F}_b\right| = G\frac{Mm_b}{\left|\vec{r}_b\right|^2}$$

$$\left|\vec{F}_b\right| = G\frac{M(3m_a)}{\left|3\vec{r}_a\right|^2}$$

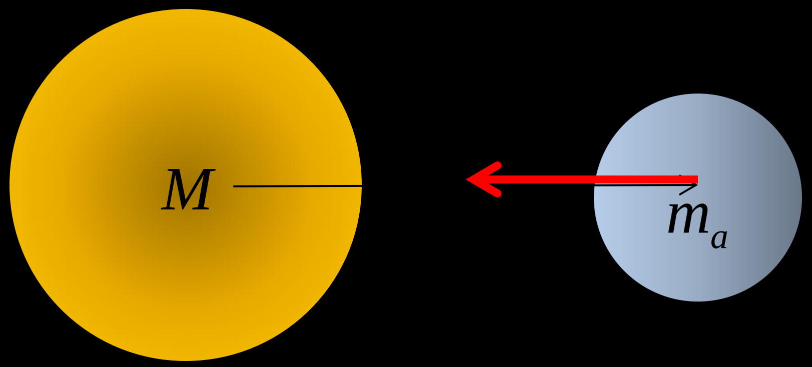
$$\left|\vec{F}_b\right| = G\frac{3M(m_a)}{9\left|\vec{r}_a\right|^2}$$

Q 3.2 a: Mass Dependence

The gravitational force exerted by a planet on one of its moons is 3e23 Newtons when the moon is at a particular location.

If the *mass* of the moon were *three times* as large what would the force on the moon be?

A. 1e23 N
B. 3e23 N
C. 6e23 N
D. 9e23 N



Q 3.2 b: Distance Dependence

The gravitational force exerted by a planet on one of its moons is 3e23 Newtons when the moon is at a particular location.

If the *distance* between the moon and the planet was *doubled*, what would the force on the moon be?

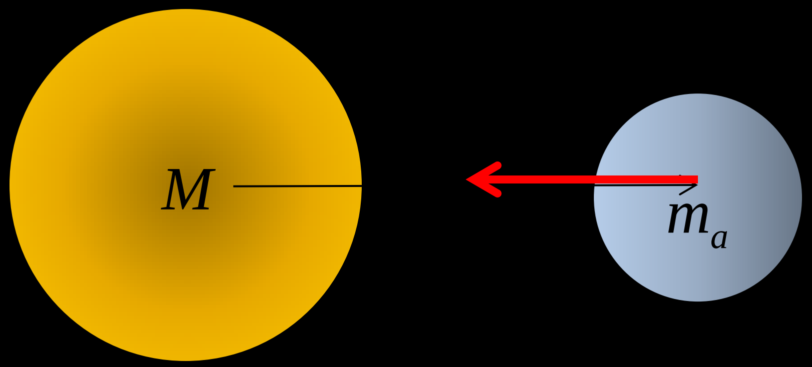
1.5e23 N

3e23 N

6e23 N

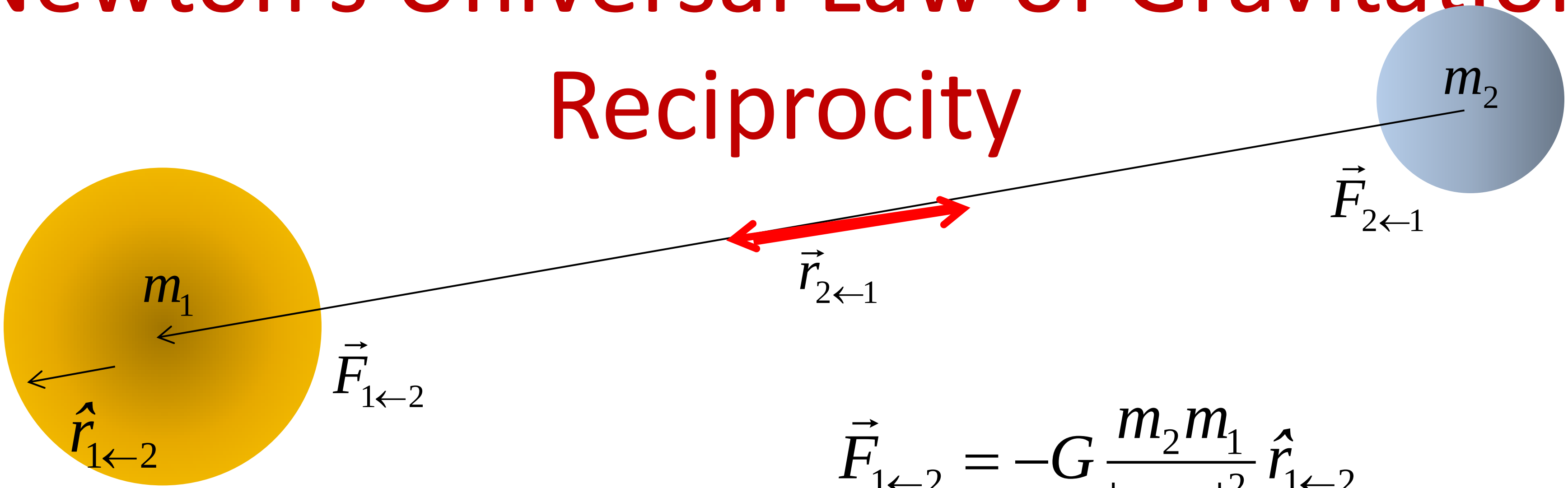
0.75e23 N

0.33e23 N



Newton's Universal Law of Gravitation

Reciprocity



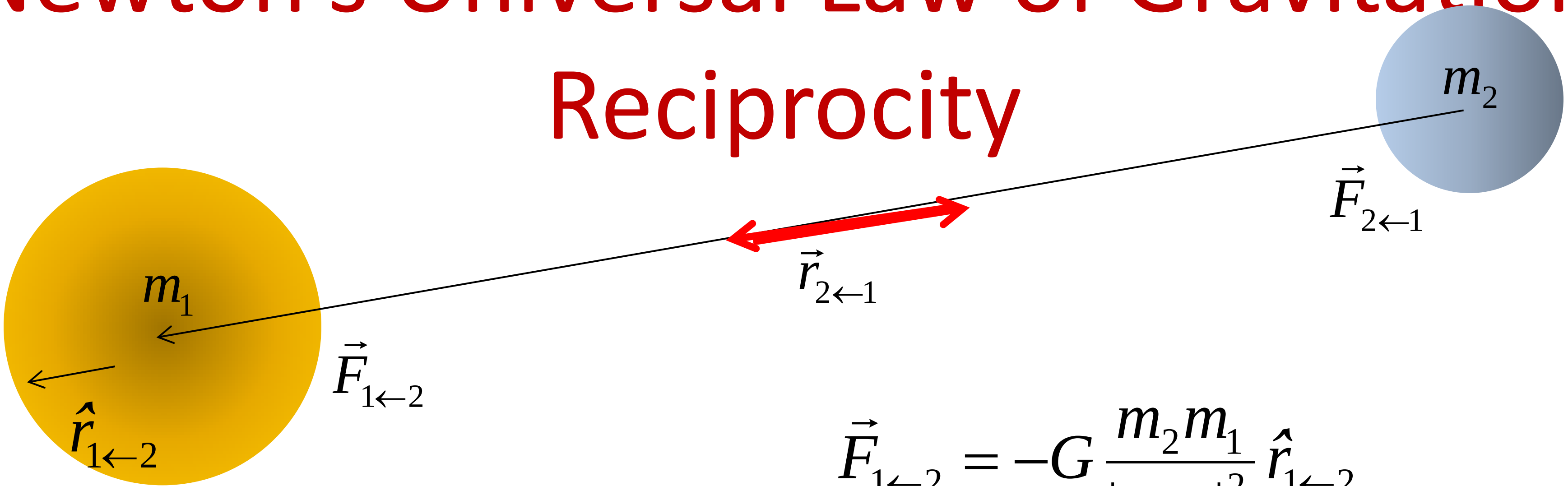
$$\hat{r}_{1\leftarrow 2} = \frac{\vec{r}_{1\leftarrow 2}}{\left|\vec{r}_{1\leftarrow 2}\right|}$$

$$\vec{F}_{1\leftarrow 2} = -G\,\frac{m_2 m_1}{\left|\vec{r}_{1\leftarrow 2}\right|^2}\,\hat{r}_{1\leftarrow 2}$$

$$\vec{F}_{1\leftarrow 2} = -G\,\frac{m_2 m_1}{\left|\vec{r}_{1\leftarrow 2}\right|^2}\left(-\hat{r}_{2\leftarrow 1}\right)$$

Newton's Universal Law of Gravitation

Reciprocity



$$\vec{F}_{1\leftarrow 2} = -G\frac{m_2 m_1}{\left|\vec{r}_{1\leftarrow 2}\right|^2}\hat{r}_{1\leftarrow 2}$$

Ex. 2.1b A 3 kg ball and a 5 kg ball are 2 m apart, center to center. If the magnitude of the gravitational force that the 3 kg ball exerts on the 5 kg ball is 2.5×10^{-9}N, what is the magnitude of the force that the 5 kg ball exerts on the 3 kg ball?

$$\vec{F}_{1\leftarrow 2} = -G\frac{m_2 m_1}{\left|\vec{r}_{1\leftarrow 2}\right|^2}\left(-\hat{r}_{2\leftarrow 1}\right)$$

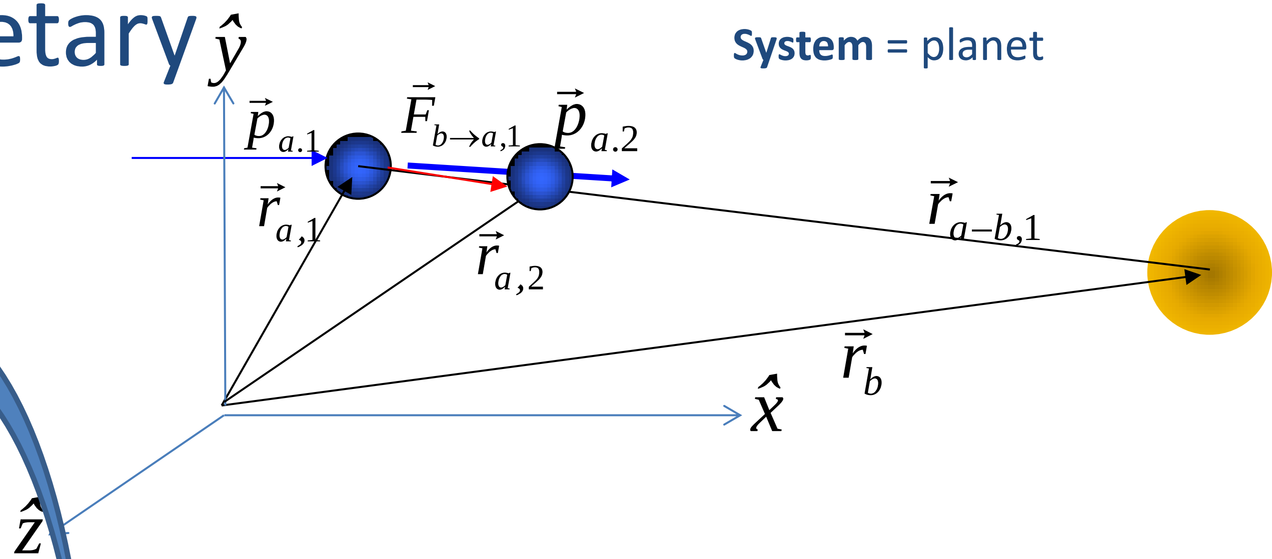
$$\vec{F}_{1\leftarrow 2} = -\left(-G\frac{m_1 m_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}\right)$$

$$\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1}$$

Modeling Nature

- Goal & System
- Approximate / Simplify
- Quantify your Approximate System
  - Initial Conditions & Known Quantities
  - Applicable Principles & Relations
  - Plan Algorithm / Algebra
- Execute
- Test
- Refine as desired

Modeling Planetary Motion



System = planet

Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

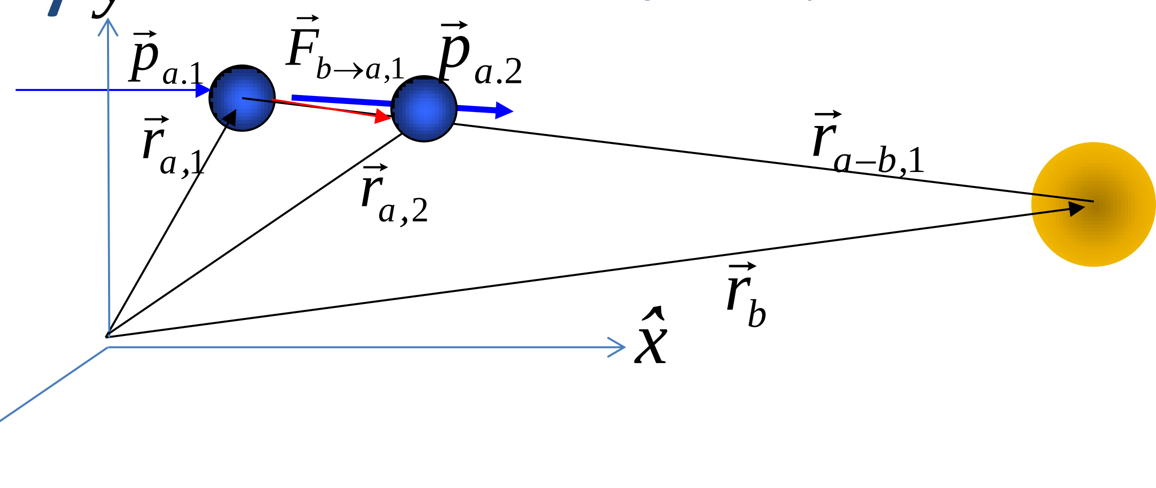
$$t_2 = t_1 + \Delta t$$

Approximations/Simplifications

- One planet, no moons
- Star's Stationary
- Point Masses or Perfect Spheres
- *v*<<c
- Finite rather than infinitesimal time step

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

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$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Planet's Initial Momentum

The planet initially has a velocity of < -1e4, 2e4, 0 > m/s. What is the initial momentum of the planet?

Star's mass: 1e30 kg

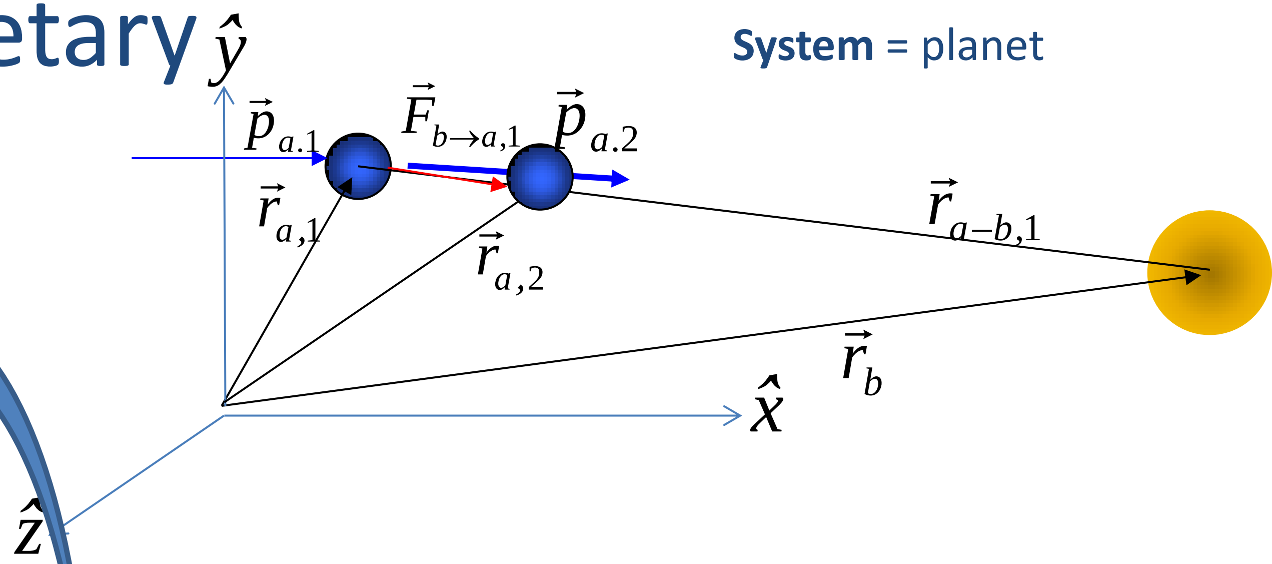
Planet's mass: 5e24 kg

***G* = 6.7e-11 N·m²/kg².**

- **a. < 5e28, -1e29, 0 > kg m/s**
- **b. < -5e28, 1e29, 0 > kg m/s**
- **c. < 1e30, 5e24, 0 > kg m/s**
- **d. < -1e4, 2e4, 0 > kg m/s**
- **e. <-2e-21,4e-21,0> kg m/s**

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Relative Position Vector

Fixed star position: < 0.5e11, 1e11, 0 > m

Initial planet position: < 2e11, 1.5e11, 0 > m

Calculate the vector that points from the star to the planet.

a. < 1e22, 1.5e22, 0 > m

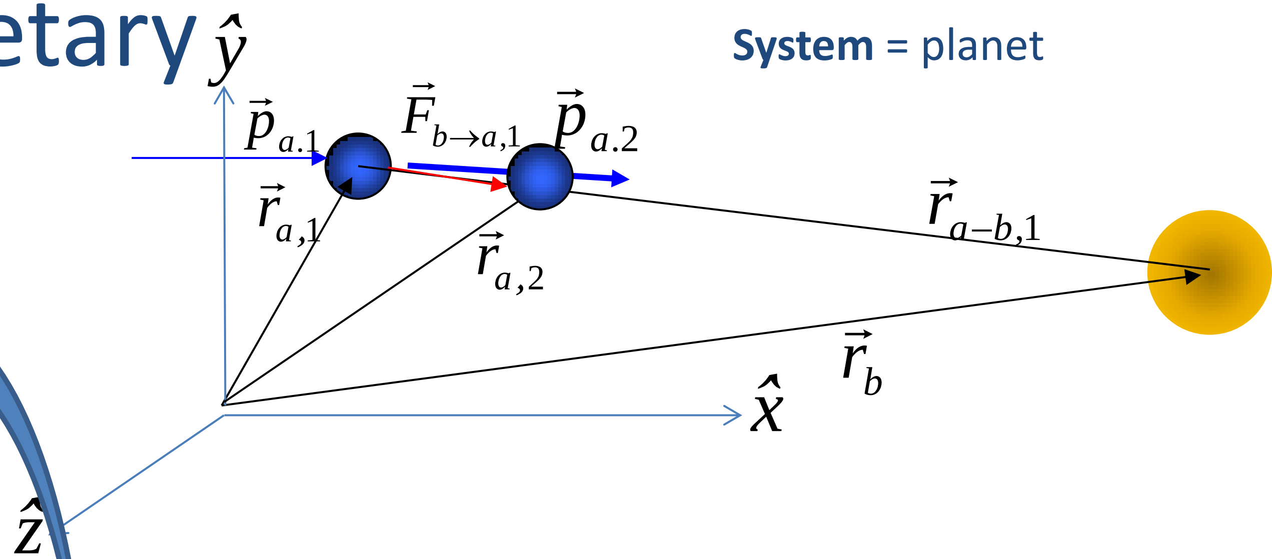
b. < 1.5e11, 0.5e11, 0 > m

c. < -1.5e11, -0.5e11, 0 > m

d. < 2.5e11, 2.0e11, 0 > m

e. We don’t have enough information to find the vector

Modeling Planetary Motion



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Distance Apart

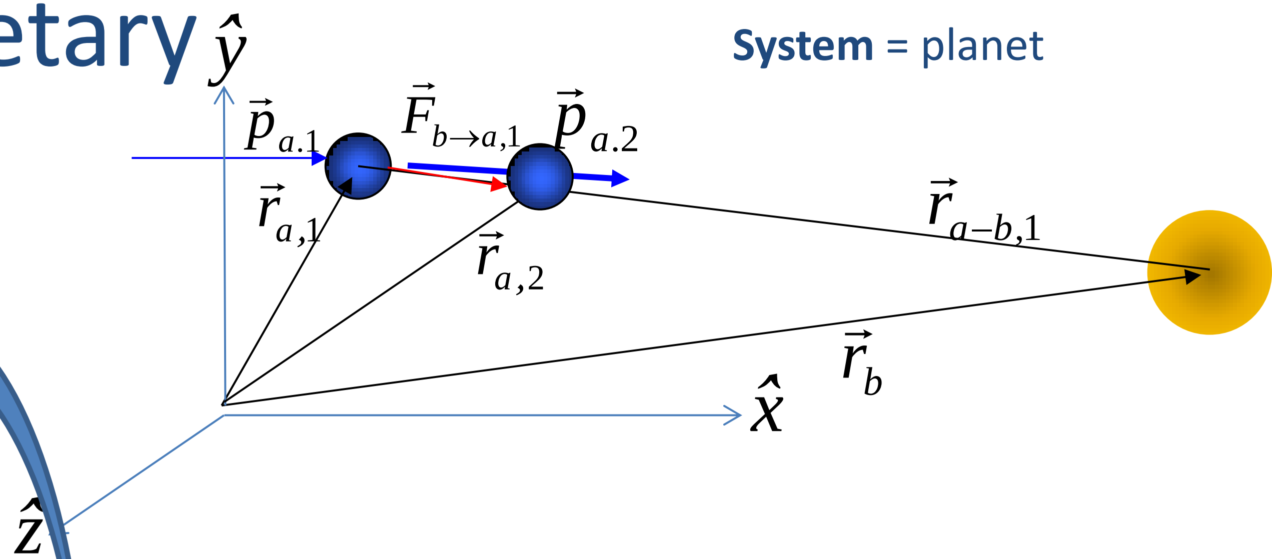
Relative position vector from star to planet is
< 1.5e11, 0.5e11, 0 > m

What is the distance between the star and the planet?

a. 0.5e11 m
b. 1.50e11 m
c. 1.58e11 m
d. 2.00e11 m
e. 2.50e11 m

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Direction

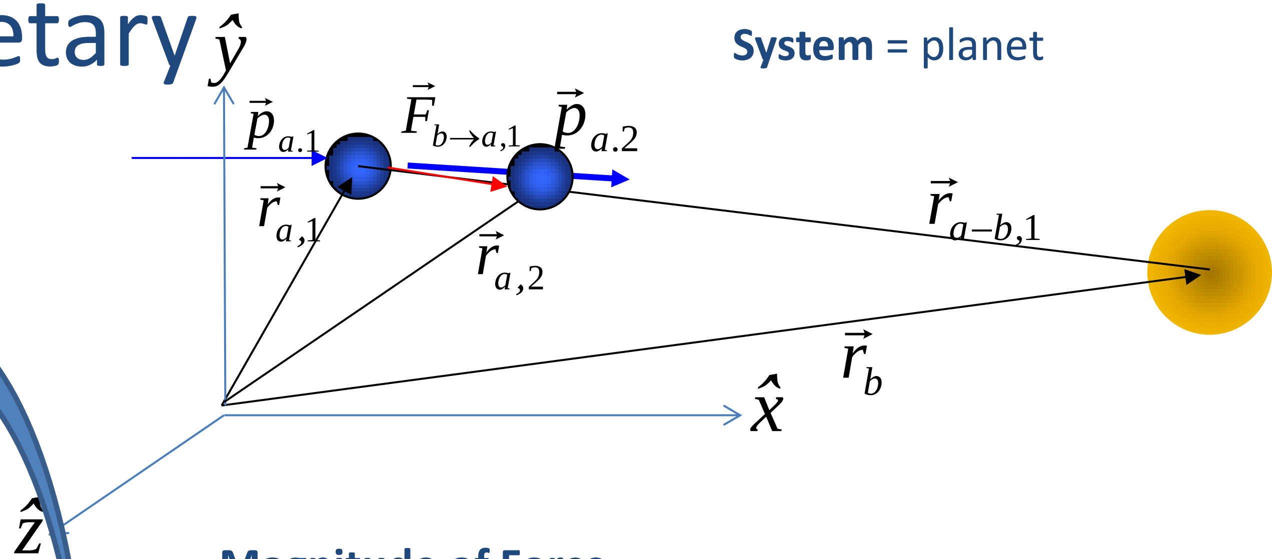
Relative position vector from star to planet is < 1.5e11, 0.5e11, 0 > m . Distance from star to planet is 1.58e11 m

Find the unit vector pointing from the star to the planet.

a. **< 1, 0, 0 >**
b. **< 1, 1, 0 >**
c. **< 0.949, 0.316, 0 >**
d. **< 1.5e11,0.5e11, 0 >**
e. **<1.58e11,0,0>**

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Magnitude of Force

Distance from star to planet: 1.58e11 m

Star's mass: 1e30 kg

Planet's mass: 5e24 kg

***G* = 6.7e-11 N·m²/kg²**

Calculate the magnitude of the gravitational force that the star exerts on the planet.

a. 1.34e-8 N

b. 2.68e-2 N

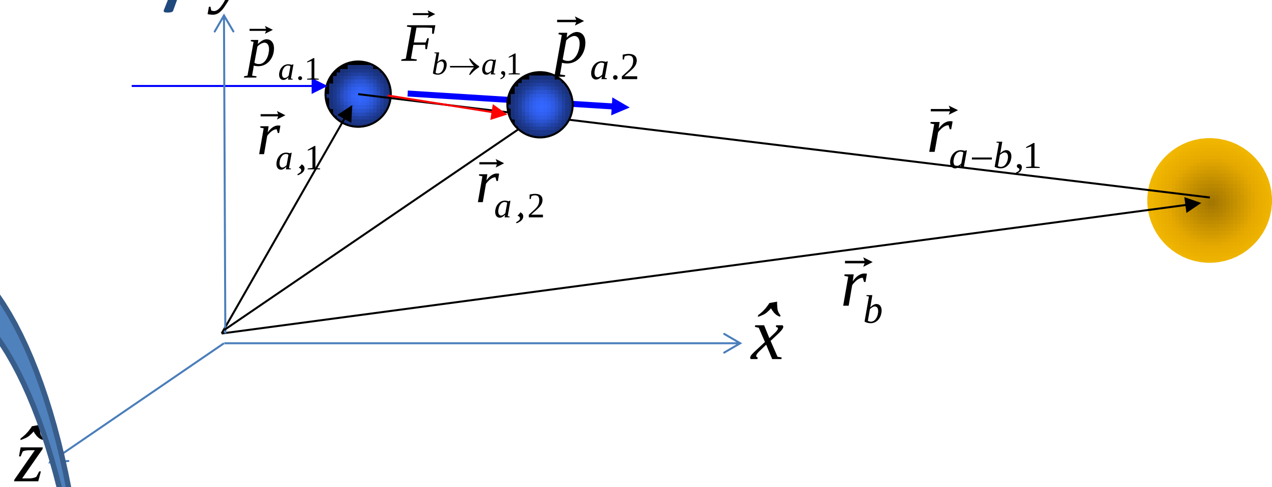
c. 1.34e22 N

d. 2.12e33 N

e. 5.3e55 N

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

$$\vec{r}_{a\leftarrow b,1} = \vec{r}_{a,1} - \vec{r}_b$$

$$\hat{r}_{a\leftarrow b,1} = \frac{\vec{r}_{a\leftarrow b,1}}{\left|\vec{r}_{a\leftarrow b,1}\right|}$$

$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Force

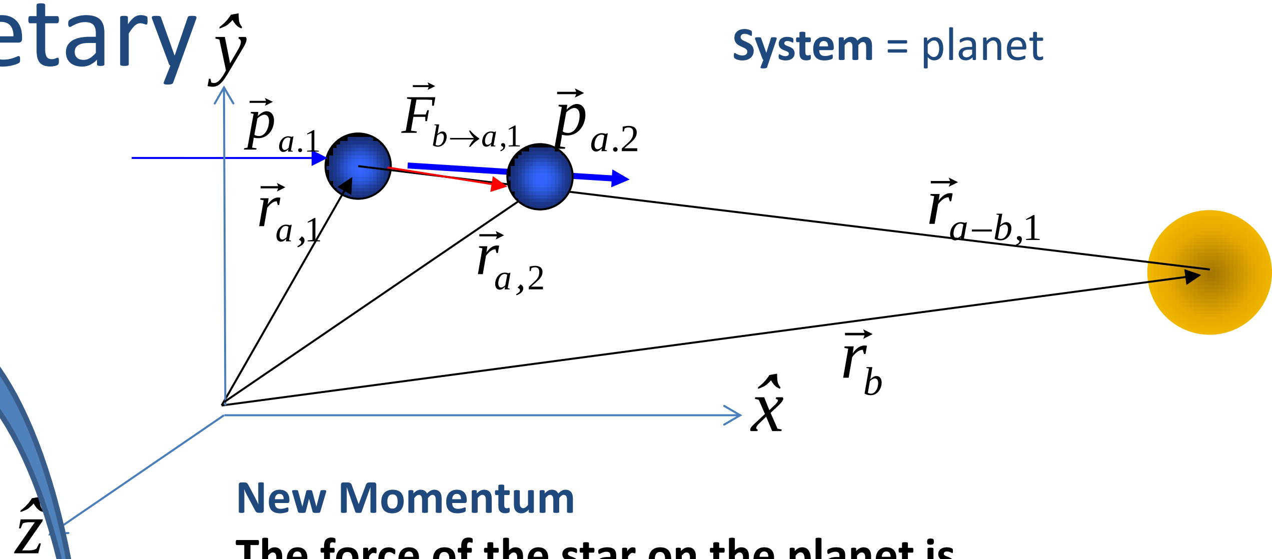
The unit vector *from* the star *to* the planet is < 0.949, 0.316, 0 >, and its magnitude is 1.34e22 N.

Calculate the gravitational force exerted *by* the star *on* the planet .

a. < 1e22, 1.5e22, 0 > N
b. < 1.5e11, 0.5e11, 0 > N
c. < -1.5e11,- 0.5e11, 0 > N
d. < 2.5e11,2.0e11, 0 > N
e. <-1.27e22,-4.2e21,0> N

Modeling Planetary Motion

System = planet



Initially

t_1

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$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

New Momentum

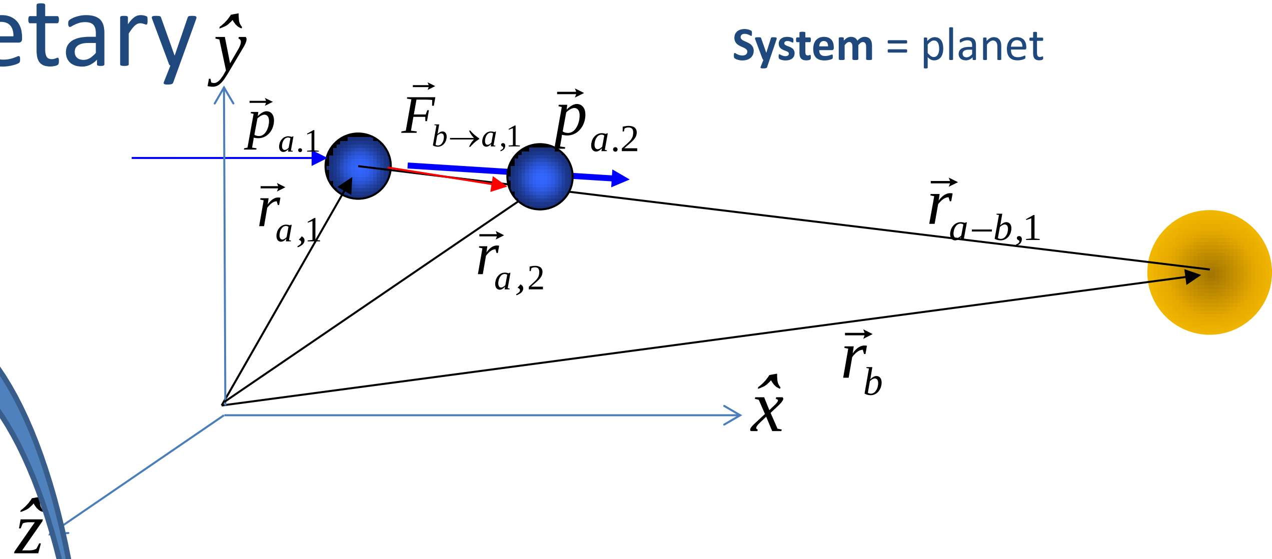
The force of the star on the planet is <-1.27e22,-4.2e21,0> N.
The planet's initial momentum is < -5e28, 1e29, 0 > kg m/s
and its mass is 5e24 kg.

After 1 day (24*60*60 s), what is the new momentum of the planet?

a. < 5.11e28, -9.97e28, 0 > kg·m/s
b. < -5.11e28, 9.97e28, 0 > kg·m/s
c. < -1.10e27, -3.66e26, 0 > kg·m/s
d. < -1e4, 2e4, 0 > kg·m/s
e. < -2e-21, 4e-21, 0 > kg·m/s

Modeling Planetary Motion

System = planet



Initially

t_1

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$\vec{r}_{a,1}$

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$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

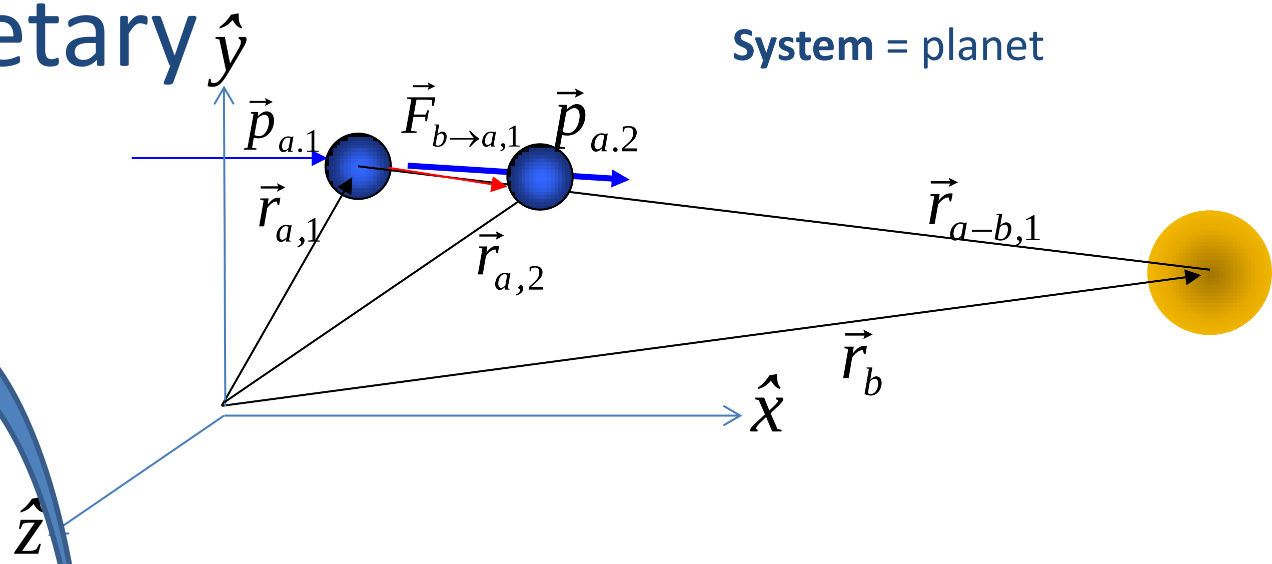
New Position

The planet was originally at < 2e11, 1.5e11, 0 > m.
It's new momentum is < -5.11e28, 9.97e28, 0 > kg·m/s

After 1 day (24*60*60 s), what is the new position of the planet?

Modeling Planetary Motion

System = planet



Initially

t_1

$\vec{p}_{a.1}$

$\vec{r}_{a,1}$

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$$\vec{F}_{a\leftarrow b.1} = -G\frac{m_a m_b}{\left|\vec{r}_{a\leftarrow b,1}\right|^2}\hat{r}_{a\leftarrow b,1}$$

Updates

$$\vec{p}_{a.2} \approx \vec{p}_{a.1} + \vec{F}_{a\leftarrow b.1}\Delta t$$

$$\vec{r}_{a.2} \approx \vec{r}_{a.1} + \frac{\vec{p}_{a.2}}{m}\Delta t$$

$$t_2 = t_1 + \Delta t$$

Next Iteration

We've applied the Momentum Principle and updated position.

Now, if we want to continue modeling the planet's motion for another step, which quantities have changed and must be recalculated?

a. Relative position vector

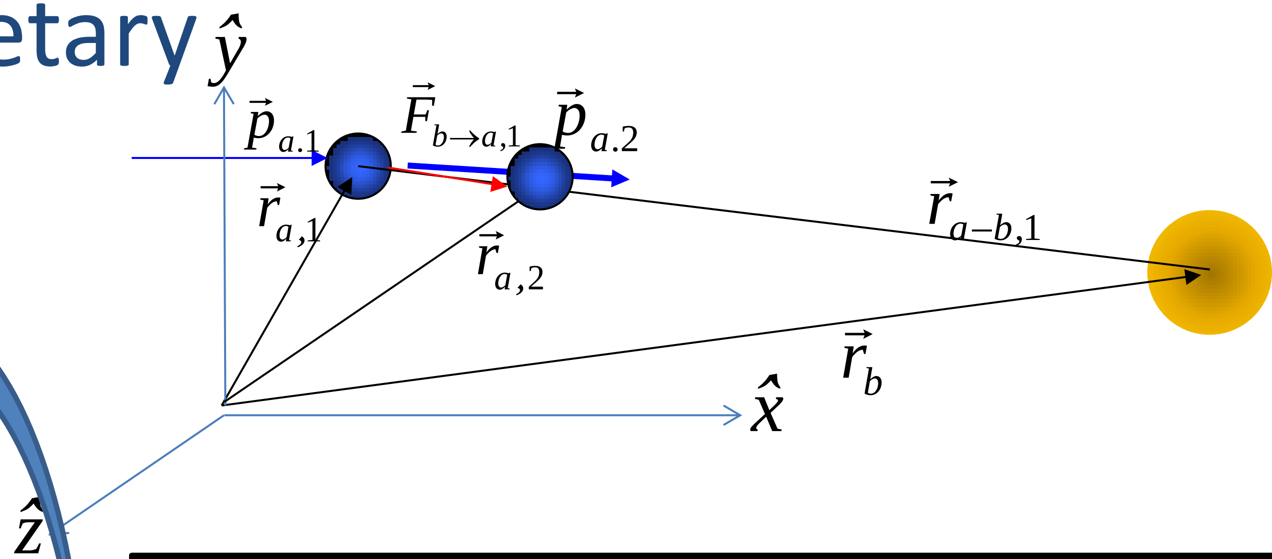
b. Unit vector r-hat

c. Force on planet by star

d. All of these

e. None of these

Modeling Planetary Motion



Initially

t_1

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$\vec{r}_{a,1}$

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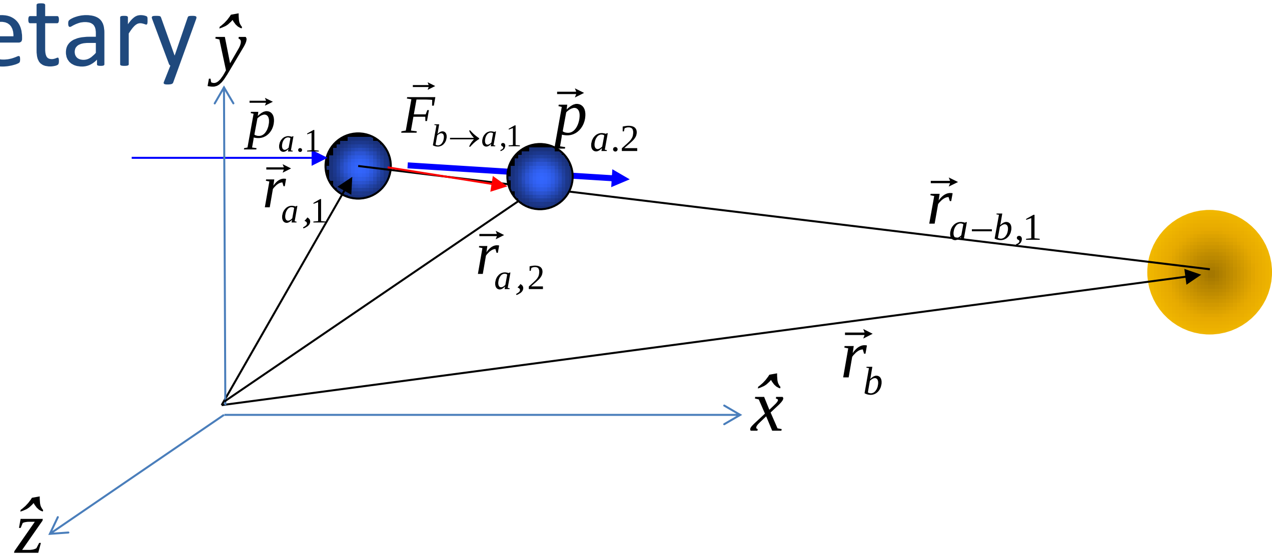
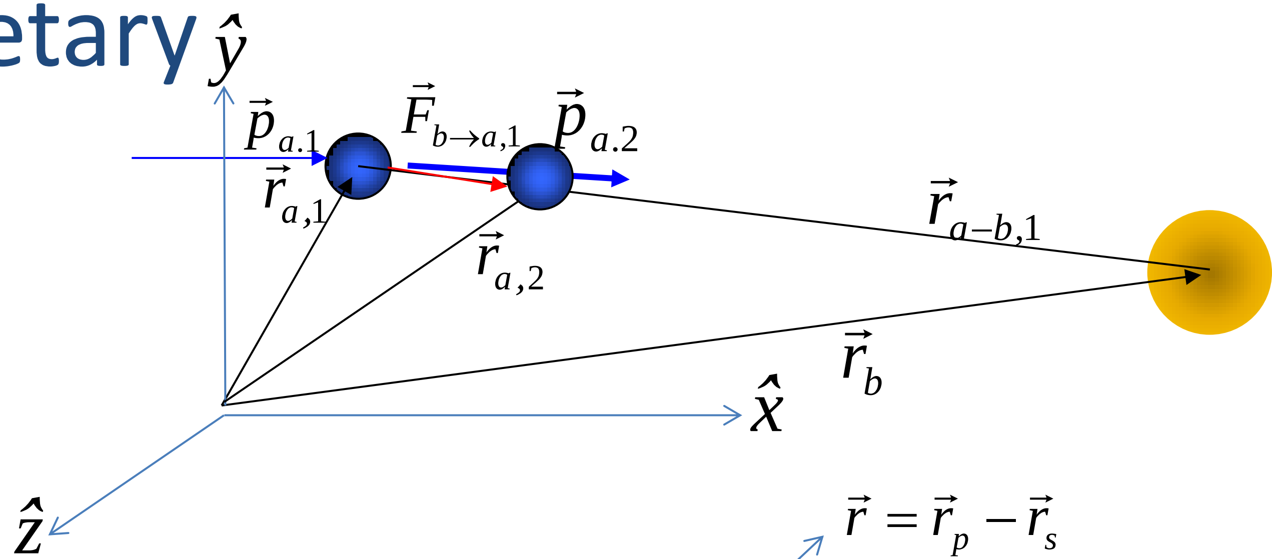
$$t_2 = t_1 + \Delta t$$

Modeling Planetary Motion




```
from visual import *
# Eric Hill

#Definitions and initial conditions
star = sphere(pos=vector(0,0,0), radius=7e9, color=color.yellow)
star.m = 2e30

planet = sphere(pos=vector(1.5e11,0,0), radius=6.4e9,
                             color=color.blue, make_trail = true)
planet.m = 6e24
planet.p =planet.m*vector(0,2.98e4,0)

G=6.7e-11
deltat = 3.16e4
tyear = 3.16e7

t=0
```

Modeling Planetary Motion



$$\vec{r} = \vec{r}_p - \vec{r}_s$$

```
while (t < tyear*5):
    r = planet.pos - star.pos
    mag = sqrt(r.x**2 + r.y**2 + r.z**2)
    rhat = r/mag
    F = -G*planet.m*star.m*rhat/(mag**2)
    planet.p = planet.p + F*deltat
    planet.pos = planet.pos + planet.p*deltat/planet.m
    t = t + deltat
```

Universal Law of Gravitation ⟹ Near-Earth Approximation

$$\left|\vec{F}_{Earth\to you}\right| = G\frac{M_{Earth}m_{you}}{\left|r_{Earth\to you}\right|^2} \quad \Rightarrow \quad \mathrm{g}m_{you}$$

Advising Moment:
Workload Expectations and Management

Expectations:

"Carnegie Unit" = Federal mandate: 1unit / 40hrs of work (in+out of class) a semester for a typical student to succeed…

4unit class means 160 hrs a semester; about 12hrs/full-week

Given our in-class/lab time, about 5-6 hrs out-of-class each week

Surveys and estimates put this class closer to 6 hrs per week.

Management:

Preview the problems before reading and lecture

Start early

Don't spend a lot of time 'stuck' - Get help when you've got problems
- Work with classmates
- Drop by Tutoring (SMT Th 7:30-9:30 upstairs)
- Drop by my office

Mon. Tues Wed. Lab Fri.	3.1 – .5, .14-.15 Fundamental F's, Gravitation 3.6-.10 Elect & Strong Force; Quiz 2 L3: Predicting Motion under Non-Constant F 3.11 –.13 Conservation of P & Multiple Particles	RE 3.a EP 2, HW2: Ch 2 Pr's 40, 57, 63, 67 & CP RE 3.b bring laptop, smartphone, pad,… bring headphones if you want RE3.c
Mon. Tues	4.1-.5 Atomic nature of matter / springs	RE 4.a EP 3, HW3: Ch 3 Pr's 42, 46, 58, 65, 72 & CP