Mon.	5.5 -.7 Curving Motion	RE 5.b
Tues.		EP 5, HW5: Ch5 Pr’s 16, 19, 45, 48, 64(c&d)
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Ch. 5 – Rate of change (if any) of momentum

$$\sum_{all} \vec{F}_{\rightarrow system} = \frac{d\vec{p}}{dt}$$

Special Cases

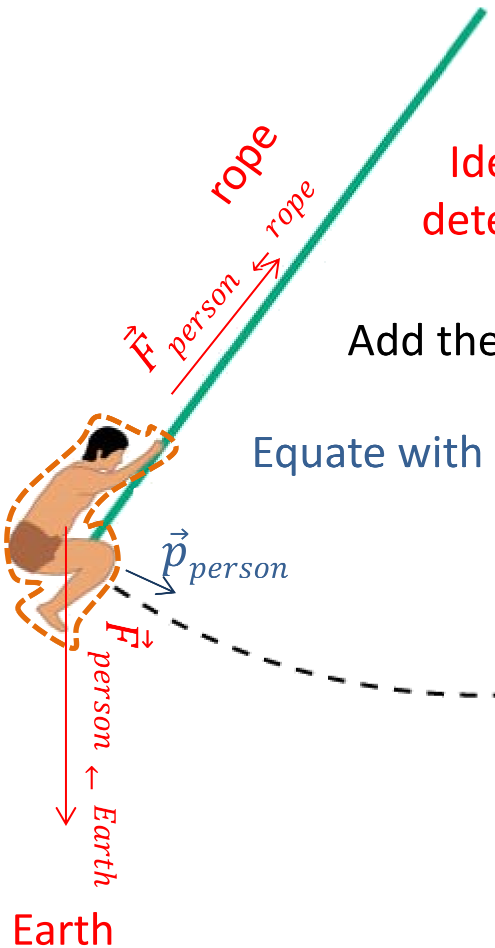
Equilibium

$$\sum_{all} \vec{F}_{\rightarrow system} = \frac{d\vec{p}}{dt} = 0$$

Uniform Circular Motion

$$\frac{d\left|\vec{p}\right|}{dt} = 0$$

The process for force problems



Define and *stick with* your system

Identify what objects are interacting with it and determine the corresponding forces acting upon it

Add the forces to get the net force acting on the system

Equate with rate of change of momentum via momentum principle

$$\sum_{all} \vec{F}_{system \leftarrow} = \frac{d\vec{p}_{sys}}{dt}$$

Solve for unknowns

Changing Momentum: Magnitude *and* Direction

$$\vec{F}_{net\to obj} = \frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \underbrace{\frac{d|\vec{p}|}{dt}\hat{p}}_{\text{Speeding/slowing}} + \underbrace{|\vec{p}|\frac{d\hat{p}}{dt}}_{\text{changing direction}}$$

Should point… Parallel to momentum vector | Perpendicular to momentum vector

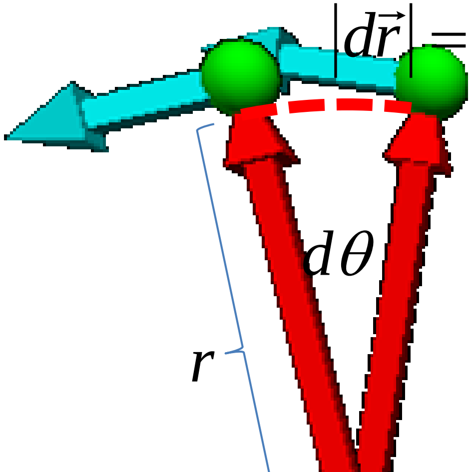
Special Case: Uniform Circular Motion (only direction changing)

$$\frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

similarly

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d|\vec{r}|}{dt}\hat{r} + |\vec{r}|\frac{d\hat{r}}{dt}$$

comparing



$$|d\vec{r}| = r\,d\theta$$

$$v = \left|\frac{d\vec{r}}{dt}\right| = \left|\frac{r\,d\theta}{dt}\right|$$

$$\left|\frac{d\vec{r}}{dt}\right| = |\vec{r}|\left|\frac{d\theta}{dt}\right|$$

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\theta}{dt}\right| = \frac{1}{|\vec{r}|}v$$

Rate of change of position vector's direction

Changing Momentum: Magnitude *and* Direction

$$\vec{F}_{net\to obj} = \frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \underbrace{\frac{d|\vec{p}|}{dt}\hat{p}}_{\text{Speeding/slowing}} + \underbrace{|\vec{p}|\frac{d\hat{p}}{dt}}_{\text{changing direction}}$$

Should point… Parallel to momentum vector — Perpendicular to momentum vector ✓

Special Case: Uniform Circular Motion

$$\frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \cancel{\frac{d|\vec{p}|}{dt}\hat{p}} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\theta}{dt}\right| = \frac{1}{|\vec{r}|}v$$

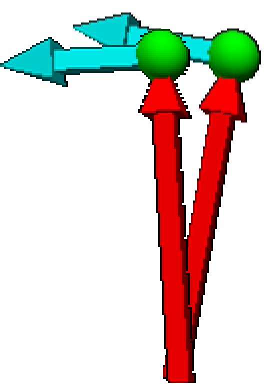
Rate of change of position vector's direction *Equals* Rate of change of momentum vector's direction

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\hat{p}}{dt}\right| = \frac{1}{|\vec{r}|}v$$

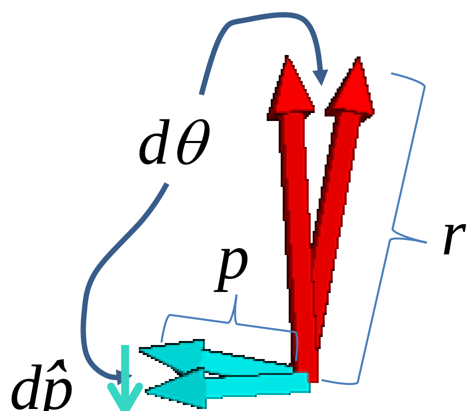
direction?

Opposite of r. $-\hat{r}$

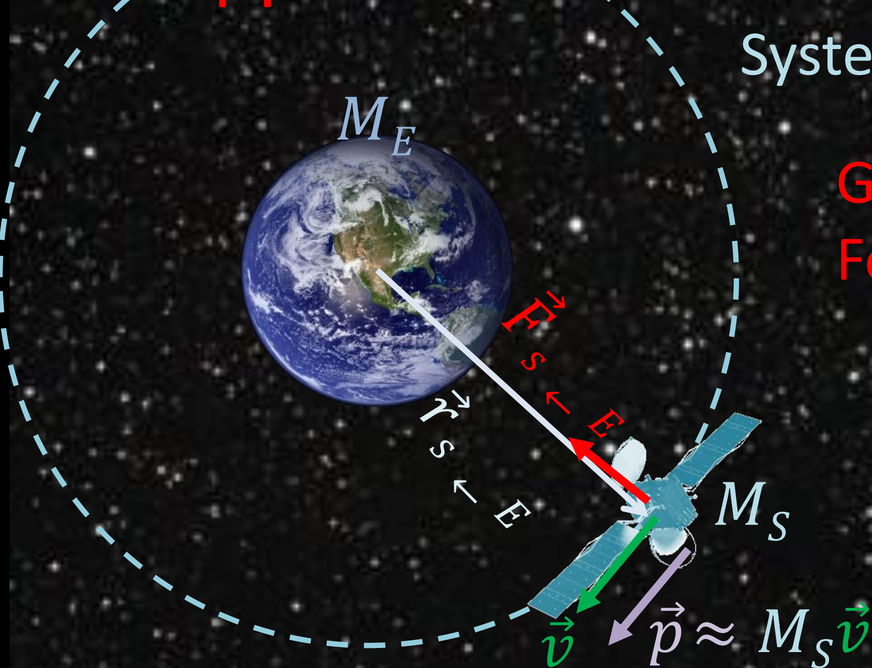
$$\frac{d\vec{p}}{dt} = -|\vec{p}|\frac{v}{|\vec{r}|}\hat{r}$$



See Vpython example



Application: Circular Gravitational Orbits



System: Satellite

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Gravitational Force

Circular Motion

$$-G\frac{M_E M_S}{|r_{s\leftarrow E}|}\hat{r}_{s\leftarrow E} = -|\vec{p}|\frac{|v|}{|r_{s\leftarrow E}|}\hat{r}_{s\leftarrow E}$$

$$G\frac{M_E M_S}{|r_{s\leftarrow E}|} = |\vec{p}||v|$$

$$G\frac{M_E \cancel{M_S}}{|r_{s\leftarrow E}|} = \cancel{M_S}|v||v|$$

$$G\frac{M_E}{|r_{s\leftarrow E}|} = v^2$$

Example: Geosynchronous Orbit

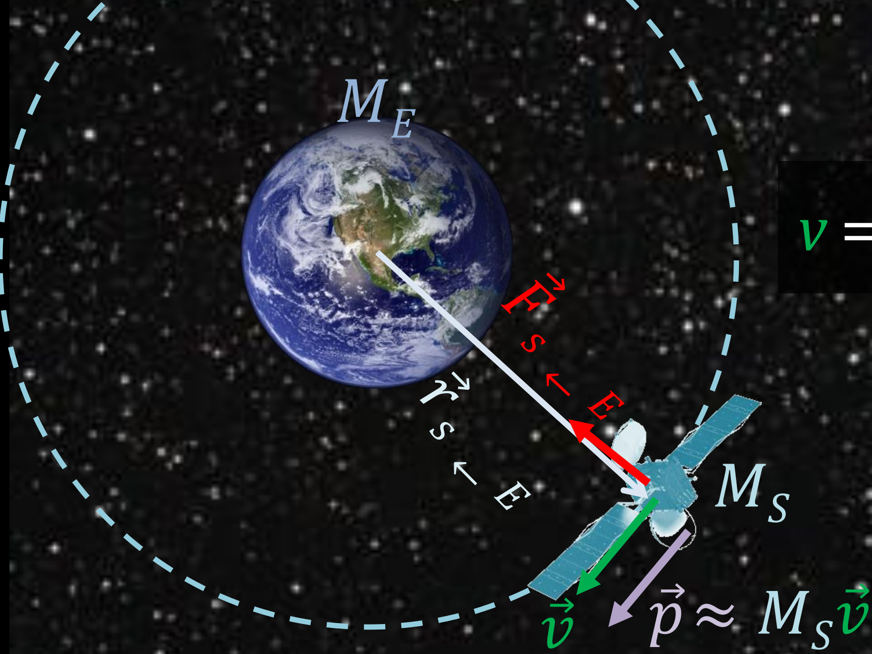
There's only one orbital radius for satellites that 'stay put' in the sky – orbit with the same period as the Earth spins: T = 1 day. What's the orbital radius?

$$v = \frac{distance}{time} = \frac{Circumferance}{Period} = \frac{2\pi r_{s\leftarrow E}}{T}$$

$$G\frac{M_E}{|r_{s\leftarrow E}|} = \left(\frac{2\pi r_{s\leftarrow E}}{T}\right)^2$$

$$r_{s\leftarrow E} = \left(GM_E\left(\frac{T}{2\pi}\right)^2\right)^{\frac{1}{3}} = \left(\left(6.7\times10^{-11}\frac{Nm^2}{kg^2}\right)(6\times10^{24}kg)\left(\frac{86{,}400s}{2\pi}\right)^2\right)^{\frac{1}{3}} = 4.2\times10^7 m$$

Kepler's 3rd Law of Planetary Motion



$$v = \frac{distance}{time} = \frac{Circumferance}{Period} = \frac{2\pi r_{s\leftarrow E}}{T}$$

$$G\frac{M_E}{|r_{s\leftarrow E}|} = v^2$$

$$G\frac{M_E}{|r_{s\leftarrow E}|} = \left(\frac{2\pi r_{s\leftarrow E}}{T}\right)^2$$

$$r_{s\leftarrow E} = \left(GM_E\left(\frac{T}{2\pi}\right)^2\right)^{\frac{1}{3}}$$

$$(r_{s\leftarrow E})^3 = GM_E\left(\frac{T}{2\pi}\right)^2$$

$$(r_{s\leftarrow E})^3 = \frac{GM_E}{(2\pi)^2}T^2$$

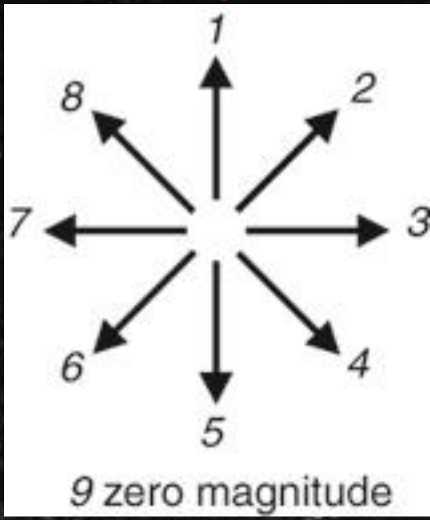
Orbital radius's relation to its period

Note: depends on mass of object orbit*ed* not orbit*ing*

Application: Circular Gravitational Orbits

The Moon travels in a nearly circular orbit around the Earth, at nearly constant speed.

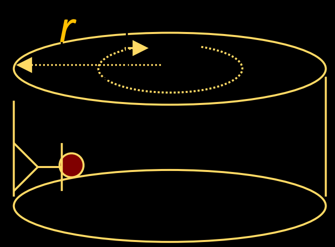
what is the direction of $\frac{d\vec{p}}{dt}$?



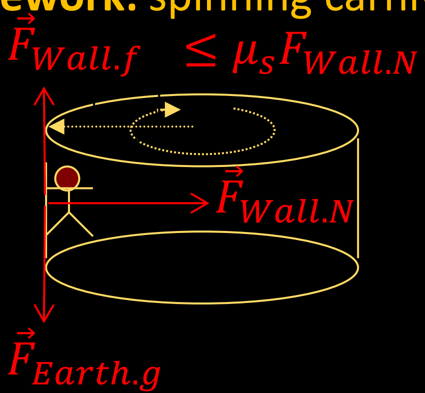
A geosynchronous satellite has an orbital radius of 4.2×10^7m.
If the moon's period were 30 days (it's really about 27), what would be its orbital radius?

a) 13×10^7m
b) 41×10^7m
c) 126×10^7m
d) 690×10^7m

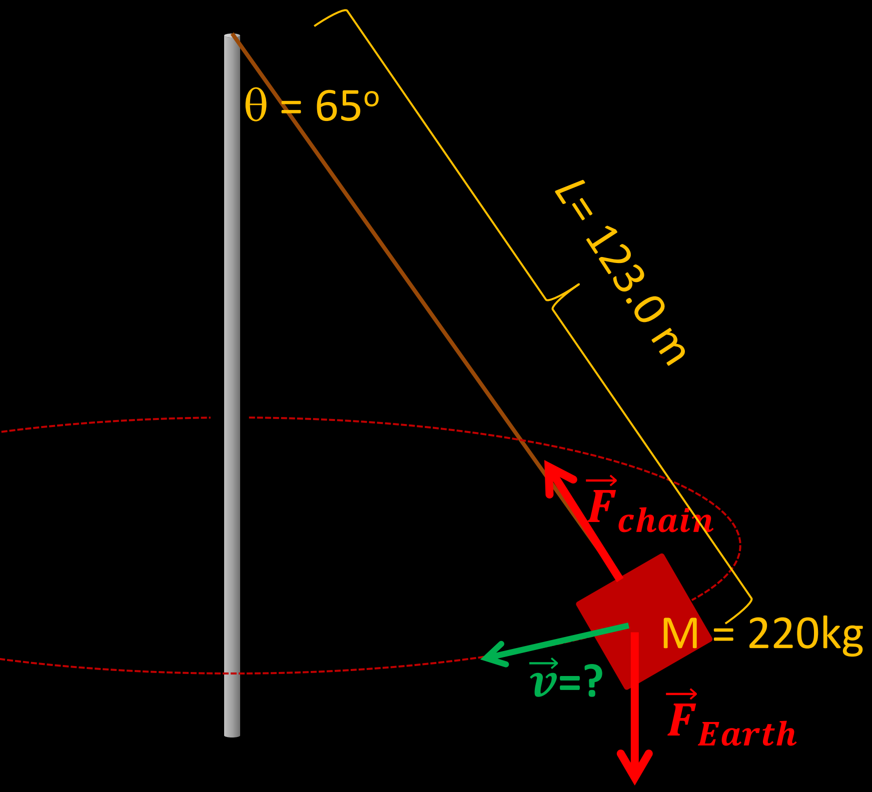
Circular Motion Example: Say you had a 1 km radius space-station, with what period must it spin to provide a normal force equal to that on Earth?



Homework: spinning carnival ride – friction holds you up



(Know and care what interactions provide net force) Person rides on a "swing" ride at a carnival. The man rides in a swing at the end of a 12.0 m chain which hangs out at 65° up from vertical. Assuming uniform circular motion, and a combined, chair+rider, mass of 220 kg, (a) what is the tension in the chain? (b) what is tangential speed of the chair?



Circular Motion: Banked Curves

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

y-component of forces and change in momentum

$$\hat{y}:\ F_{c\leftarrow E} + F_{c\leftarrow R.y} = 0$$

$$-mg + F_{c\leftarrow R.y} = 0$$

$$F_{c\leftarrow R}\cos(\theta) = mg$$

$$F_{c\leftarrow R} = \frac{mg}{\cos(\theta)}$$

x-component of forces and change in momentum

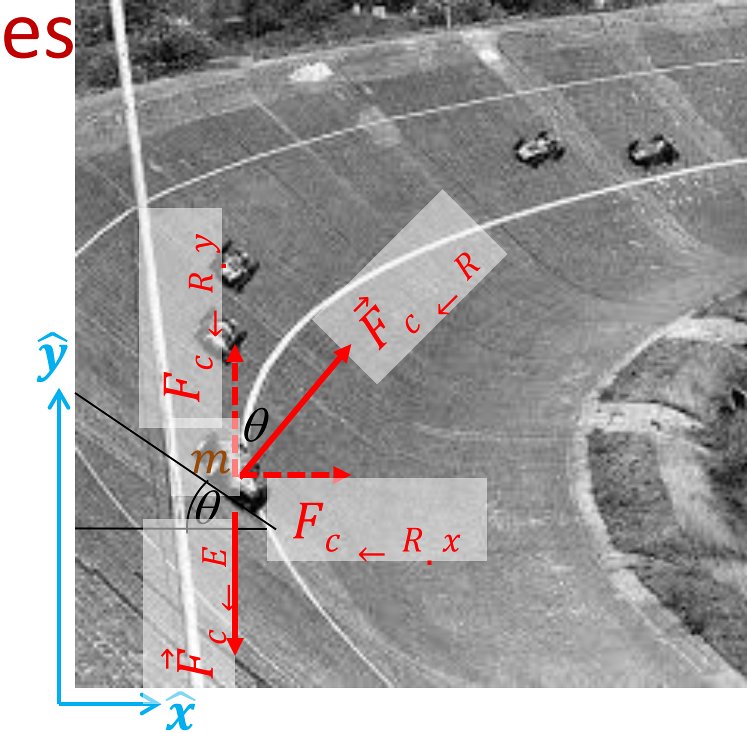
$$\hat{x}:\ Fc_{\leftarrow R.x} = |\vec{p}|\,\frac{|v|}{|r|}$$

$$\vec{p} \approx m\vec{v}$$

$$F_{c\leftarrow R.x} = m\frac{v^2}{|r|}$$

$$F_{c\leftarrow R}\sin(\theta) = m\frac{v^2}{|r|}$$

$$\frac{mg}{\cos(\theta)}\sin(\theta) = m\frac{v^2}{|r|}$$

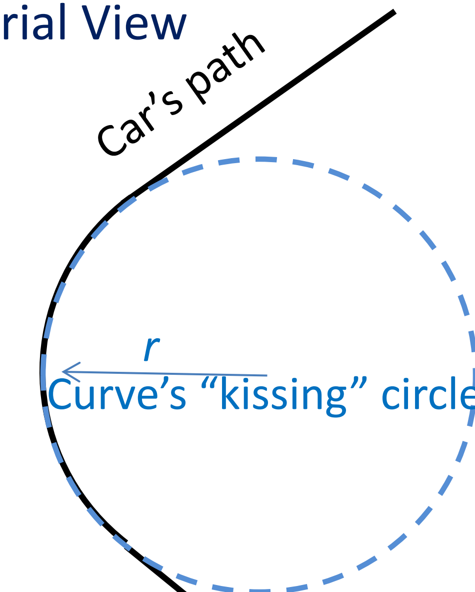


Why is the track banked?

For a given radius of curvature and expected speed, there's a particular banking angle.

$$g\tan(\theta) = \frac{v^2}{|r|}$$

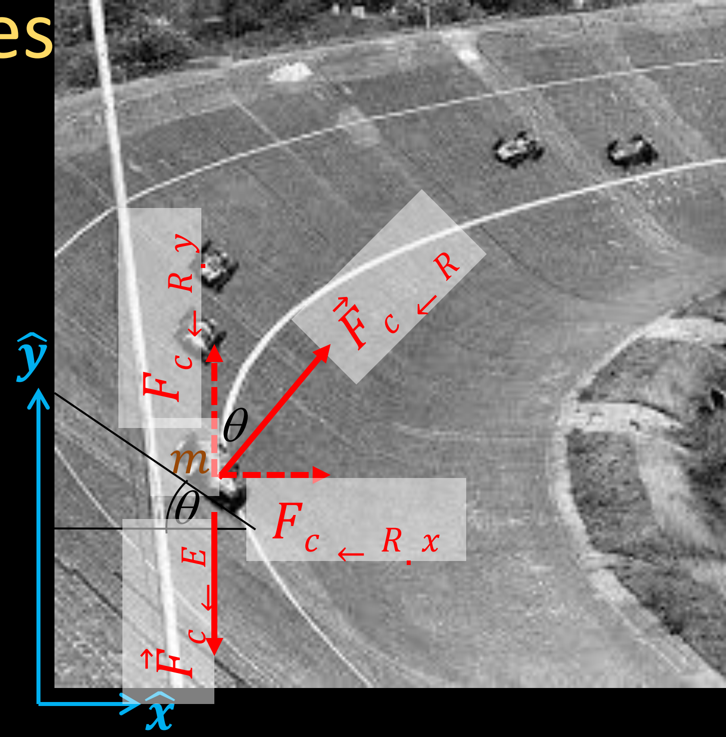
Aerial View



Circular Motion: Banked Curves

$$g\tan(\theta)=\frac{v^2}{|r|}$$

For a given radius of curvature and expected speed, there's a particular banking angle.



If you're taking the interchange between I-10 West and I-125 South, you may well be going 65 mph around a curve with a radius of curvature of 1/8 mi. If it was designed for this speed, at what angle should it be banked?

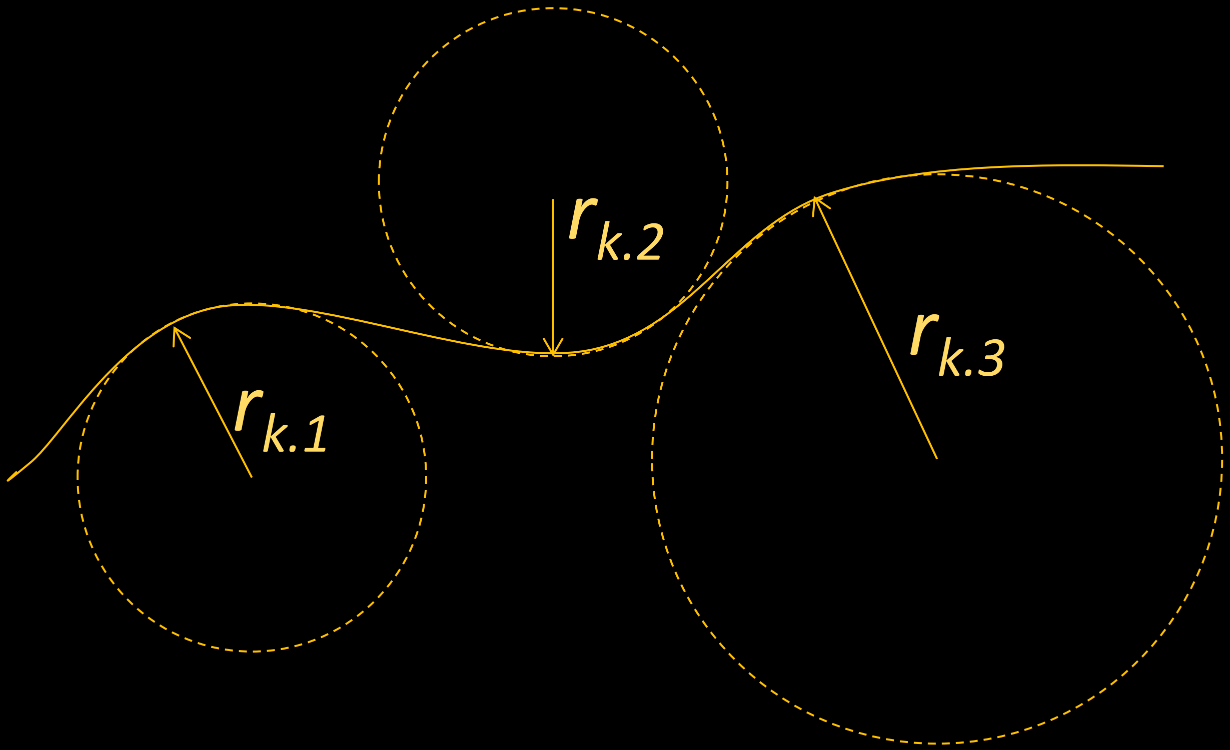
If your tires have a coefficient of static friction of 0.9, how fast could you take the curve without sliding sideways?

Circular Motion: Banked Curves

Why do planes bank when they turn?

Circular Motion: Generalized – “Kissing” Circles

$$\vec{F}_{net\to obj} = \frac{d\left(\left|\vec{p}\right|\hat{p}\right)}{dt} = \frac{d\left(\left|\vec{p}\right|\right)}{dt}\hat{p} + \left|\vec{p}\right|\frac{d\left(\hat{p}\right)}{dt}$$



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