Wed. Lab Fri.	5.1-.5 Rate of Change & Components Quiz 4 Review for Exam 1(Ch 1-4) Exam 1 (Ch 1-4) – *Accommodations?*	RE 5.a bring laptop, smartphone, pad,… Practice Exam 1 (due beginning of lab)
Mon. Tues. Fri.	5.5 -.7 Curving Motion 3pm – Visit from Columbia Rep	RE 5.b EP 5, HW5: Ch5 Pr’s 16, 19, 45, 48, 64(c&d)

Ch. 5 – Rate of change (if any) of momentum

$$\sum_{all} \vec{F}_{\to system} = \frac{d\vec{p}}{dt}$$

Today: Special Cases

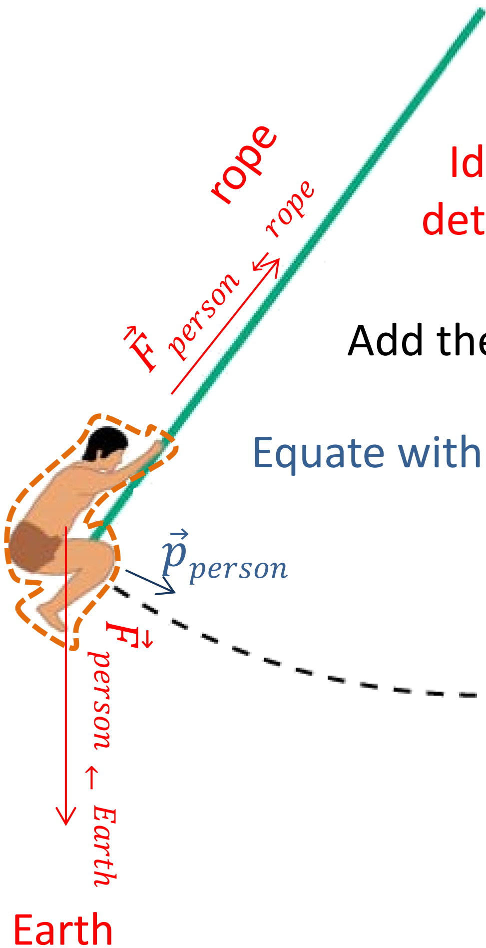
Equilibium

$$\sum_{all} \vec{F}_{\to system} = \frac{d\vec{p}}{dt} = 0$$

Uniform Circular Motion

$$\frac{d|\vec{p}|}{dt} = 0$$

The process for force problems



Define and *stick with* your system

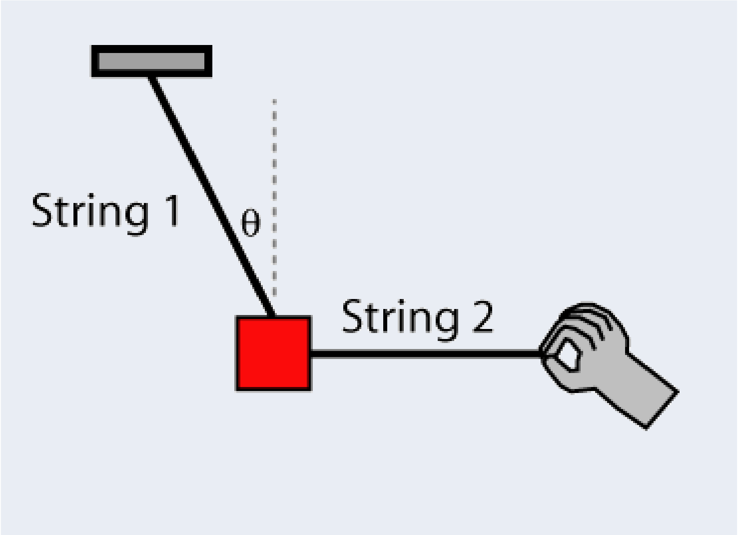
Identify what objects are interacting with it and determine the corresponding forces acting upon it

Add the forces to get the net force acting on the system

Equate with rate of change of momentum via momentum principle

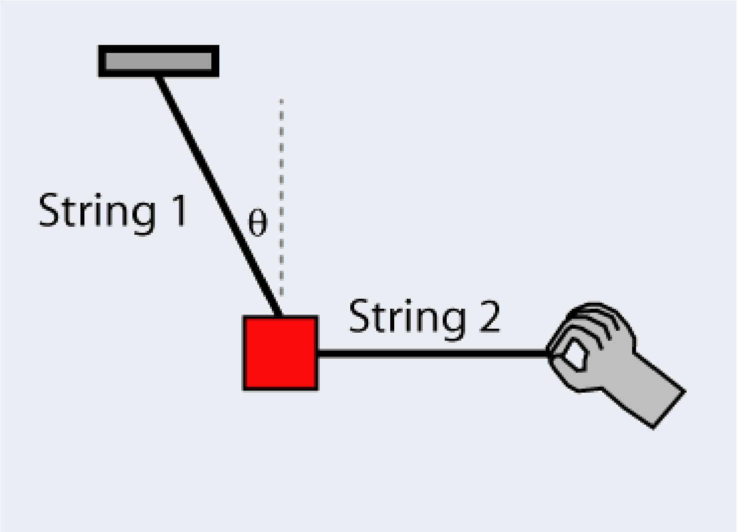
$$\sum_{all} \vec{F}_{system \leftarrow} = \frac{d\vec{p}_{sys}}{dt}$$

Solve for unknowns



What objects exert significant forces on the red block?

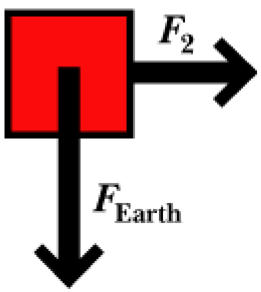
a. Earth, String 1, String 2
b. Earth, String 1, String 2, Hand
c. Earth, String 1, String 2, Hand, Ceiling
d. Earth, Hand, Ceiling

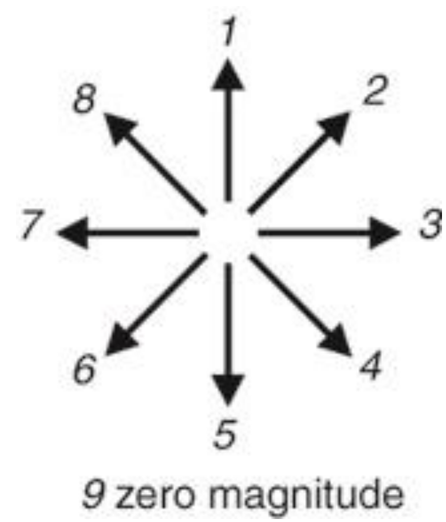


What objects exert significant forces on the red block?

a. Earth, String 1, String 2
b. Earth, String 1, String 2, Hand
c. Earth, String 1, String 2, Hand, Ceiling
d. Earth, Hand, Ceiling

Here is an incomplete force diagram for the system of the red block.

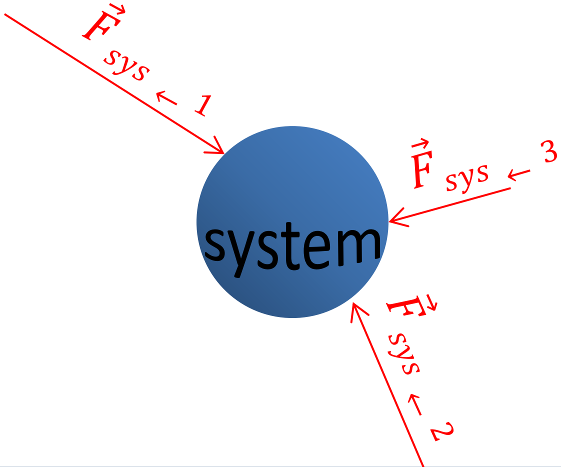




To complete it we need to draw the force due to String 1. Which arrow best indicates the direction of this force?

Equilibrium

$$\sum_{all} \vec{F}_{system\leftarrow} = \frac{d\vec{p}}{dt} = 0$$



$$\vec{F}_{sys\leftarrow 1} + \vec{F}_{sys\leftarrow 2} + \vec{F}_{sys\leftarrow 3} = 0$$

$$\hat{x}:\ F_{sys\leftarrow 1.x} + Fsy_{s\leftarrow 2.x} + F_{sys\leftarrow 3.x} = 0$$

$$\hat{y}:\ F_{sys\leftarrow 1.y} + F_{sys\leftarrow 2.y} + F_{sys\leftarrow 3.y} = 0$$

$$\hat{z}:\ F_{sys\leftarrow 1.z} + F_{sys\leftarrow 2.z} + F_{sys\leftarrow 3.z} = 0$$

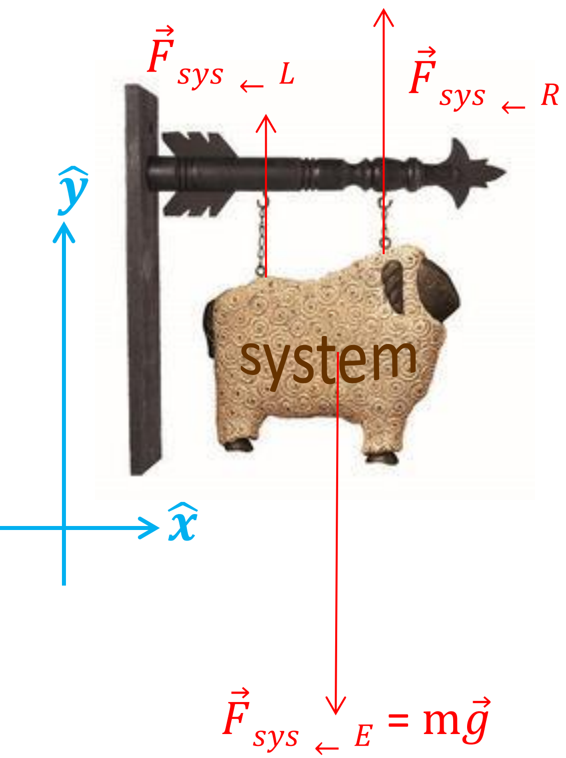
Q: Can an object ever be in equilibrium if the object is acted on by only (a) a single nonzero force, (b) two forces that point in mutually perpendicular directions, and (c) two forces that point in directions that are not perpendicular?

1 (a)
2 (b)
3 (c)
4 (a) & (b)
5 (a) & (c)
6 (b) & (c)
7 (a), (b), & (c)

Equilibrium

1-D Example.
A 4 kg sheep sign hangs outside a woolen factory; if 2/3 of the weight is born by the right chain, what is the tension in the left chain?

$$|F_{sys\leftarrow R.y}| = {}^{2}/_{3}\,|F_{sys\leftarrow E.y}|$$



$$\sum_{all} \vec{F}_{system\leftarrow} = \frac{d\vec{p}}{dt} = 0$$

$$\vec{F}_{sys\leftarrow L} + \vec{F}_{sys\leftarrow R} + \vec{F}_{sys\leftarrow E} = 0$$

$$\hat{x}:\ \sout{F_{sys\leftarrow L.x} + F_{sys\leftarrow R.x} + F_{sys\leftarrow E.x} = 0}$$

$$\hat{y}:\ F_{sys\leftarrow L.y} + F_{sys\leftarrow R.y} + F_{sys\leftarrow E.y} = 0$$

$$\hat{z}:\ \sout{F_{sys\leftarrow L.z} + F_{sys\leftarrow R.z} + F_{sys\leftarrow E.z} = 0}$$

$$F_{sys\leftarrow L.y} + F_{sys\leftarrow R.y} - mg = 0$$

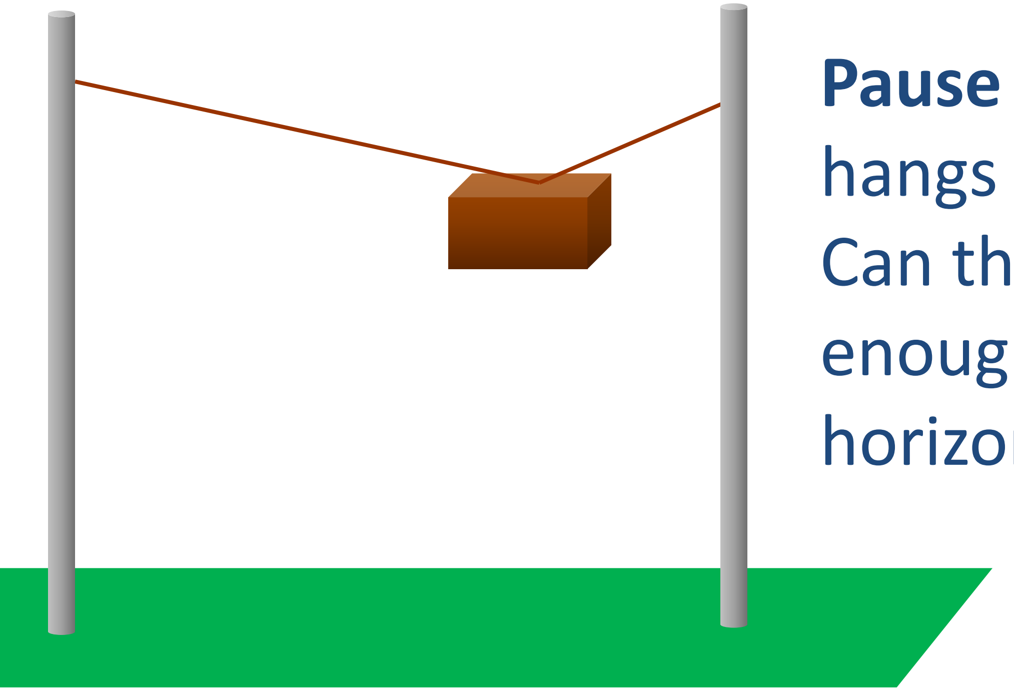
$$F_{sys\leftarrow L.y} + {}^{2}/_{3}mg - mg = 0$$

$$F_{sys\leftarrow L.y} - {}^{1}/_{3}\,mg = 0$$

$$F_{sys\leftarrow L.y} = {}^{1}/_{3}mg = {}^{1}/_{3}\,(4\text{kg})(9.8\,\tfrac{\text{m}}{\text{s}^2}) = 13.1\ \text{kg}\,\tfrac{\text{m}}{\text{s}^2} = 13.1\ N$$

Young's Modulus Tie-in: If material, radius, and initial length of wires were given, could find how much the stretch holding the sheep.

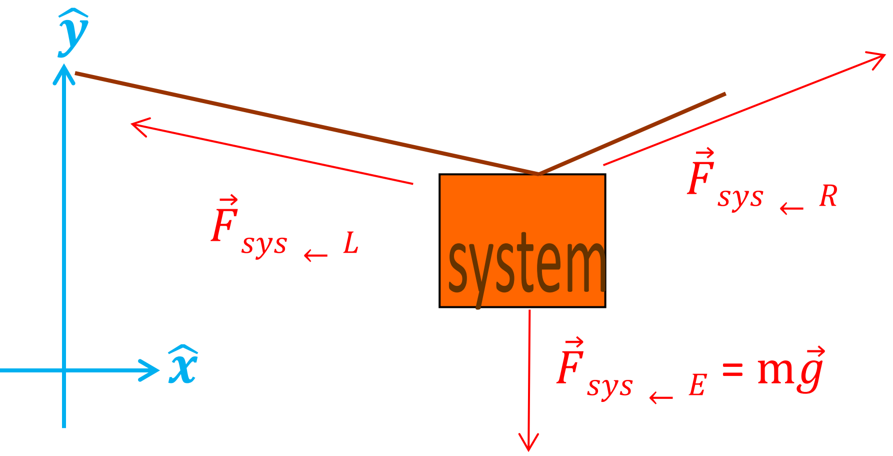
Equilibrium



Pause and Consider: A box hangs from a rope as illustrated. Can the rope be pulled tight enough to be completely horizontal?

Equilibrium

2-D Relations. A box hangs from a rope as illustrated.



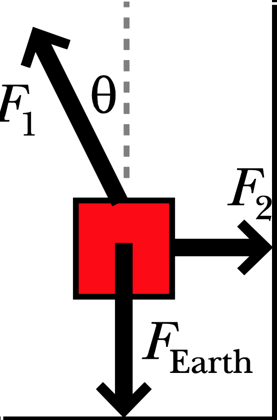
$$\sum_{all} \vec{F}_{system\leftarrow} = \frac{d\vec{p}}{dt} = 0$$

$$\vec{F}_{sys \leftarrow L} + \vec{F}_{sys \leftarrow R} + \vec{F}_{sys \leftarrow E} = 0$$

$\hat{x}$: $F_{sys \leftarrow L.x} + F_{sys \leftarrow R.x} + F_{sys \leftarrow E.x} = 0$

$\hat{y}$: $F_{sys \leftarrow L.y} + F_{sys \leftarrow R.y} + F_{sys \leftarrow E.y} = 0$

$\hat{z}$: $F_{sys \leftarrow L.z} + F_{sys \leftarrow R.z} + F_{sys \leftarrow E.z} = 0$



The object hangs in equilibrium. $|F_1|$ and $|F_2|$ are magnitudes of forces.

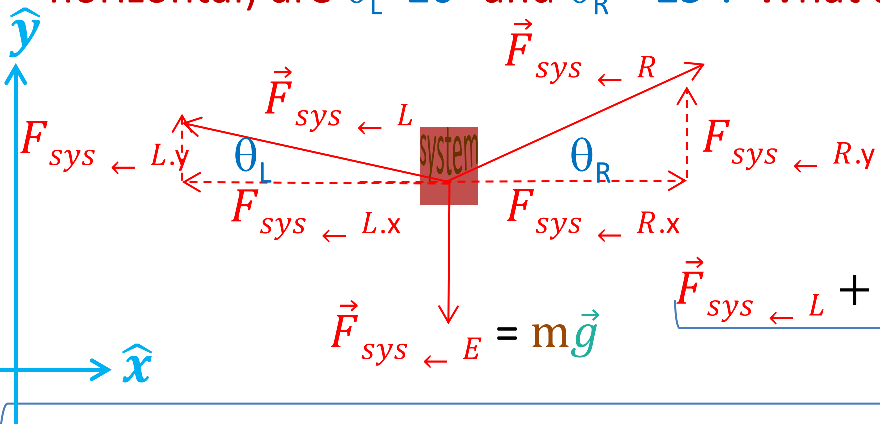
Which equation correctly states that $dp_y/dt = Fnet_y$?

a. $0 = |F_1| - mg$
b. $0 = |F_1| + |F_2| - mg$
c. $0 = -|F_1|*\cos(\theta) + |F_2|$
d. $0 = |F_1|*\cos(\theta) - mg$

Equilibrium

2-D Example.

Say the box is 10 kg and the angles of the two ropes (from the horizontal) are $\theta_L = 20°$ and $\theta_R = 25°$. What are the *magnitudes* of the tensions in each rope?



$$\vec{F}_{sys\leftarrow L} + \vec{F}_{sys\leftarrow R} + \vec{F}_{sys\leftarrow E} = 0$$

$\hat{y}$: $F_{sys\leftarrow L.y} + F_{sys\leftarrow R.y} = mg$

$$F_{sys\leftarrow Ls}in(\theta_L) + F_{sys\leftarrow R}\sin(\theta_R) = mg$$

$$F_{sys\leftarrow R}\frac{\cos(\theta_R)}{\cos(\theta_L)}\sin(\theta_L) + F_{sys\leftarrow R}sin(\theta_R) = mg$$

$$F_{sys\leftarrow Rco}s(\theta_R)\tan(\theta_L) + F_{sys\leftarrow R}sin(\theta_R) = mg$$

$$F_{sys\leftarrow R}(\cos(\theta_R)\tan(\theta_L) + \sin(\theta_R)) = mg$$

$$F_{sys\leftarrow R} = \frac{mg}{\cos(\theta_R)\tan(\theta_L) + \sin(\theta_R)}$$

$$F_{sys\leftarrow R} = \frac{(10kg)(9.8\frac{m}{s^2})}{\cos(25°)\tan(20°) + \sin(25°)}$$

$$F_{sys\leftarrow R} = 130N$$

$\hat{x}$: $F_{sys\leftarrow L.x} + F_{sys\leftarrow R.x} = 0$

$$F_{sys\leftarrow Lc}os(\theta_L) - F_{sys\leftarrow R}cos(\theta_R) = 0$$

$$F_{sys\leftarrow L} = F_{sys\leftarrow R}\frac{\cos(\theta_R)}{\cos(\theta_L)}$$

$$F_{sys\leftarrow L} = 130N\frac{\cos(25°)}{\cos(20°)}$$

$$F_{sys\leftarrow L} = 125N$$

Equilibrium

Hanging Bar Example

What’s the force at the hinge in terms of the measurable angles, tension, and weights?

(Non-static) Equilibrium

Elevator Example

What's the normal force read by scale you're standing on in an elevator that's rising at a constant rate?

Non-Equilibrium

What's the normal force read by scale you're standing on in an elevator that's accelerating up to speed?

Changing Momentum: Magnitude *and* Direction

$$\vec{F}_{net\to obj} = \frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \underbrace{\frac{d|\vec{p}|}{dt}\hat{p}}_{\text{Speeding/slowing}} + \underbrace{|\vec{p}|\frac{d\hat{p}}{dt}}_{\text{changing direction}}$$

Speeding/slowing — changing direction

Should point… Parallel to momentum vector — Perpendicular to momentum vector

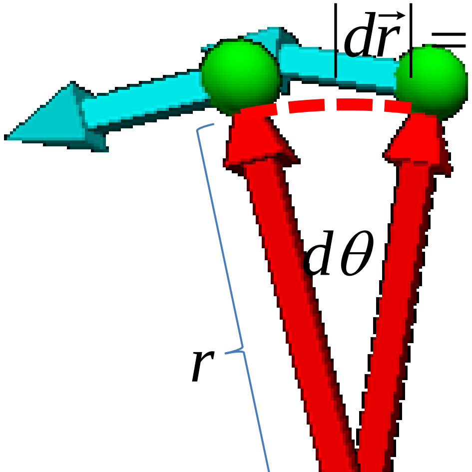
Special Case: Uniform Circular Motion (only direction changing)

$$\frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

similarly

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d|\vec{r}|}{dt}\hat{r} + |\vec{r}|\frac{d\hat{r}}{dt}$$

comparing



$$|d\vec{r}| = rd\theta$$

$$v = \left|\frac{d\vec{r}}{dt}\right| = \left|\frac{rd\theta}{dt}\right|$$

$$\left|\frac{d\vec{r}}{dt}\right| = |\vec{r}|\left|\frac{d\theta}{dt}\right|$$

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\theta}{dt}\right| = \frac{1}{|\vec{r}|}v$$

Rate of change of position vector’s direction

Changing Momentum: Magnitude *and* Direction

$$\vec{F}_{net\to obj} = \frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \underbrace{\frac{d|\vec{p}|}{dt}\hat{p}}_{} + \underbrace{|\vec{p}|\frac{d\hat{p}}{dt}}_{}$$

	Speeding/slowing	changing direction
Should point…	Parallel to momentum vector	Perpendicular to momentum vector ✓

Special Case: Uniform Circular Motion

$$\frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\hat{p}\right)}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\theta}{dt}\right| = \frac{1}{|\vec{r}|}v$$

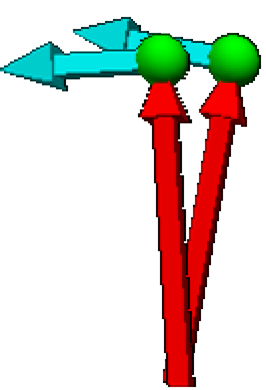
Rate of change of position vector's direction *Equals* Rate of change of momentum vector's direction

$$\left|\frac{d\hat{r}}{dt}\right| = \left|\frac{d\hat{p}}{dt}\right| = \frac{1}{|\vec{r}|}v$$

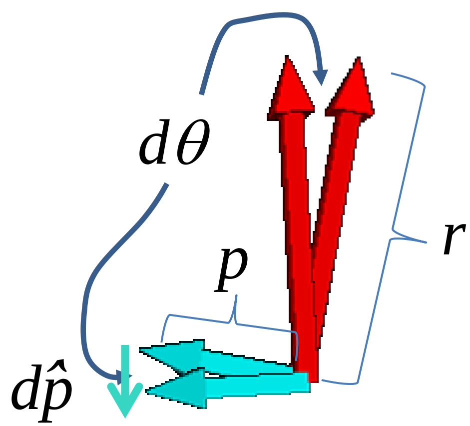
direction?

Opposite of r. $-\hat{r}$

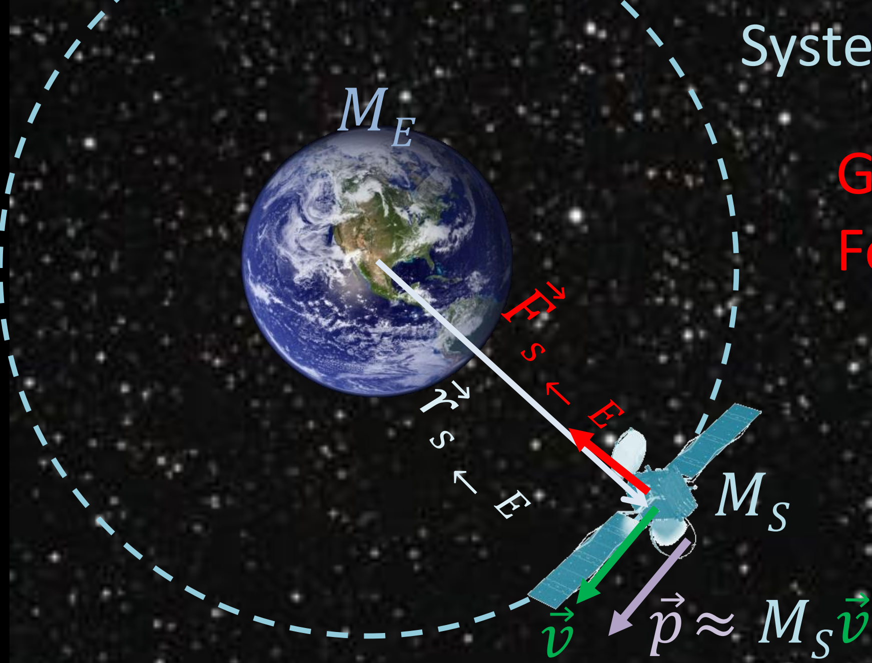
$$\frac{d\vec{p}}{dt} = -|\vec{p}|\frac{v}{|\vec{r}|}\hat{r}$$



See Vpython example



Application: Circular Gravitational Orbits



System: Satellite

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Gravitational Force | Circular Motion

$$-G\frac{M_E M_S}{|r_{s\leftarrow E}|^2}\hat{r}_{s\leftarrow E} = -|\vec{p}|\frac{|v|}{|r_{s\leftarrow E}|}\hat{r}_{s\leftarrow E}$$

$$G\frac{M_E M_S}{|r_{s\leftarrow E}|} = |\vec{p}||v|$$

$$G\frac{M_E M_S}{|r_{s\leftarrow E}|} = M_S|v||v|$$

$$G\frac{M_E}{|r_{s\leftarrow E}|} = v^2$$

Example: Geosynchronous Orbit

There's only one orbital radius for satellites that 'stay put' in the sky – orbit with the same period as the Earth spins: T = 1 day. What's the orbital radius?

$$v = \frac{distance}{time} = \frac{Circumferance}{Period} = \frac{2\pi r_{s\leftarrow E}}{T}$$

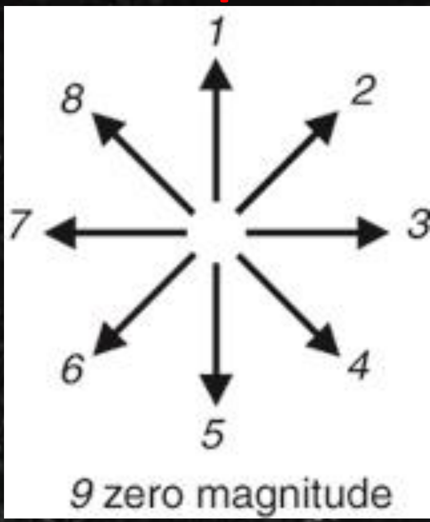
$$G\frac{M_E}{|r_{s\leftarrow E}|} = \left(\frac{2\pi r_{s\leftarrow E}}{T}\right)^2$$

$$r_{s\leftarrow E} = \left(GM_E\left(\frac{T}{2\pi}\right)^2\right)^{\frac{1}{3}} = \left(\left(6.7\times10^{-11}\frac{Nm^2}{kg^2}\right)\left(6\times10^{24}kg\right)\left(\frac{86{,}400s}{2\pi}\right)^2\right)^{\frac{1}{3}} = 4.2\times10^7 m$$

Application: Circular Gravitational Orbits

The Moon travels in a nearly circular orbit around the Earth, at nearly constant speed.

what is the direction of $\frac{d\vec{p}}{dt}$?



A geosynchronous satellite has an orbital radius of 4.2×10^7m.
If the moon's period were 30 days (it's really about 27), what would be its orbital radius?

a) 13×10^7m
b) 41×10^7m
c) 126×10^7m
d) 690×10^7m

Kepler's 3rd Law of Planetary Motion