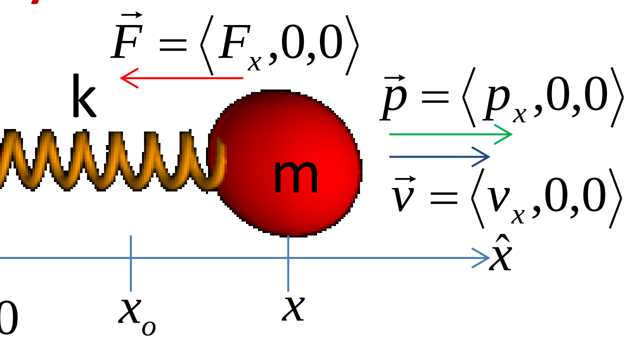
Mon.	4.8, .13 Friction and Buoyancy & Suction	RE 4.d
Tues		EP 4, HW4: Ch 4 Pr's 46, 50, 81, 88 & CP
Wed.	5.1-.5 Rate of Change & Components Quiz 4	RE 5.a bring laptop, smartphone, pad,…
Lab	L4b: Buoyancy, Review for Exam 1(Ch 1-4)	Practice Exam 1 (due beginning of lab)
Fri.	Exam 1 (Ch 1-4)	

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

System: Ball



$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Solution

$$x(t) = X \cos(\omega t) + x_o$$

where: $\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

Concisely tells us…

$$x(t) = X \cos(\omega t) + x_o$$

- **Sinusoidally oscillates**
- **About the equilibrium**
- **With a period that…**
  - **Shortens with greater stiffness**
  - **Lengthens with larger masses**

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

  - **Doesn't care about amplitude**

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Period dependence on Stiffness:

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we double the spring stiffness? (We could use a stiffer spring, or we could attach the mass to two springs.)

a. T = 0.5 s
b. T = 0.7 s
c. T = 1.0 s
d. T = 1.4 s
e. T = 2.0 s

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

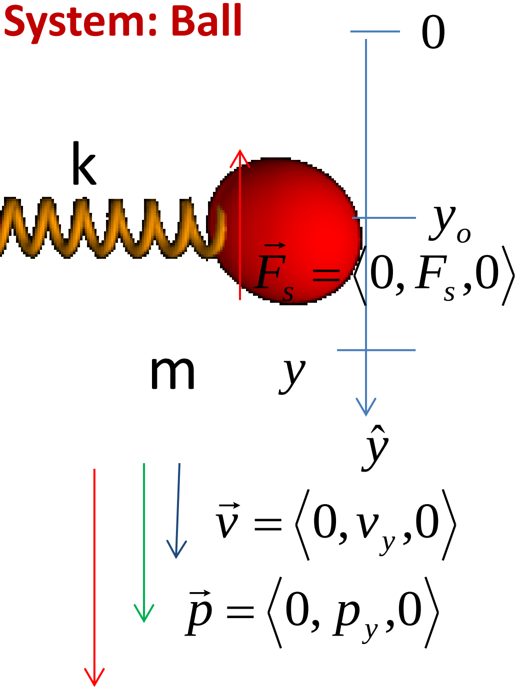
Period Dependence on Amplitude: Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we increase the amplitude to 10 cm, so that the total distance traveled in one period is twice as large?	1) T = 0.5 s 2) T = 0.7 s 3) T = 1.0 s 4) T = 1.4 s 5) T = 2.0 s

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

How does gravitational interaction change behavior?

System: Ball



$$\vec{F}_E = \langle 0, mg, 0 \rangle$$

Note: I've defined *down* as +y direction
So Earth's pull has + sign

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + \frac{k}{k} mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + k\left[\frac{mg}{k}\right]$$

$$F_{net.y}(t) = -k * \left[y(t) - y_o - \frac{mg}{k}\right]$$

$$F_{net.y}(t) = -k * \left[y(t) - \left\{y_o + \frac{mg}{k}\right\}\right]$$

$$F_{net.y}(t) = -k * [y(t) - y_o']$$

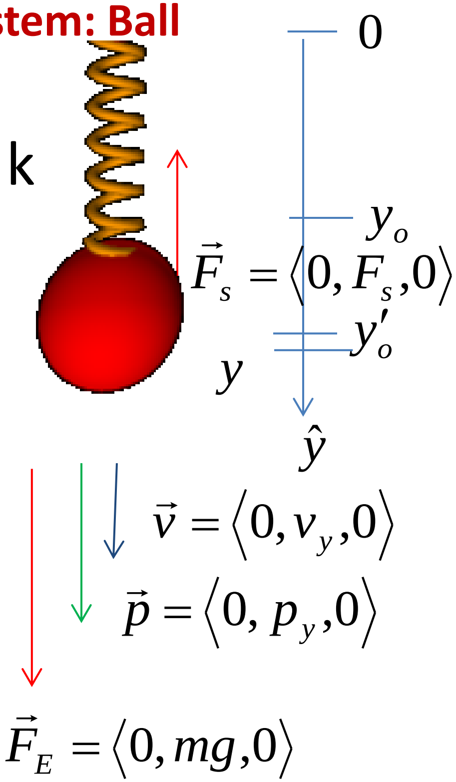
where $y_o' \equiv y_o + \frac{mg}{k}$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

How does gravitational interaction change behavior?

System: Ball



$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

$$\vdots$$

$$F_{net.y}(t) = -k * [y(t) - y'_o]$$ where $y'_o \equiv y_o + \frac{mg}{k}$

- Exact same form as for horizontal mass-spring, but shifted equilibrium

$$m \frac{d^2}{dt^2} y(t) = -k * [y(t) - y'_o]$$

HW pr 81 hint: write similar expression in terms of r, 'read off' what plays role of "k", and find corresponding T.

Note: I've defined *down* as +y direction

So Earth's pull has + sign

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

Solution:

$$y(t) = Y \cos(\omega t) + y'_o$$

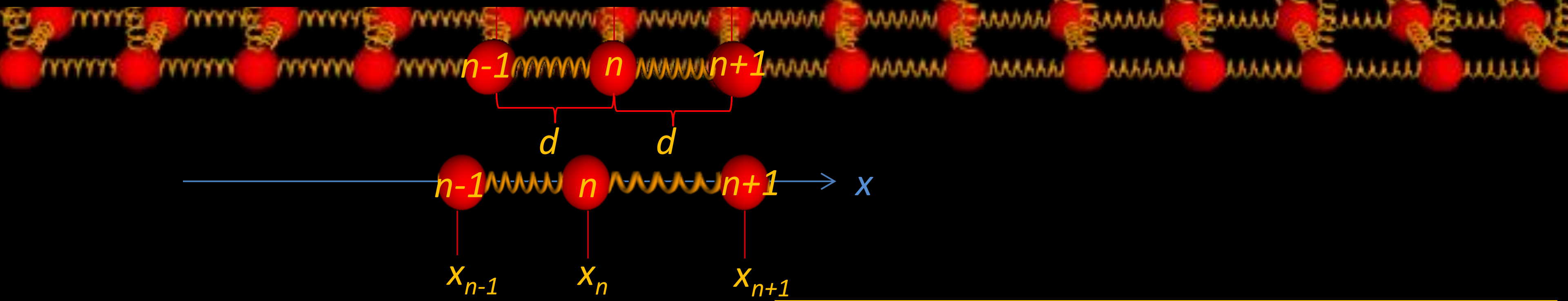
$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

Period dependence on *g*:

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we take the oscillator to a massive planet where g = 19.6 N/kg?

1) T = 0.5 s
2) T = 0.7 s
3) T = 1.0 s
4) T = 1.4 s
5) T = 2.0 s

Speed of Sound in a Solid: the logic



$$F_{n,net} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{dp_n}{dt} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{d(mv_n)}{dt} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{d\left(\frac{dx_n}{dt}\right)}{dt} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = \frac{k_s}{m} d\left(\frac{(x_{n+1} - x_n)}{d} - \frac{(x_n - x_{n-1})}{d}\right)$$

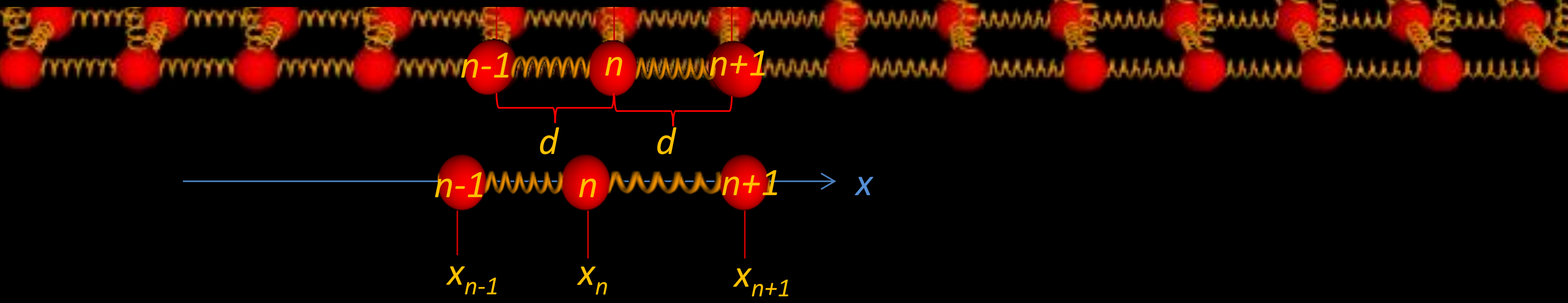
$$\frac{\mathrm{d}^2 \varepsilon_n}{\mathrm{d}t^2} \approx -\frac{k_s}{m} d^2 \frac{\left(\frac{\mathrm{d}x_{n+1}}{\mathrm{d}x} - \frac{\mathrm{d}x_n}{\mathrm{d}x}\right)}{d}$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} \approx -\frac{k_s}{m} d^2 \frac{\mathrm{d}^2 x_{n+1}}{\mathrm{d}x^2}$$

Speed of Sound in a Solid: the result

$$v = \sqrt{\frac{k_s}{m}d}$$

Speed of Sound in a Solid



$$v = \sqrt{\frac{k_s}{m}}\,d$$

Stiffer, for a given atomic displacement, greater force pulling it so greater velocity achieved.

More distance between atoms means further the distortion can propagate just through the light weight spring /bond without encountering the resistance of massive atoms.

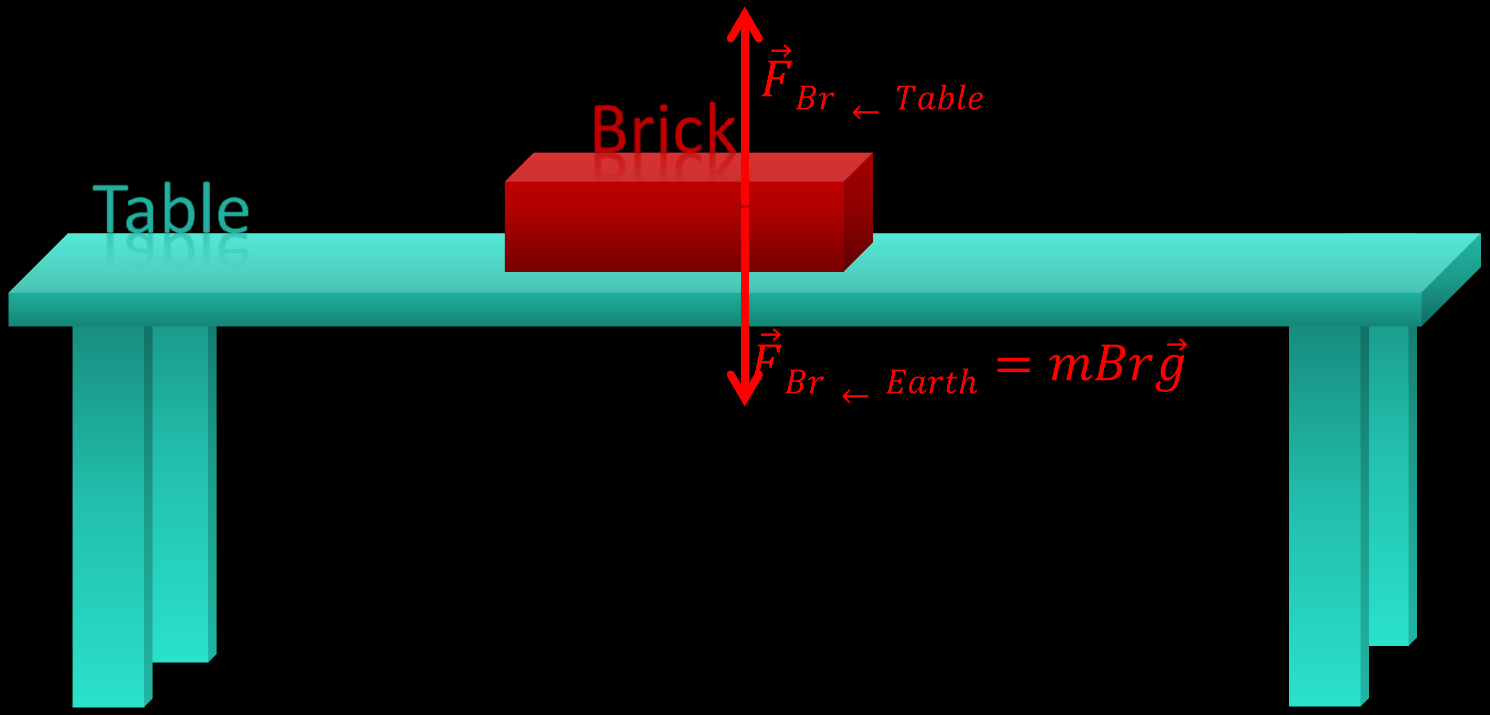
More massive, more inertial resistance to applied force, less velocity achieved.

Compression (Normal) Force

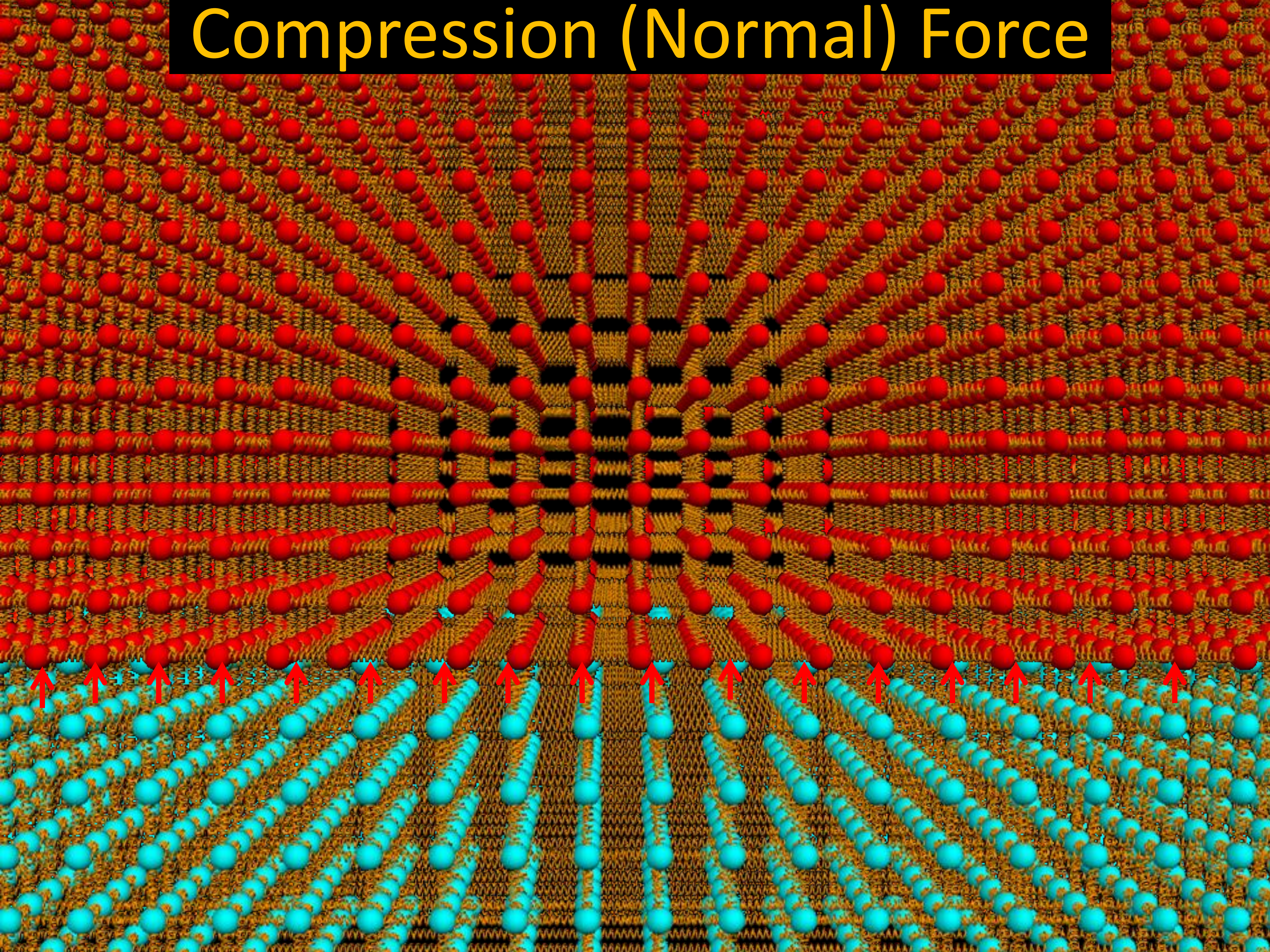
$$\vec{F}_{Br \leftarrow net} = \frac{d\vec{p}_{Br}}{dt}$$

$$\vec{F}_{Br \leftarrow Table} + \vec{F}_{Br \leftarrow Earth} = 0$$

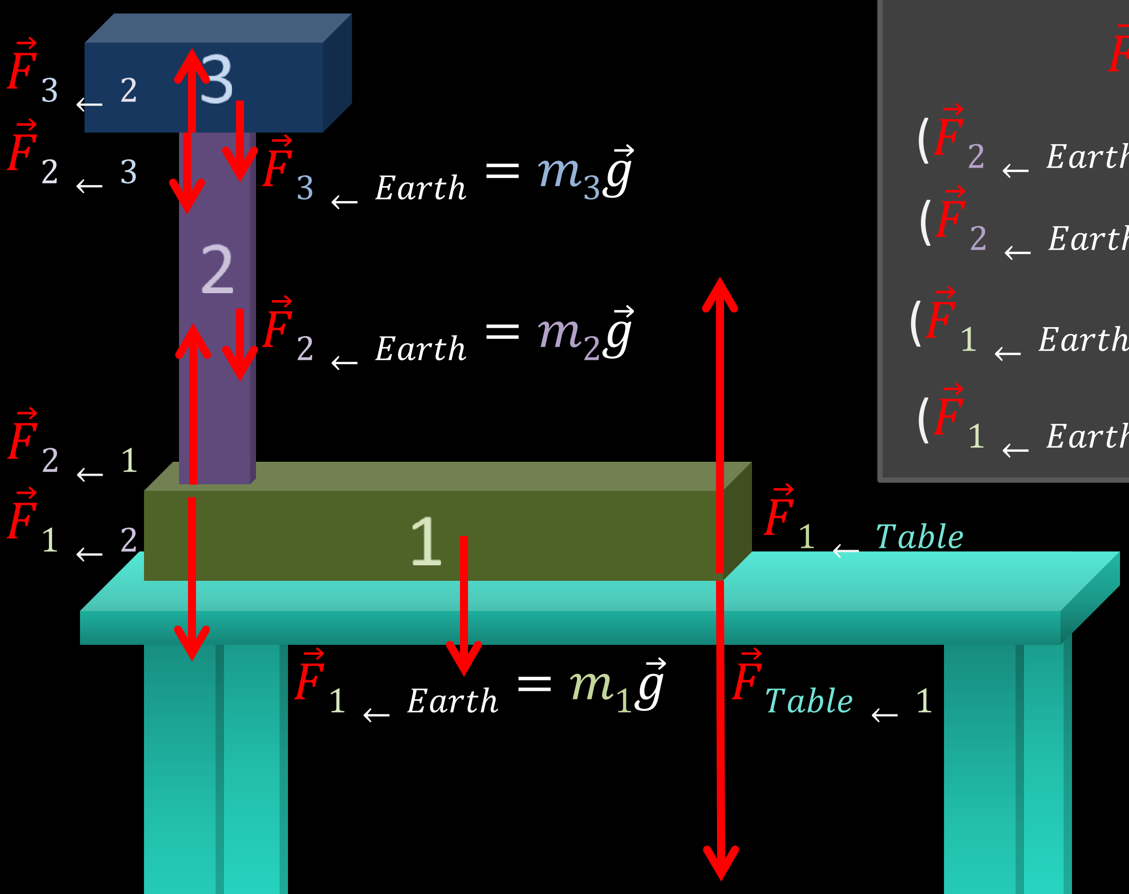
$$\vec{F}_{Br \leftarrow Table} = -\vec{F}_{Br \leftarrow Earth}$$



Compression (Normal) Force



Compression (Normal) Force
Stacked Objects



By Momentum Update (Newton's 2nd Law)

$$\vec{F}_{3 \leftarrow net} = \frac{d\vec{p}_3}{dt}$$

$$\vec{F}_{3 \leftarrow Earth} + \vec{F}_{3 \leftarrow 2} = 0$$

By Reciprocity (Newton's 3rd Law)

$$\vec{F}_{3 \leftarrow Earth} = -\vec{F}_{3 \leftarrow 2} = \vec{F}_{2 \leftarrow 3}$$

$$(\vec{F}_{2 \leftarrow Earth} + \vec{F}_{2 \leftarrow 3}) = -\vec{F}_{2 \leftarrow 1} = \vec{F}_{1 \leftarrow 2}$$

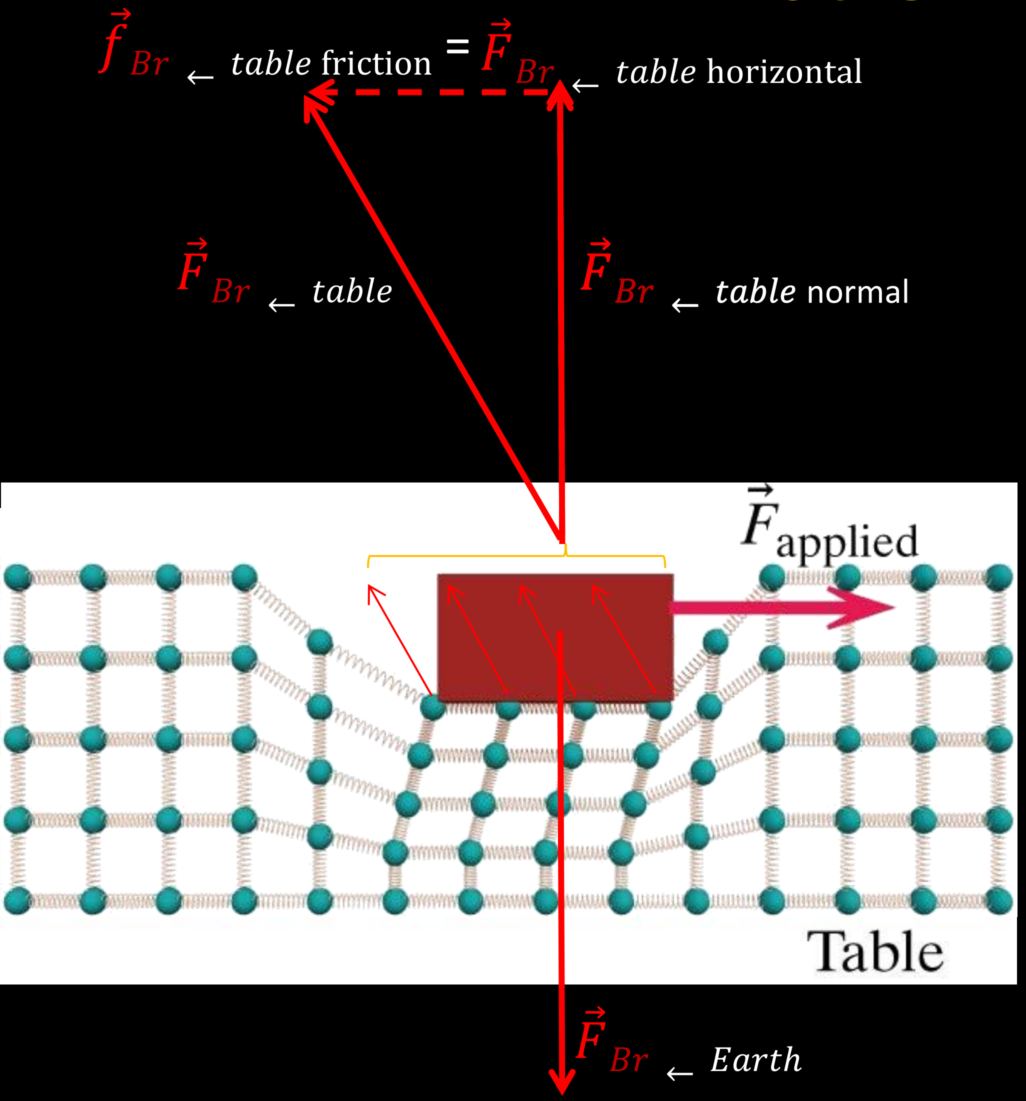
$$(\vec{F}_{2 \leftarrow Earth} + \vec{F}_{3 \leftarrow Earth}) = \vec{F}_{1 \leftarrow 2}$$

$$(\vec{F}_{1 \leftarrow Earth} + \vec{F}_{1 \leftarrow 2}) = -\vec{F}_{1 \leftarrow Table} = \vec{F}_{Table \leftarrow 1}$$

$$(\vec{F}_{1 \leftarrow Earth} + \vec{F}_{2 \leftarrow Earth} + \vec{F}_{3 \leftarrow Earth}) = \vec{F}_{Table \leftarrow 1}$$

$$(m_1 + m_2 + m_3)\,\vec{g} = \vec{F}_{Table \leftarrow 1}$$

Friction Force



Experiment

$f_{friction} \propto F_{normal}$

sliding (kinetic)	stationary (static)
$f_{friction} \approx \mu_k F_{normal}$	$f_{friction} \leq \mu_s F_{normal}$

sliding (kinetic)	stationary (static)
$f_{friction} \approx \mu_k F_{normal}$	$f_{friction} \leq \mu_s F_{normal}$

You push a 100 kg mass on the floor with a horizontal force of 400 N. It doesn't move. The coefficient of static friction is 0.6.	What is the magnitude of the frictional force on the block by the floor? a. 980 N b. 588 N c. 400 N d. Can't tell

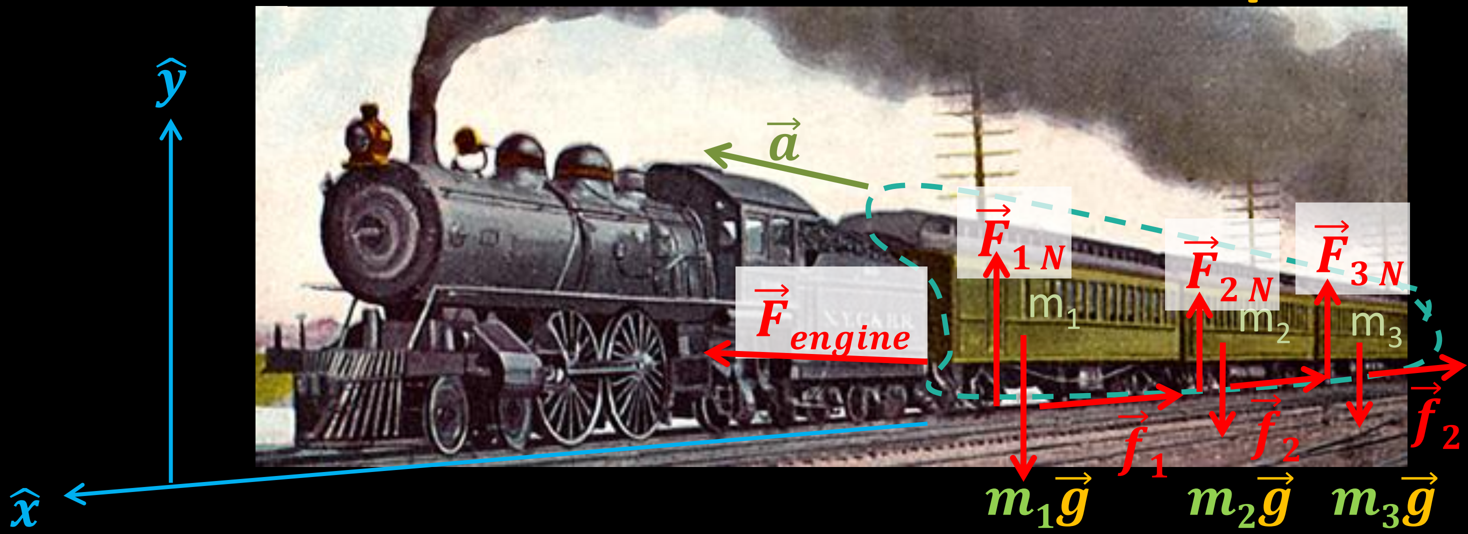
sliding (kinetic)	stationary (static)
$f_{friction} \approx \mu_k F_{normal}$	$f_{friction} \leq \mu_s F_{normal}$

You push a 100 kg mass on the floor with a horizontal force of 400 N, and it's moving in the direction you are pushing. The coefficient of static friction is 0.3.	What happens to the speed of the block while you push it? a. The speed increases b. The speed decreases c. The speed does not change d. Can't tell

sliding (kinetic)	stationary (static)
$f_{friction} \approx \mu_k F_{normal}$	$f_{friction} \le \mu_s F_{normal}$

You push a 100 kg mass on the floor with a horizontal force, and it's moving in the direction you are pushing at a constant speed. The coefficient of kinetic friction is 0.3.	How much force are you exerting on the block? a. 980 N b. 294 N c. 490 N d. Can't tell

Friction Force Example



a) What's the acceleration of the whole train in terms of the masses, coefficient of friction, and the force exerted by the engine?

System = train cars (excluding the engine)

μ_k = kinetic coefficient of friction

$f_1 = \mu_k F_{1N}$

No vertical acceleration

$\vec{F}_{1N} + m_1\vec{g} = 0$

$F_{1N} = m_1 g$

$f_1 = \mu_k m_1 g$

Similarly, $f_2 = \mu_k m_2 g$ *and* $f_3 = \mu_k m_3 g$

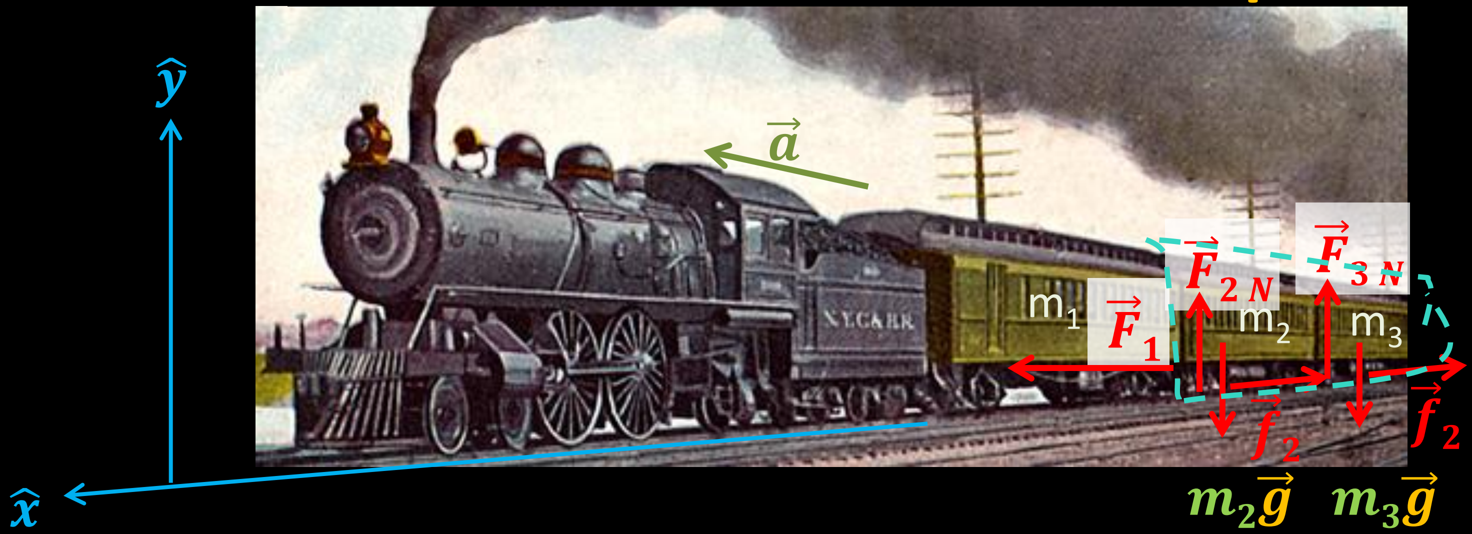
$$\vec{F}_{train \leftarrow net} \approx m_{train}\vec{a}$$

$$\vec{F}_{engine} + \vec{f}_1 + \vec{f}_2 + \vec{f}_3 \approx m_{train}\vec{a}$$

$$F_{engine} - (f_1 + f_2 + f_3) \approx (m_1 + m_2 + m_3)a$$

$$\frac{F_{engine} - (f_1 + f_2 + f_3)}{(m_1 + m_2 + m_3)} \approx a \approx \frac{F_{engine} - (\mu_k m_1 g + \mu_k m_2 g + \mu_k m_3 g)}{(m_1 + m_2 + m_3)} = \frac{F_{engine}}{(m_1 + m_2 + m_3)} - \mu_k g$$

Friction Force Example



a) What's the acceleration of the whole train in terms of the masses, coefficient of friction, and the force exerted by the engine?

$$a \approx \frac{F_{engine}}{(m_1+m_2+m_3)} - \mu_k g$$

$$f_2 = \mu_k m_2 g \text{ and } f_3 = \mu_k m_3 g$$

b) What's the force the first car exerts on the second?

System = last two train cars

$$\vec{F}_1 + \vec{f}_2 + \vec{f}_3 \approx m_{system}\vec{a}$$

$$F_1 - (f_2 + f_3) \approx (m_2 + m_3)a$$

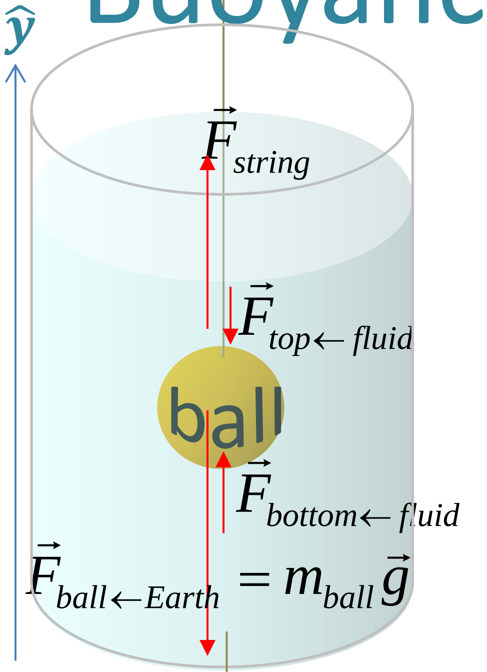
$$F_1 \approx (m_2 + m_3)a + (f_2 + f_3)$$

$$F_1 \approx (m_2 + m_3)\left(\frac{F_{engine}}{(m_1+m_2+m_3)} - \mu_k g\right) + (\mu_k m_2 g + \mu_k m_3 g)$$

$$F_1 \approx \frac{(m_2+m_3)F_{engine}}{(m_1+m_2+m_3)} - \mu_k g(m_2 + m_3) + \mu_k g(m_2+m_3)$$

$$F_1 \approx \frac{(m_2+m_3)}{(m_1+m_2+m_3)} F_{engine}$$

Buoyancy and Archimedes' Principle



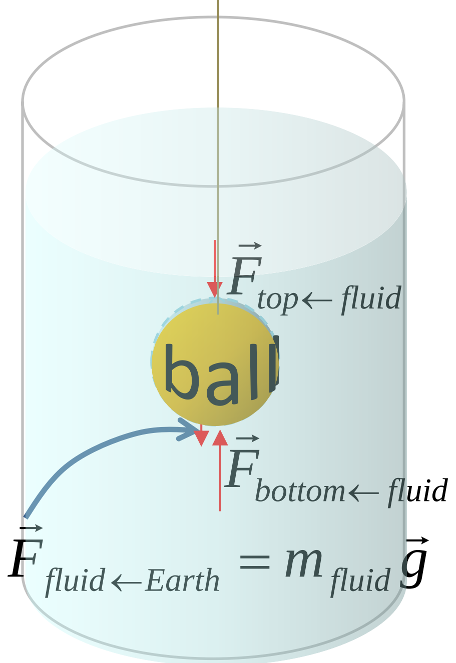
System: brass ball

$$\frac{dp_{ball.y}}{dt} = F_{net.y}$$

$$0 = F_{string} - m_{ball}g + \left(F_{bottom \leftarrow fluid} - F_{top \leftarrow fluid}\right)$$

$$F_{Buoy} \equiv F_{bottom \leftarrow fluid} - F_{top \leftarrow fluid}$$

$$0 = F_{string} - m_{ball}g + F_{Buoy}$$



System: displaced-volume of fluid 'ball'

$$\frac{dp_{fluid.y}}{dt} = F_{net.y}$$

$$0 = -m_{fluid.displaced}\,g + \left(F_{bottom \leftarrow fluid} - F_{top \leftarrow fluid}\right)$$

$$0 = -m_{fluid.displaced}\,g + F_{Buoy}$$

$$F_{Buoy} = m_{fluid.displaced}\,g$$

Archimedes' Principle:

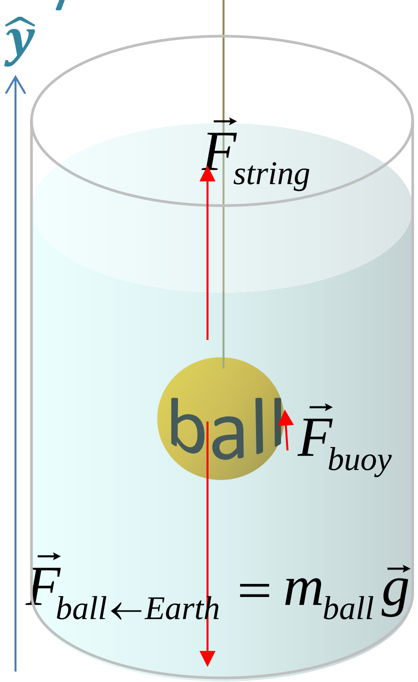
Buoyant force = weight of the fluid displaced

Buoyancy and Archimedes' Principle

In terms of volumes and densities

density $\rho \equiv \dfrac{m}{V}$ mass / Volume

System: brass ball



$$0 = F_{string} - m_{ball} g + F_{Buoy}$$

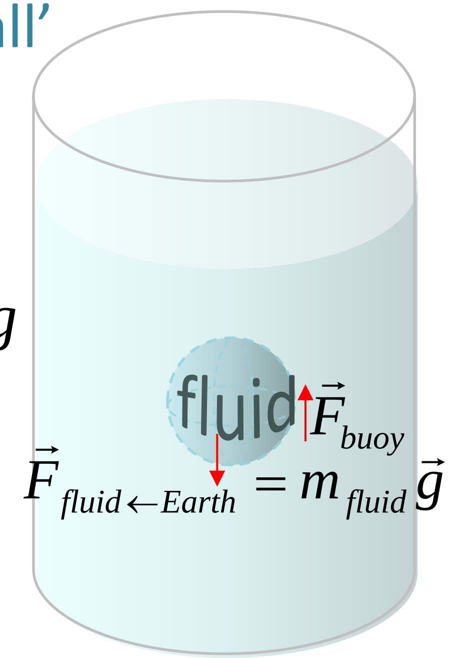
$$m_{ball} = V_{ball}\rho_{ball}$$

$$F_{Buoy} = m_{fluid.displaced}\, g$$

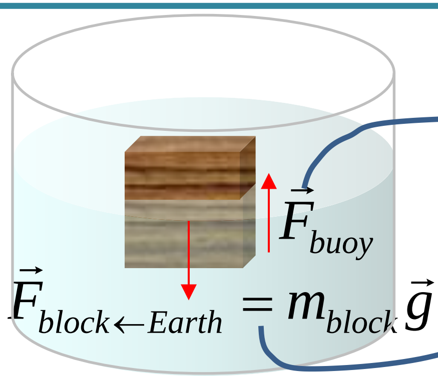
$$m_{fluid.displaced} = V_{displaced}\,\rho_{fluid}$$

$$0 = F_{string} - V_{ball}\rho_{ball} g + V_{displaced}\,\rho_{fluid}\, g$$

System: displaced-volume fluid 'ball'



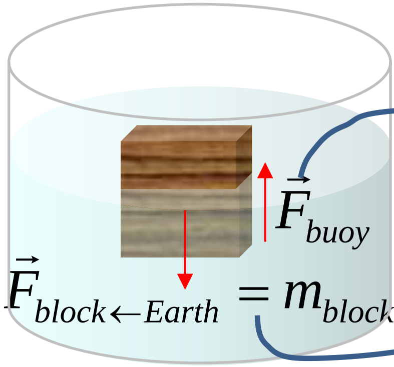
Special case - floating



$$0 = -V_{ball}\rho_{ball} g + V_{displaced}\,\rho_{fluid}\, g$$

$$V_{ball}\rho_{ball} = V_{displaced}\,\rho_{fluid}$$

Special case - floating



$\vec{F}_{buoy}$

$$\vec{F}_{block \leftarrow Earth} = m_{block}\vec{g}$$

$$0 = -V_{ball}\rho_{ball}g + V_{displaced}\rho_{fluid}g$$

$$V_{ball}\rho_{ball} = V_{displaced}\rho_{fluid}$$

Say it's only $^2/_3$ submerged and $^1/_3$ above water. So, if the density of water is 1g/cm^3, then what is the density of the wood?

Ex. Hot Air balloon

$\vec{F}_{top}$

$V_{balloon} = V_{air\ disp}$

$\uparrow \vec{F}_{buoy} = m_{air\ disp}.\vec{g}$

$\vec{F}_{bottom}$

$\vec{F}_{Earth} = m_{balloon}\vec{g}$

Total mass: basket, air in balloon, payload, etc.

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