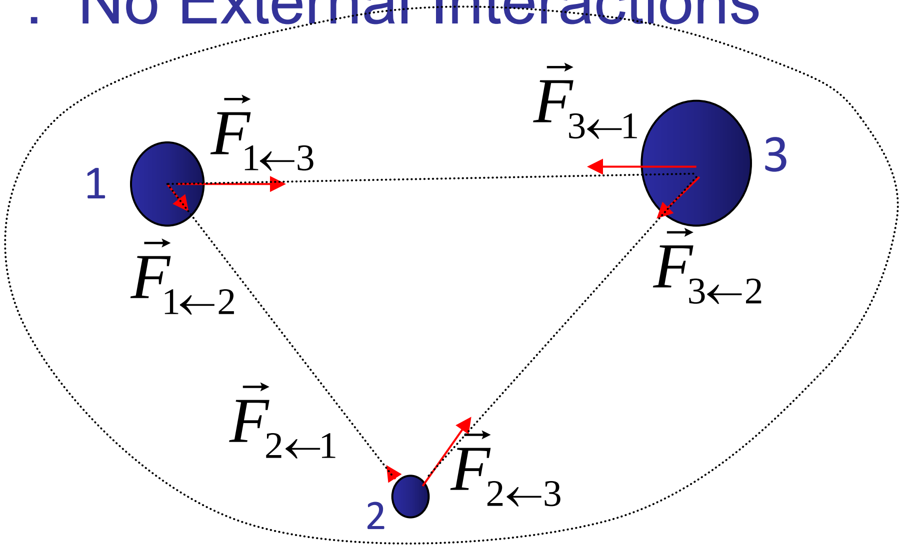
Fri.	3.11 –.13 Conservation of Momentum & Multiple Particles	RE3.c
Mon. Tues Wed.	4.1-.5 Atomic nature of matter / springs *InStove, here at noon (counts for EP)* *Summer Research Poster Session, Hedco - 7-9pm (counts for EP)*	RE 4.a EP 3, HW3: Ch 3 Pr's 42, 46, 58, 65, 72 & CP

Multi particle Systems

- Momentum Principle
- Center of Mass

"Closed / Isolated System": No External Interactions



Reciprocity

$$\vec{F}_{3\to 1} = -\vec{F}_{1\to 3}$$
$$\vec{F}_{2\to 1} = -\vec{F}_{1\to 2}$$
$$\vec{F}_{3\to 2} = -\vec{F}_{2\to 3}$$

$$\frac{\Delta \vec{p}_1}{\Delta t} = \vec{F}_{net.1} = \vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3}$$

$$\frac{\Delta \vec{p}_2}{\Delta t} = \vec{F}_{net.2} = \vec{F}_{2\leftarrow 1} + \vec{F}_{2\leftarrow 3}$$

$$\frac{\Delta \vec{p}_3}{\Delta t} = \vec{F}_{net.3} = \vec{F}_{3\leftarrow 1} + \vec{F}_{3\leftarrow 2}$$

$$\frac{\Delta \vec{p}_1}{\Delta t} + \frac{\Delta \vec{p}_2}{\Delta t} + \frac{\Delta \vec{p}_3}{\Delta t} = (\vec{F}_{1\leftarrow 3} + \vec{F}_{3\leftarrow 1}) + (\vec{F}_{3\leftarrow 2} + \vec{F}_{2\leftarrow 3}) + (\vec{F}_{1\leftarrow 2} + \vec{F}_{2\leftarrow 1})$$

$$\frac{\Delta}{\Delta t}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) = (0) + (0) + (0)$$

$$\frac{\Delta \vec{p}_{total}}{\Delta t} = 0$$

Momentum is conserved

Example: You and a friend each hold a lump of wet clay. Each lump has a mass of 20 grams. You each toss your clay into the air where they collide and stick together. Just before impact, the velocity of one lump was <5,2,-3> m/s and the velocity of the other was <-3,0,-2> m/s.

a. What was the total momentum of both lumps just before the collision?

b. What is the velocity of the stuck-together lumps just after the collision?

Your Try: A system consists of a 3 kg block moving with velocity $\langle 11, 14, 0 \rangle$ m/s and a 5 kg block moving with velocity $\langle -4, 3, 0 \rangle$ m/s.

(a) What is the momentum of this two-block system?

(b) Next, due to interactions between the two blocks, each of their velocities change, but the two-block system is nearly isolated from the surroundings. Now what is the momentum of the two-block system?

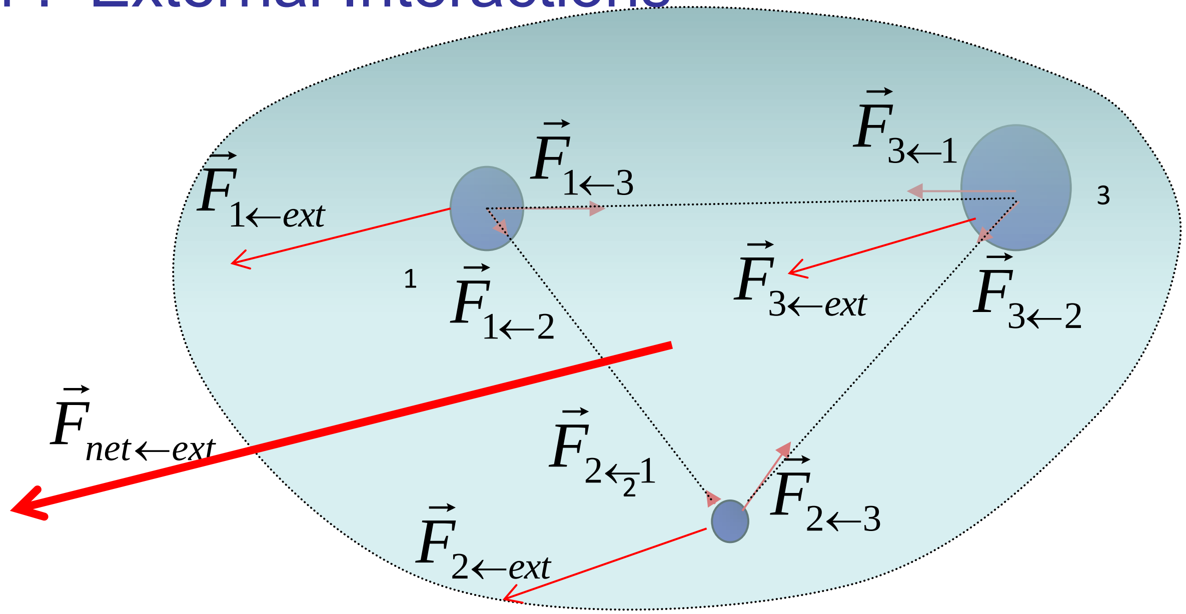
"Open System": External Interactions

Reciprocity

$$\vec{F}_{3\to1} = -\vec{F}_{1\to3}$$

$$\vec{F}_{2\to1} = -\vec{F}_{1\to2}$$

$$\vec{F}_{3\to2} = -\vec{F}_{2\to3}$$



$$\frac{\Delta\vec{p}_1}{\Delta t} = \vec{F}_{net.1} = \vec{F}_{ext.1} + \vec{F}_{1\leftarrow2} + \vec{F}_{1\leftarrow3}$$

$$\frac{\Delta\vec{p}_2}{\Delta t} = \vec{F}_{net.2} = \vec{F}_{ext.2} + \vec{F}_{2\leftarrow1} + \vec{F}_{2\leftarrow3}$$

$$\frac{\Delta\vec{p}_3}{\Delta t} = \vec{F}_{net.3} = \vec{F}_{ext.3} + \vec{F}_{3\leftarrow1} + \vec{F}_{3\leftarrow2}$$

$$\frac{\Delta\vec{p}_1}{\Delta t} + \frac{\Delta\vec{p}_2}{\Delta t} + \frac{\Delta\vec{p}_3}{\Delta t} = \vec{F}_{ext.1} + \vec{F}_{ext.2} + \vec{F}_{ext.3} + (\vec{F}_{1\leftarrow3} + \vec{F}_{3\leftarrow1}) + (\vec{F}_{3\leftarrow2} + \vec{F}_{2\leftarrow3}) + (\vec{F}_{1\leftarrow2} + \vec{F}_{2\leftarrow1})$$

$$\frac{\Delta}{\Delta t}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3) = \vec{F}_{ext.1} + \vec{F}_{ext.2} + \vec{F}_{ext.3} + (0) + (0) + (0)$$

$$\frac{\Delta\vec{p}_{total}}{\Delta t} = \vec{F}_{ext.net}$$

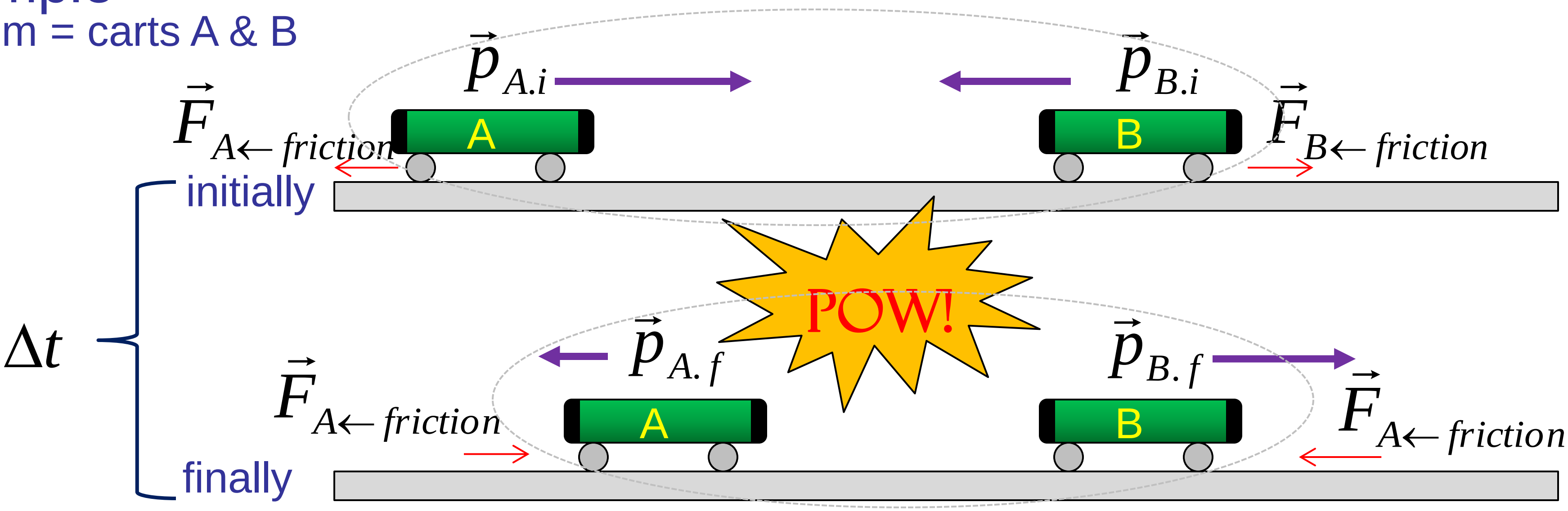
Identical as for a single object

Collision: Operational definition: quick enough/strong enough interaction that all others are negligible

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

Example

System = carts A & B



$$\Delta\vec{p}_{system} = \vec{F}_{system\leftarrow net.ext}\Delta t$$

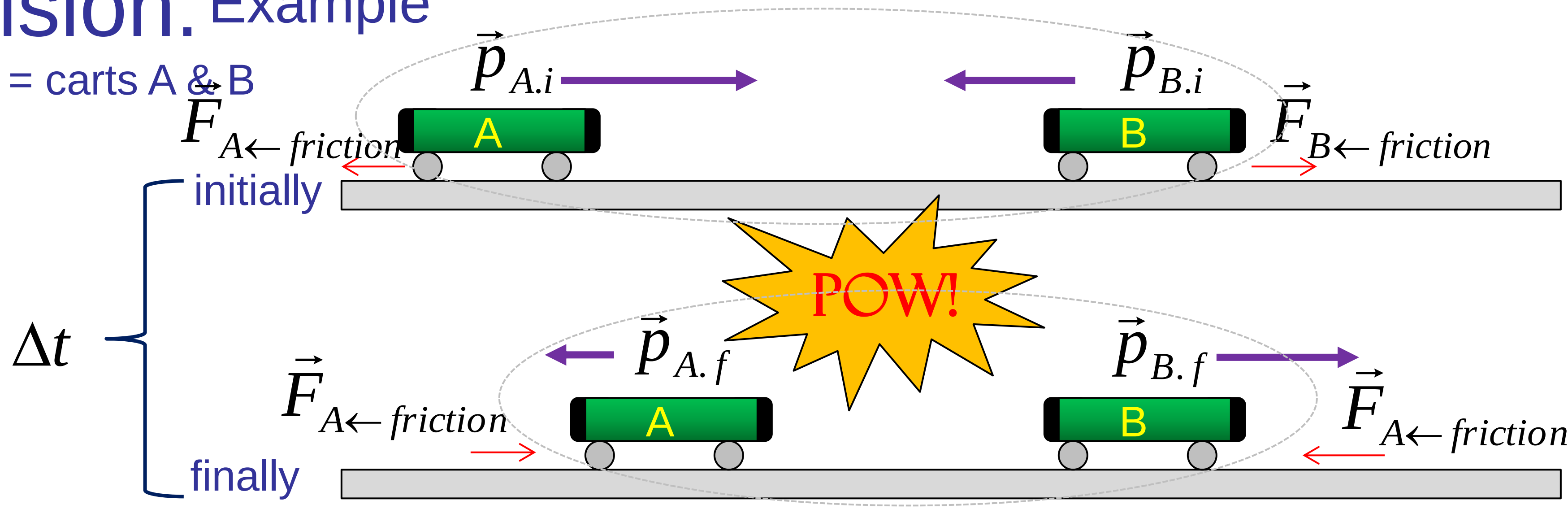
$$\Delta\vec{p}_A + \Delta\vec{p}_B = (\vec{F}_{A\leftarrow friction} + \vec{F}_{B\leftarrow friction} + \ldots)\Delta t$$

$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0$$

If we look *right* before and *right* after collision

Collision: Example

System = carts A & B



$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0 \qquad (v's << c)$$

$$\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right) \approx 0$$

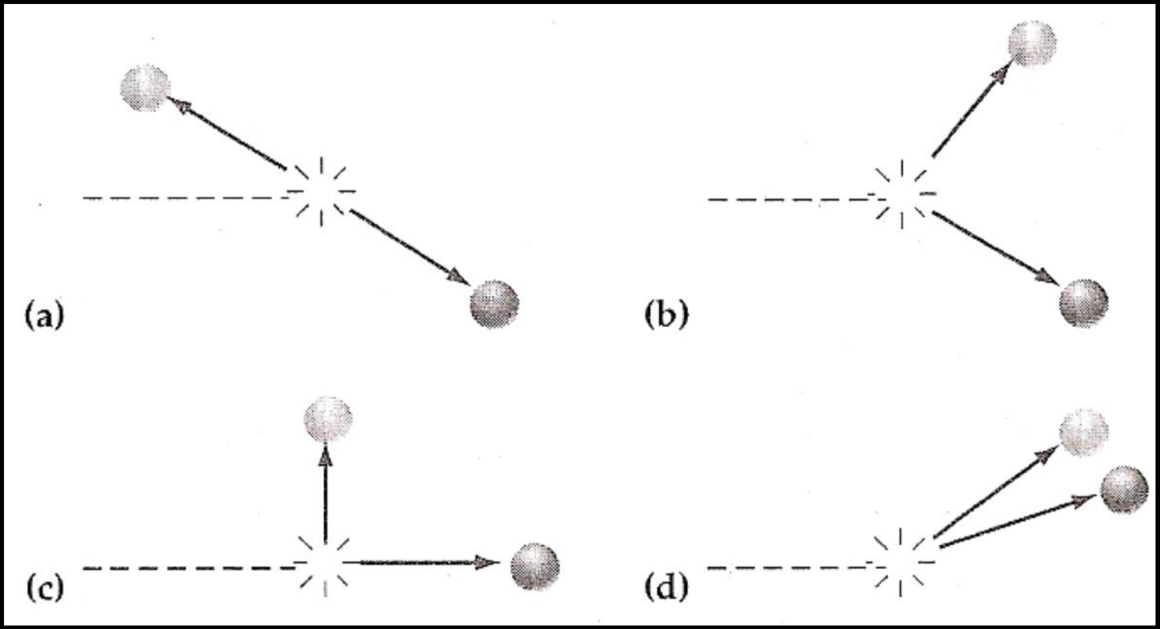
Rearranging

A space satellite of mass 500 kg has velocity < 12, 0, –8 > m/s just before being struck by a rock of mass 3 kg with velocity < –3000, 0, 900 > m/s.

After the collision the rock's velocity is < 700, 0, –300 > m/s. Now what is the velocity of the space satellite?

a. < –5100, 0, –400 > m/s
b. < –10.2, 0, –0.8 > m/s
c. < 10.2, 0, 0.8 > m/s
d. < –3688, 0, 1191 > m/s
e. < 3688, 0, –1192 > m/s

The following diagrams show hypothetical results for collisions between two identical balls floating in space. The white ball was initially moving to the right along the dotted line before it hit the gray ball, which was initially at rest. The collision is not necessarily head-on. The arrows depict the balls' final velocities. Which outcome is physically possible?



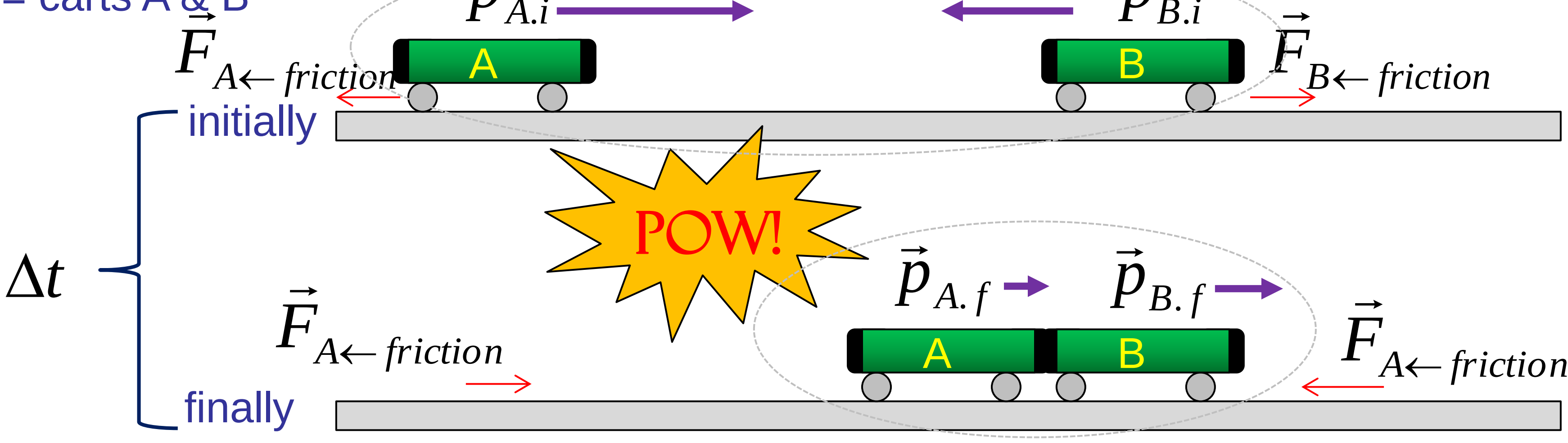
Suppose you have two bullets with equal masses. One is made of metal and the other is made of rubber. If the two bullets can be shot at the same speed, which of the following gives a wooden block a higher speed after the collision?

(a) The metal bullet gets embedded in the block.

(b) The rubber bullet bounces off of the block.

Collision: Perfectly Inelastic (hit & stick)

System = carts A & B



$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0 \qquad (v's << c)$$

$$\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right) \approx 0$$

Specializing

A bullet of mass 0.04 kg traveling horizontally at a speed of 800 m/s embeds itself in a block of mass 0.50 kg that is sitting at rest on a very slippery sheet of ice.

Which equation will correctly give the final speed v_{f_BLOCK} of the block?

a) (0.04 kg)*(800 m/s) = (0.50 kg) *v_{f_BLOCK}
b) (0.04 kg)*(800 m/s) = (0.04 kg) *v_{f_BLOCK}
c) (0.04 kg)*(800 m/s) = (0.50 kg) *v_{f_BLOCK} + (0.04 kg)*(800 m/s)
d) (0.04 kg)*(800 m/s) = (0.54 kg) *v_{f_BLOCK}
e) (0.04 kg)*(800 m/s) = (0.5 kg) *v_{f_BLOCK} + (0.04 kg)*v_{f_bullet}

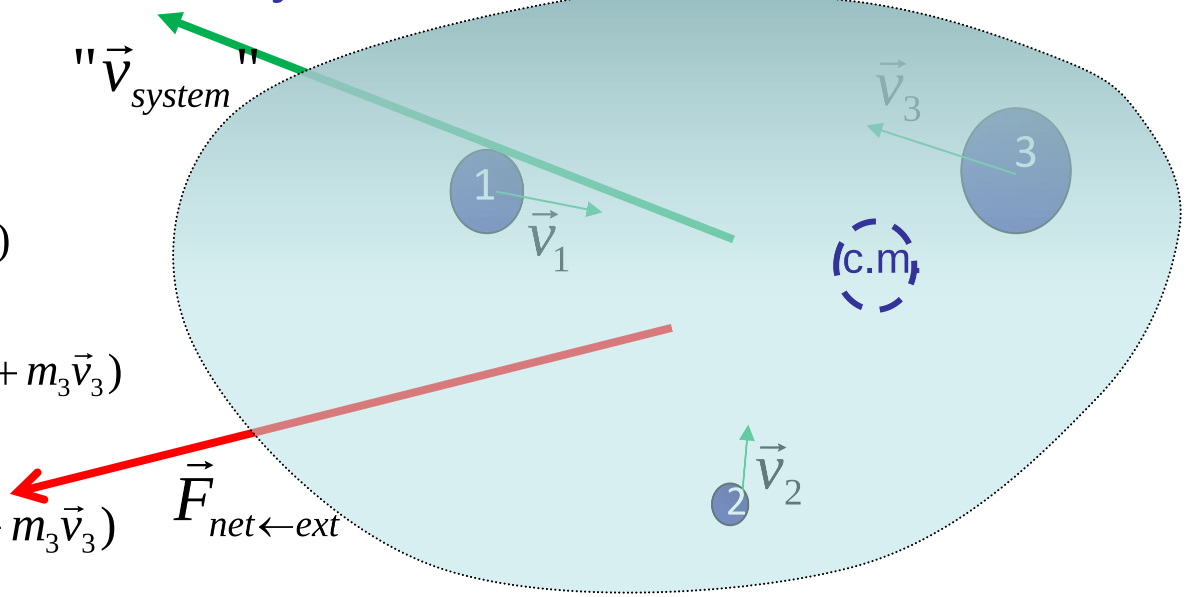
Whole System's Velocity and Position

$$\frac{\Delta M_{total}\vec{v}_{system}}{\Delta t} \approx \frac{\Delta \vec{p}_{total}}{\Delta t} = \vec{F}_{ext.net}$$

$$\frac{\Delta M_{total}\vec{v}_{system}}{\Delta t} \approx \frac{\Delta}{\Delta t}\left(\vec{p}_1 + \vec{p}_2 + \vec{p}_3\right)$$

$$\frac{\Delta\left(m_1 + m_2 + m_3\right)\vec{v}_{system}}{\Delta t} \approx \frac{\Delta}{\Delta t}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right)$$

$$\left(m_1 + m_2 + m_3\right)\vec{v}_{system} \approx \left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right)$$



$$\vec{v}_{system} \approx \frac{\left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right)}{\left(m_1 + m_2 + m_3\right)}$$

$$\frac{\Delta \vec{r}_{system}}{\Delta t} \approx \frac{\left(m_1\frac{\Delta \vec{r}_1}{\Delta t} + m_2\frac{\Delta \vec{r}_2}{\Delta t} + m_3\frac{\Delta \vec{r}_3}{\Delta t}\right)}{\left(m_1 + m_2 + m_3\right)}$$

$$\frac{\Delta \vec{r}_{system}}{\Delta t} \approx \frac{\Delta}{\Delta t}\left(\frac{\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right)}{\left(m_1 + m_2 + m_3\right)}\right)$$

$$\vec{r}_{system} \approx \frac{\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right)}{\left(m_1 + m_2 + m_3\right)}$$

Follow reasoning back *up:*
center of mass responds to force
As would a point with all the
System's mass

Mass-averaged position: Center of Mass

Center of Mass Examples: Batons and Binary-Stars

- **Q:** In this simulation, the blue star’s mass is 2e30 kg and it starts out at the origin; the yellow star’s mass is 1e30kg and it starts out at <1.5e11, 0, 0> m

- Initially, where’ the center of mass?

Example: A 17 kg ball is located at ‹ 6, 0, 0 › m, and a 3 kg ball is located at ‹ 13, 3, 0 › m. Find the center of mass of the two-ball system. You should find that the center of mass is close to the heavier ball.

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