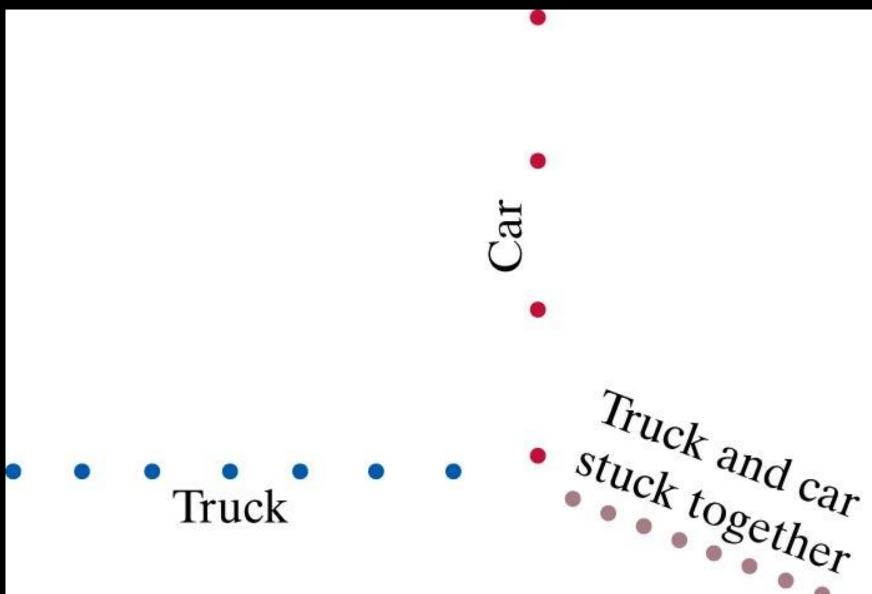
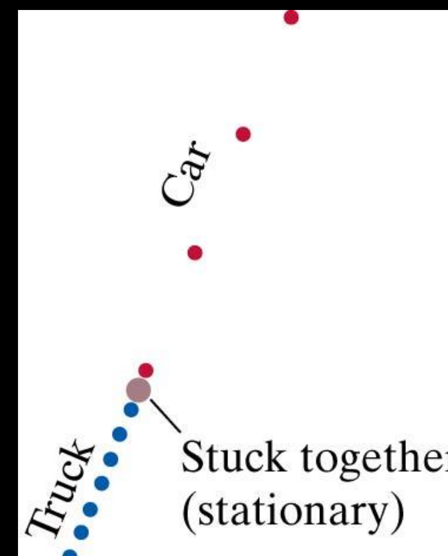
Ved.,	10.5, .11 Different Reference Frames	
hurs		Washinton's 3/2 visits Thurs night 7pr
ri.,	1.1 Translational Angular Momentum Quiz 10	RE 11.a; HW10: Pr's 13*, 21, 30, "39'

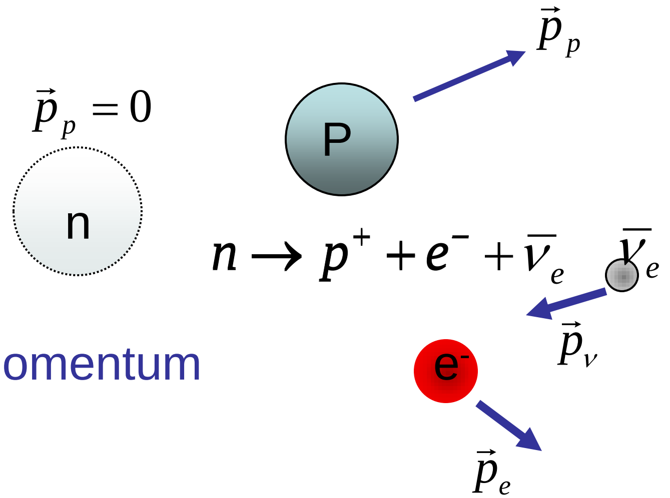
* For *each part* of these problems, be very careful about what you choose as the system and what you are using as initial and final states.

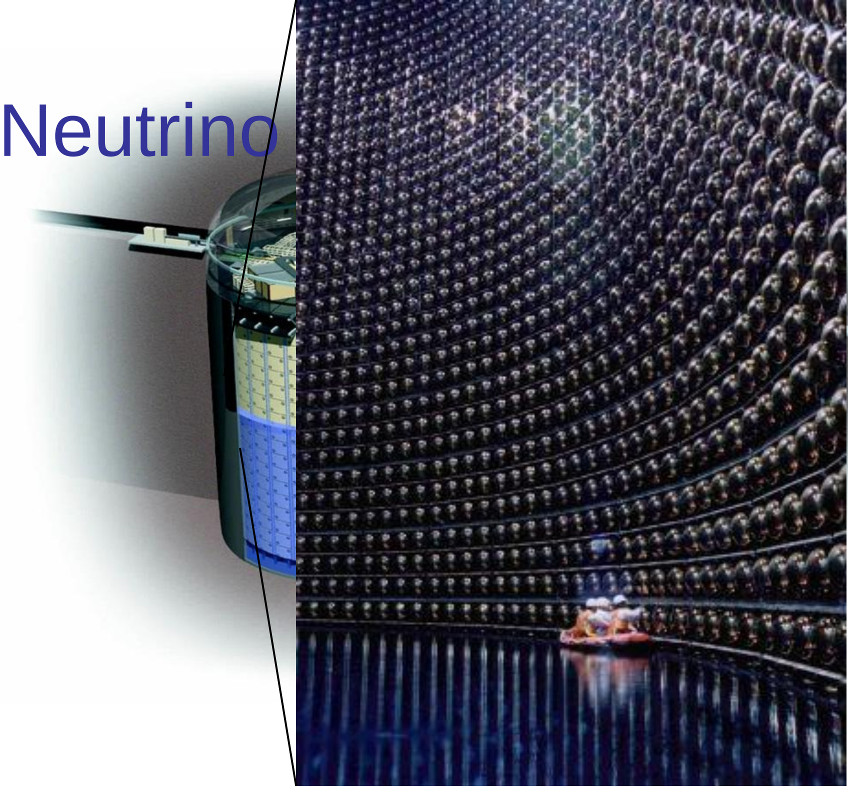
Collisions & Center of Mass





But First:
Deducing the invisible particle: Neutrino





Conservation of Momentum

$$\vec{p}_p + \vec{p}_e \cancel{\neq} 0$$

$$\vec{p}_p + \vec{p}_e + \vec{p}_\nu = 0$$

Must be another particle with missing energy & momentum

Charge is conserved and detectors can't see it – must be neutral

Sometimes energy and momentum *almost* conserved – must be nearly massless

Conservation of Energy

$$m_n c^2 \cancel{\neq} \sqrt{\left(p_e c\right)^2 + \left(m_e c^2\right)^2} + \sqrt{\left(p_p c\right)^2 + \left(m_p c^2\right)^2}$$

$$m_n c^2 = \sqrt{\left(p_e c\right)^2 + \left(m_e c^2\right)^2} + \sqrt{\left(p_p c\right)^2 + \left(m_p c^2\right)^2} + \sqrt{\left(p_\nu c\right)^2 + \left(m_\nu c^2\right)^2}$$

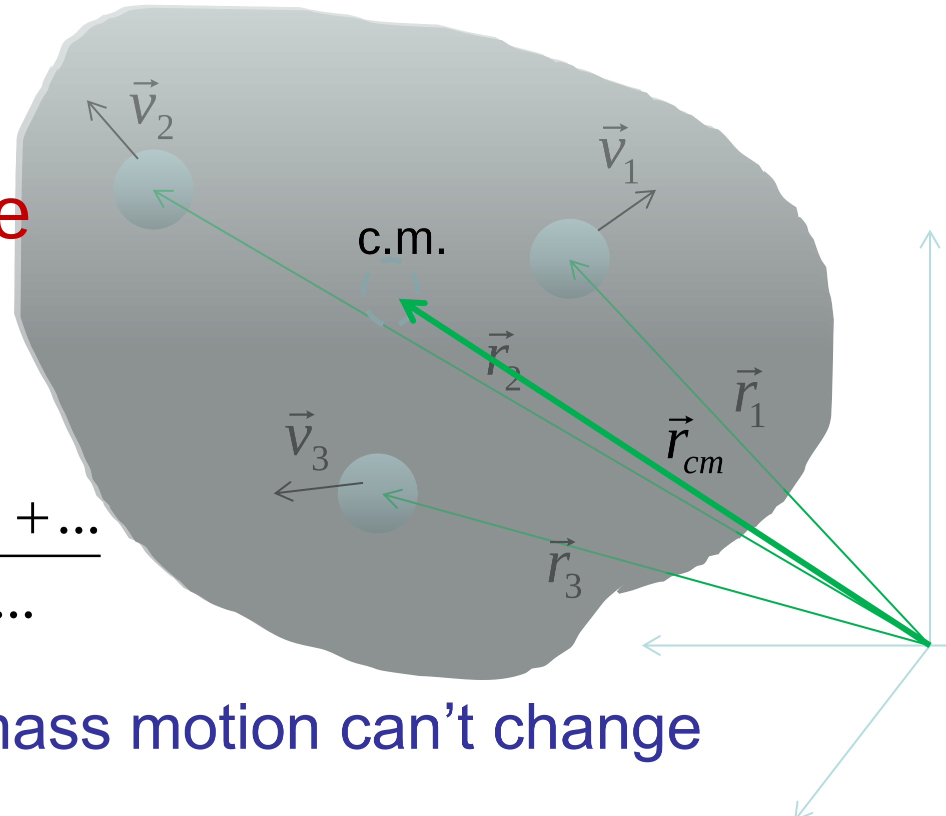
Memory Lane:
Multi-Particle System's Momentum and Center of Mass

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

If no external force

$$\vec{p}_{system} \approx m_{system}\vec{v}_{cm} = \text{constant}$$

$$\vec{v}_{cm} \approx \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$



If no external force, center-of-mass motion can't change

Often simpler to just focus on relative motion

lab3 binary w cm.py

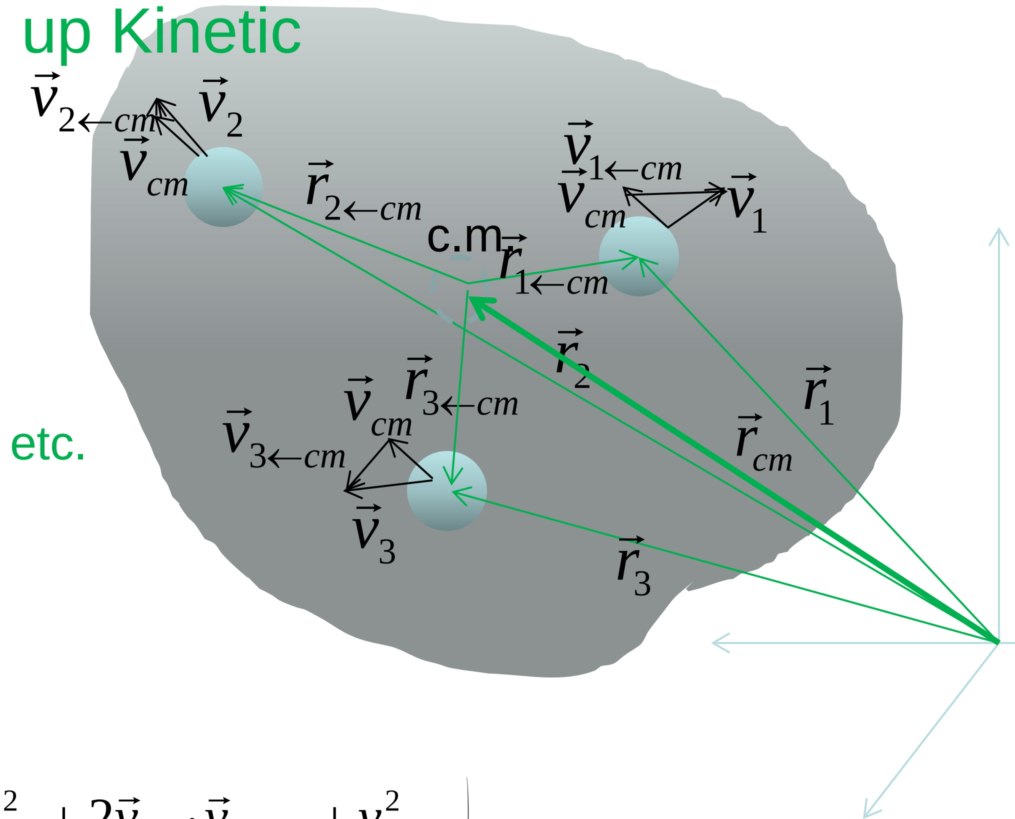
RotateVibrateTranslate (e edit).py

Memory Lane: Multi-Particle System's Energy

Splitting up Kinetic

$$K_{total} = K_1 + K_2 + K_3 + \ldots$$

$$K_{total} \approx \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \ldots$$



Where $\vec{v}_1 = \dfrac{d}{dt}\vec{r}_1$, ect.

But $\vec{r}_1 = \vec{r}_{cm} + \vec{r}_{1\leftarrow cm}$, etc.

so $\vec{v}_1 = \dfrac{d}{dt}\vec{r}_{cm} + \dfrac{d}{dt}\vec{r}_{1\leftarrow cm}$

Or $\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm}$, etc.

so

$$\tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2} m_1 \left| \vec{v}_{cm} + \vec{v}_{1\leftarrow cm} \right|^2 = \tfrac{1}{2} m_1 \left| v_{cm}^2 + 2\vec{v}_{cm} \cdot \vec{v}_{1\leftarrow cm} + v_{1\leftarrow cm}^2 \right|$$

$$= \tfrac{1}{2} m_1 v_{cm}^2 + m_1 \vec{v}_{cm} \cdot \vec{v}_{1\leftarrow cm} + \tfrac{1}{2} m_1 v_{1\leftarrow cm}^2$$

then $K_{total} \approx \tfrac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \vec{v}_{cm} \cdot \underbrace{\left(\sum_i m_i \vec{v}_{1\leftarrow cm}\right)} + \sum_i \left| \tfrac{1}{2} m_i v_{i\leftarrow cm}^2 \right|$

$$K_{total} \approx \tfrac{1}{2} m_{total} v_{cm}^2 + \sum_i \left| \tfrac{1}{2} m_i v_{i\leftarrow cm}^2 \right|$$

recall $\vec{v}_{cm} = \dfrac{\sum_i m_i \vec{v}_i}{m_{total}}$

recall $\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm}$

so $\vec{v}_{1\leftarrow cm} = \vec{v}_1 - \vec{v}_{cm}$

$$\sum_i m_i \left| \vec{v}_{i\leftarrow cm} - \vec{v}_{cm} \right| = \sum_i m_i \vec{v}_{i\leftarrow cm} - \left(\sum_i m_i\right)\vec{v}_{cm} = m_{total}\vec{v}_{cm} - m_{total}\vec{v}_{cm} = 0$$

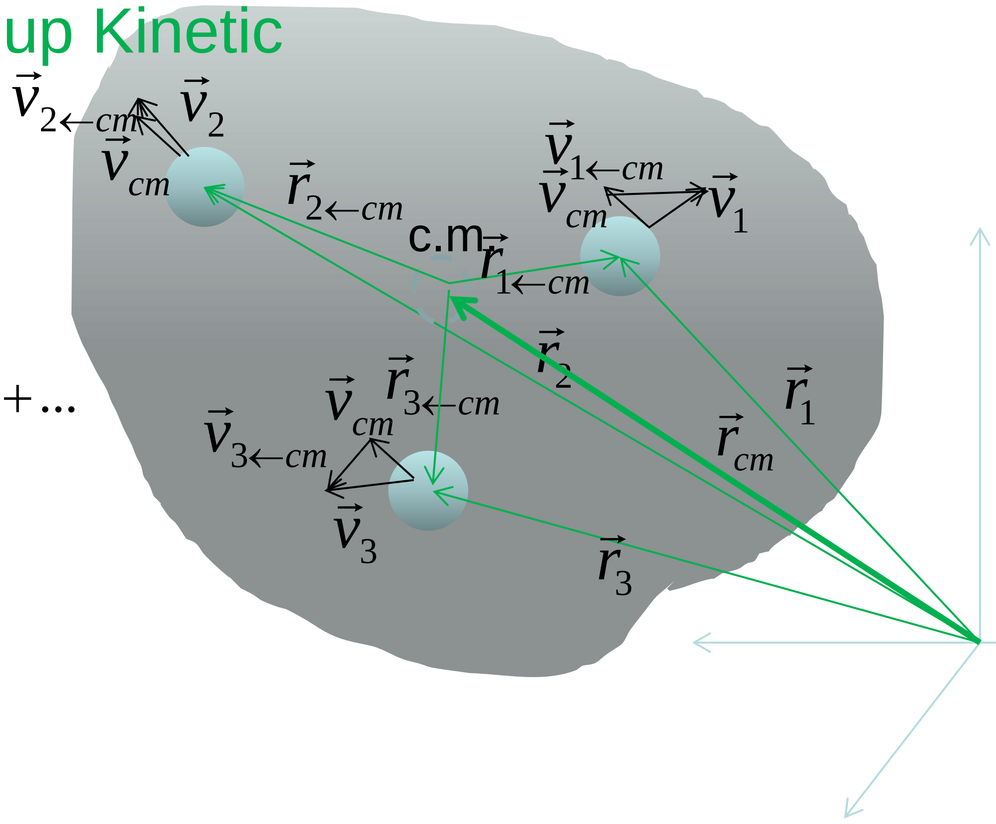
Memory Lane: Multi-Particle System's Energy

Splitting up Kinetic

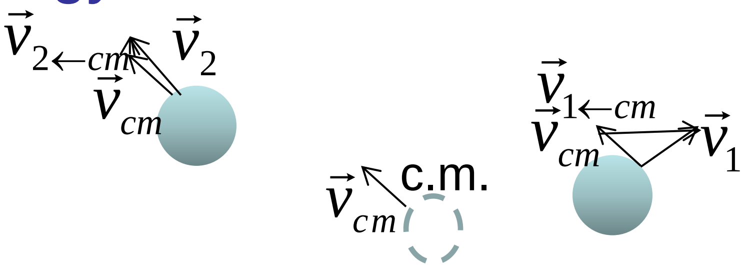
$$K_{total} = K_1 + K_2 + K_3 + \ldots$$

$$K_{total} \approx \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \ldots$$

$$K_{total} \approx \tfrac{1}{2} m_{total} v_{cm}^2 + \tfrac{1}{2} m_1 v_{1\leftarrow cm}^2 + \tfrac{1}{2} m_2 v_{2\leftarrow cm}^2 + \ldots$$



Center of Mass & Collisions
Energy



$$K_{total} = K_1 + K_2 = K_{cm} + K_{1\leftarrow cm} + K_{2\leftarrow cm}$$

or

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_{total} v_{cm}^2 + \tfrac{1}{2} m_1 v_{1\leftarrow cm}^2 + \tfrac{1}{2} m_2 v_{2\leftarrow cm}^2$$

where $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \text{constant}$

so $K_{cm} = \tfrac{1}{2} m_{total} v_{cm}^2 = \text{constant}$

Regardless of whether collision is elastic or inelastic

Before collision: $K_{1.i} + K_{2.i} = K_{cm} + K_{1\leftarrow cm.i} + K_{2\leftarrow cm.i}$

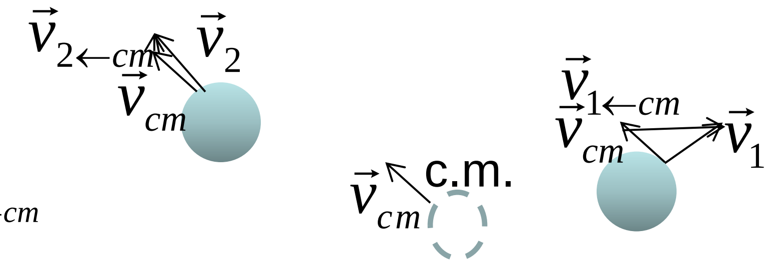
After collision: $K_{1.f} + K_{2.f} = K_{cm} + K_{1\leftarrow cm.f} + K_{2\leftarrow cm.f}$

change: $\Delta\left(K_1 + K_2\right) = \Delta\left(K_{1\leftarrow cm.f} + K_{2\leftarrow cm.f}\right)$

If no external force, center-of-mass motion can't change
Only motion *relative* to center-of-mass can change

Center of Mass & Collisions
Momentum

$$\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \vec{v}_{cm} = \text{constant}$$



where

$$\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm} \qquad \vec{v}_2 = \vec{v}_{cm} + \vec{v}_{2\leftarrow cm}$$

so

$$\frac{m_1\left(\vec{v}_{cm} + \vec{v}_{1\leftarrow cm}\right) + m_2\left(\vec{v}_{cm} + \vec{v}_{2\leftarrow cm}\right)}{m_1 + m_2} = \vec{v}_{cm}$$

or

$$\frac{m_1\vec{v}_{cm} + m_2\vec{v}_{cm}}{m_1 + m_2} + \frac{m_1\vec{v}_{1\leftarrow cm} + m_2\vec{v}_{2\leftarrow cm}}{m_1 + m_2} = \vec{v}_{cm}$$

$$\cancelto{1}{\left(\frac{m_1 + m_2}{m_1 + m_2}\right)}\cancel{\vec{v}_{cm}} + \frac{m_1\vec{v}_{1\leftarrow cm} + m_2\vec{v}_{2\leftarrow cm}}{m_1 + m_2} = \cancel{\vec{v}_{cm}}$$

so $\quad m_1\vec{v}_{1\leftarrow cm} + m_2\vec{v}_{2\leftarrow cm} = 0$

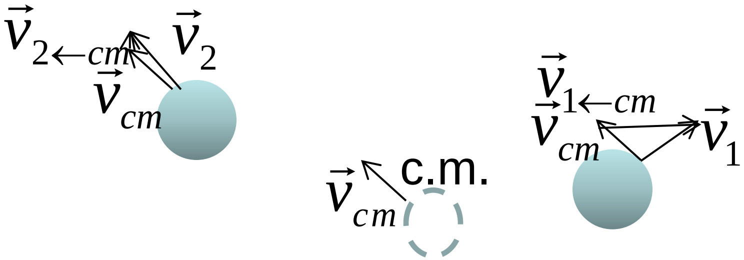
Regardless of whether collision is elastic or inelastic

Before and after the collision

$$m_1\vec{v}\big|_{1\leftarrow cm}\big|_i = -m_2\vec{v}\big|_{2\leftarrow cm}\big|_i \qquad m_1\vec{v}\big|_{1\leftarrow cm}\big|_f = -m_2\vec{v}\big|_{2\leftarrow cm}\big|_f$$

Center of Mass & Collisions
Momentum & Energy

$$\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm} \qquad \vec{v}_2 = \vec{v}_{cm} + \vec{v}_{2\leftarrow cm}$$



where

$$\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \vec{v}_{cm} = \text{constant}$$

Regardless of whether collision is elastic or inelastic

Momentum

$$\vec{p}\,|_{1\leftarrow cm}|_i = -\vec{p}\,|_{2\leftarrow cm}|_i$$

$$\vec{p}\,|_{1\leftarrow cm}|_f = -\vec{p}\,|_{2\leftarrow cm}|_f$$

Kinetic Energy

$$\Delta\left(K_1 + K_2\right) = \Delta\left(K_{1\leftarrow cm.f} + K_{2\leftarrow cm.f}\right)$$

If no external force, center-of-mass motion can’t change

Only motion *relative* to center-of-mass can change

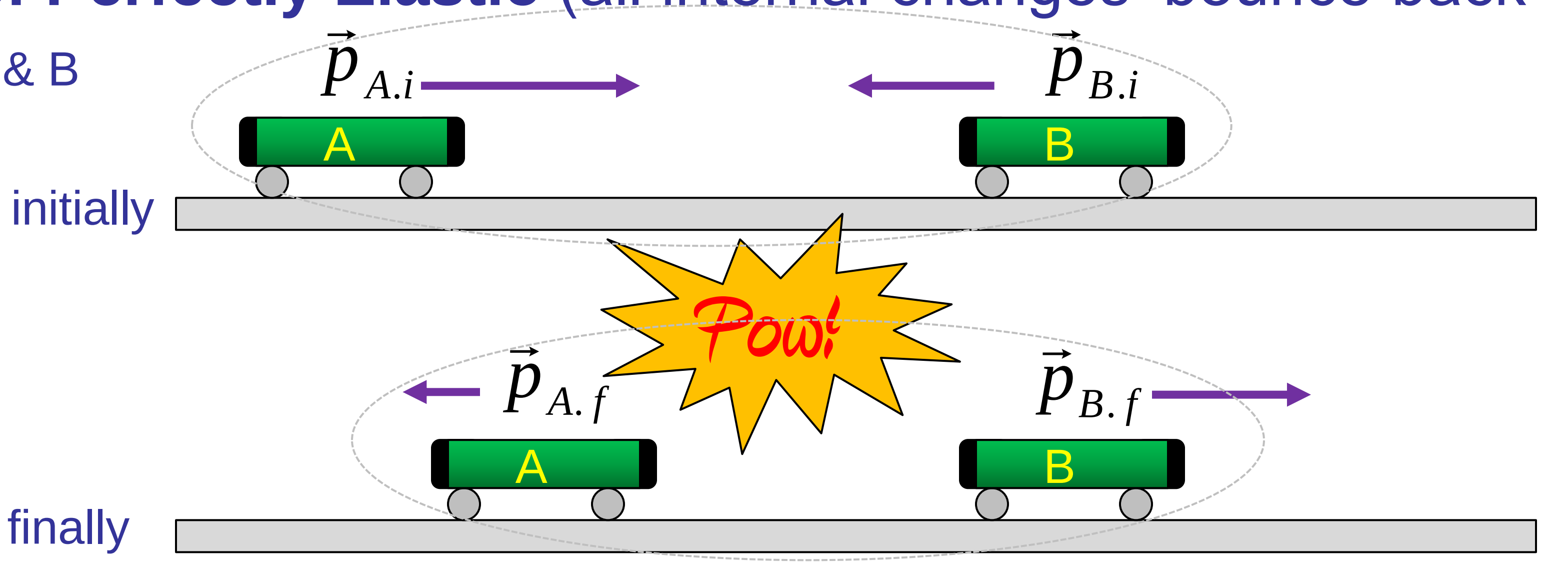
Strategy:

- Determine center of mass velocity
- subtract it out and calculate change (of relative motion)
- Then add it back in.

1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

System = carts A & B



Equation 1 $$\Delta E_{A\&B} = \Delta K_A + \cancel{\Delta E_{A.\text{int}}} + \Delta K_B + \cancel{\Delta E_{B.\text{int}}} + \cancel{\Delta U_{A\&B}} = 0$$

$$\left(\frac{p^2_{A.f}}{2m_A} - \frac{p^2_{A.i}}{2m_A}\right) + \left(\frac{p^2_{B.f}}{2m_B} - \frac{p^2_{B.i}}{2m_B}\right) \approx 0 \qquad (v's << c)$$

Focus on
relative motion

$$\left(\frac{p^2_{A.f\leftarrow cm}}{2m_A} - \frac{p^2_{A.i\leftarrow cm}}{2m_A}\right) + \left(\frac{p^2_{B.f\leftarrow cm}}{2m_B} - \frac{p^2_{B.i\leftarrow cm}}{2m_B}\right) \approx 0$$

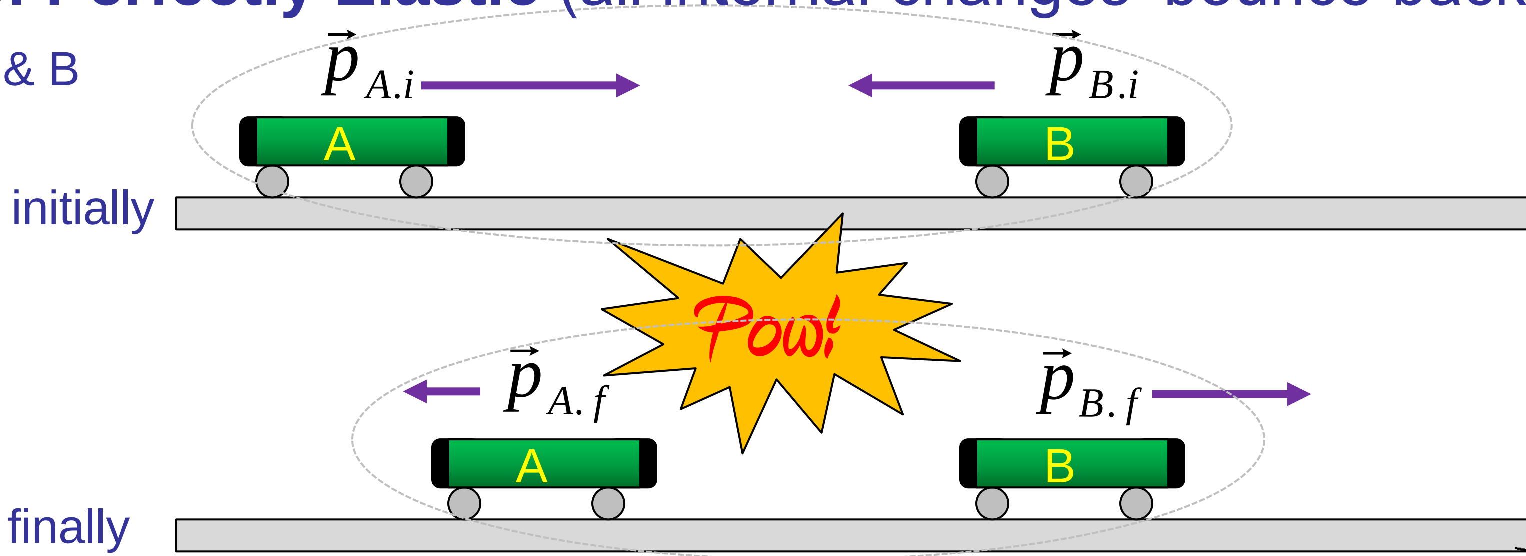
Equation 2 $$\vec{p}_{A.f} + \vec{p}_{B.f} = \vec{p}_{A.i} + \vec{p}_{B.i}$$

$$\vec{p}_{A.f\leftarrow cm} = -\vec{p}_{B.f\leftarrow cm} \qquad \vec{p}_{A.i\leftarrow cm} = -\vec{p}_{B.i\leftarrow cm}$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes 'bounce back')

System = carts A & B



$v's \ll c$

Focus on *relative motion*

$$\left(\frac{p^2_{A.f\leftarrow cm}}{2m_A}-\frac{p^2_{A.i\leftarrow cm}}{2m_A}\right)+\left(\frac{p^2_{B.f\leftarrow cm}}{2m_B}-\frac{p^2_{B.i\leftarrow cm}}{2m_B}\right)\approx 0$$

$$\vec{p}_{A.f\leftarrow cm}=-\vec{p}_{B.f\leftarrow cm} \qquad \vec{p}_{A.i\leftarrow cm}=-\vec{p}_{B.i\leftarrow cm}$$

$$\left(\frac{p^2_{A.f\leftarrow cm}}{2m_A}-\frac{p^2_{A.i\leftarrow cm}}{2m_A}\right)+\left(\frac{p^2_{A.f\leftarrow cm}}{2m_B}-\frac{p^2_{A.i\leftarrow cm}}{2m_B}\right)\approx 0$$

$$p^2_{A.f\leftarrow cm}\,\tfrac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right)=p^2_{A.i\leftarrow cm}\,\tfrac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right)$$

$$p^2_{A.f\leftarrow cm}=p^2_{A.i\leftarrow cm}$$

similarly

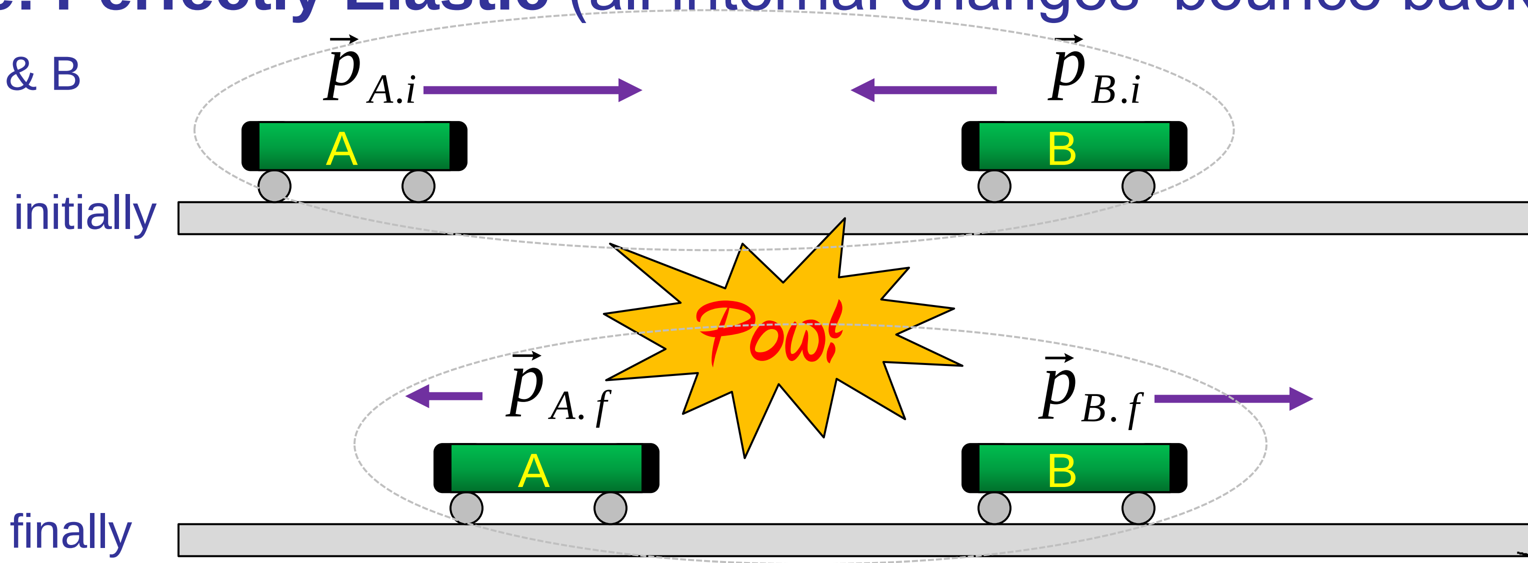
$$p^2_{B.f\leftarrow cm}=p^2_{B.i\leftarrow cm}$$

(true in 2 & 3-D too)

1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

System = carts A & B



v's $<< c$

Focus on
relative motion

$$p^2_{A.f\leftarrow cm} = p^2_{A.i\leftarrow cm} \quad \text{similarly} \quad p^2_{B.f\leftarrow cm} = p^2_{B.i\leftarrow cm}$$

In 1-D: if they actually hit

$$\vec{p}_{A.f\leftarrow cm} = -\vec{p}_{A.i\leftarrow cm} \quad \text{similarly} \quad \vec{p}_{B.f\leftarrow cm} = -\vec{p}_{B.i\leftarrow cm}$$

$$\vec{v}_{A.f\leftarrow cm} = -\vec{v}_{A.i\leftarrow cm} \qquad \vec{v}_{B.f\leftarrow cm} = -\vec{v}_{B.i\leftarrow cm}$$

Add back in
CM velocity

$$\vec{v}_{A\leftarrow cm} = \vec{v}_A - \vec{v}_{cm} \qquad \vec{v}_{B\leftarrow cm} = \vec{v}_B - \vec{v}_{cm}$$

$$\vec{v}_{A.f} - \vec{v}_{cm} = -\left(\vec{v}_{A.i} - \vec{v}_{cm}\right)$$

similarly

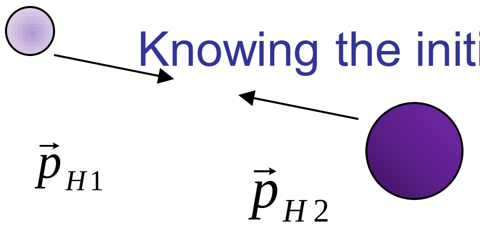
$$\vec{v}_{A.f} = 2\vec{v}_{cm} - \vec{v}_{A.i} \qquad \vec{v}_{B.f} = 2\vec{v}_{cm} - \vec{v}_{B.i}$$

Center of Mass & Collisions
High Speeds

$$^{1}H + {}^{2}H \rightarrow {}^{3}He^{*} \rightarrow {}^{3}He + \gamma$$

"gamma ray" = High energy photon

Knowing the initial momenta and masses, what's the mass of the excited He?



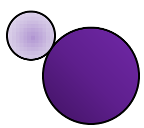
Stage 1

In center of mass frame

$$\vec{p}_{H1} + \vec{p}_{H2} = 0$$

$$E = \sqrt{\left(p_{H1}c\right)^2 + \left(m_{H1}c^2\right)^2} + \sqrt{\left(p_{H2}c\right)^2 + \left(m_{H2}c^2\right)^2}$$

$$E = \sqrt{\left(p_{H1}c\right)^2 + \left(m_{H1}c^2\right)^2} + \sqrt{\left(p_{H1}c\right)^2 + \left(m_{H2}c^2\right)^2}$$

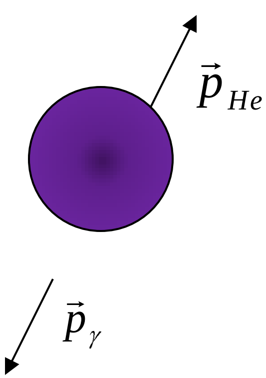


$$E = m_{He^*}c^2 \qquad E/c^2 = m_{He^*}$$

Stage 2

$$\vec{p}_{He} + \vec{p}_{\gamma} = 0$$

What's the final momentum?



$$E = \sqrt{\left(p_{He}c\right)^2 + \left(m_{He}c^2\right)^2} + \sqrt{\left(p_{\gamma}c\right)^2 + \left(\cancel{m_{\gamma}}c^2\right)^2}$$

photon is massless

$$E = \sqrt{\left(p_{\gamma}c\right)^2 + \left(m_{He}c^2\right)^2} + p_{\gamma}c$$

$$\left(E - p_{\gamma}c\right)^2 = \left(p_{\gamma}c\right)^2 + \left(m_{He}c^2\right)^2$$

$$E^2 - 2Ep_{\gamma}c + \left(p_{\gamma}c\right)^2 = \left(p_{\gamma}c\right)^2 + \left(m_{He}c^2\right)^2$$

$$\frac{E^2 - \left(m_{He}c^2\right)^2}{2Ec} = p_{\gamma}$$

Stage 3