Physics 231 – Lab 4
Macro- and Micro-scopic Springs:
Period, Young's Modulus, and the Speed of Sound

There are two parts to this lab. First you'll look at the period of a macroscopically sized spring-mass system. Next you'll investigate how the "microscopic spring"-type behavior of atomic bonds relates to a solid's Young's Modulus and the speed of sound through the solid.

(Equipment: computer, LabPro, 200-g mass, 50-g mass, spring, force probe, meter stick,)

Objectives
You will make measurements for a mass attached to a vertical spring and compare with predictions based on the Momentum Principle.

For copper, you will measure the following:
- The density
- The Young's modulus
- The speed of sound

The ball-spring model will be used to relate the macroscopic measurements for copper to its microscopic properties.

Background
Hook's Force Law says that the restoring force exerted by a spring is proportional to its displacement from equilibrium, $\vec{F}_s = -k_s(\vec{r} - \vec{r}_{eq})$. A mass hung vertically is subject to both the spring force and gravitation, so that $F_{net.y} = mg - k_s(y - y_{eq})$. The spring force alone, or in combination with a constant force (such as near-Earth gravitation), drives a spring-mass system sinusoidally such that $\vec{r} = \Delta\vec{r}_{\max}\cos\left(2\pi\frac{t}{T}\right) + \vec{r}_{eq}$ where $T = 2\pi\sqrt{\frac{m}{k_s}}$.

The Young's modulus of a material (Y) is defined as the stress divided by the strain. Although the examples in the textbook are about stretching material, the Young's modulus can also be found by compressing it. Stress is the tension (or compression) force per unit area (F/A). Strain is the fractional change in length ($\Delta L/L$). In terms of microscopic quantities, the Young's modulus is $Y = k_{s,I}/d$, where $k_{s,i}$ is the stiffness of an interatomic bond and d is the length of a bond. The speed of sound in solid predicted by the ball-spring model is $v = d\sqrt{k_{s,i}/m_a}$, where m_a is the mass of an atom.

I. Macroscopic:
Period's Mass Dependence for a Mass on a Spring

A. *Theory*
Question: Based on the expression for period in terms of mass and spring constant, if you hung 50g from one spring and separately hung 200g from an identical spring, what would be the ratio of the two mass's oscillations? Check it in WebAssign

B. *Experiment*
Next you will use LoggerPro to record the oscillations of the spring-mass systems with 50 g and 200 g masses; from that you'll determine the periods and the ratio of the periods.
- Hang the 50 gram mass from the spring.
- Plug the force probe LabPro.
- Open Force.cmbl from the PHYS231 folder in Physics Experiments on the desktop.

- Set the mass bobbing up and down and set LoggerPro recording the oscillations in the Force (which reflect the oscillations in the motion.)
- To read the plot more precisely, you can select "Examine" from the "Analyze" menu. Then you'll see a readout of the time and position data where ever you move the cursor.
- Measure the period of oscillation (you may want to measure the time for several cycles and then divide by the number of cycles.)
- Replace the mass with a 200 gram mass and measure the new period. Note: feel free to raise the spring-mass system to avoid crashing into the detector!

C. Comparison

- Determine the ratio of experimentally measured periods, $\frac{T_{200g}}{T_{50g}}$.

Period's Amplitude Dependence for a Mass on a Spring

A. Theory

Before running the experiment, consider how the *amplitude* of oscillation should effect the *period* of oscillation.

B. Experiment

- Repeat your period measurement for one of the masses but using a vastly different amplitude.

C. Comparison

Question: Qualitatively, how does the period depend on the amplitude? (in WebAssign)

Definitely *increases* with increased amplitude.
Definitely *decreases* with increased amplitude.
If there is any dependence at all, it's very small.

II. Microscopic: Speed of Sound in a Solid

Theoretically, the speed of sound in a solid depends on the materials springiness, that is, its atomic-scale spring constant and mass. First, you'll make the measurements necessary to get a theoretical value for the speed of sound, then you'll experimentally measure it and compare. Because this lab builds on itself, with later work depending on previous results, you're going to use WebAssign to double check your work a lot along the way.

You'll measure the mass-density of copper, ρ, and determine the mass of one copper atom, m_{atm}. If the 'personal volume' of one atom of copper is a cube with side length *d,* determine *d* in terms of these parameters.

Density and Bond Length & atomic mass

- Measure the mass of the copper sample by the scales (weigh all 5, and then divide by 5.)
- Use the calipers by the scale to measure and record the dimensions of the copper (I'll help you if you don't know how to use calipers.) Measuring the thickness, be sure to 'bite' the sample lengthwise instead of across (to avoid getting fooled by its slight bowing). Calculate its volume.
- Calculate the density of copper.
- One mole of copper has a mass of 0.064 kg. So determine the mass of one copper atom.
- Finally, determine the bond length based on your measurements.

Young's Modulus and Interatomic Spring Stiffness

The stretch of a wire is usually so small that it can be difficult to measure accurately.

- Use Vernier calipers to measure the diameter of the wire. WARNING: this tends to be the single largest source of error for students, so be careful to get the thinnest value you can.
- Based on that, determine the cross-sectional area of the wire.
- Measure the un-stretched length of the wire.
- Gently hang 1 kg to 7 kg from the wire (don't forget the 1kg mass of the hanger itself) and use the gauge to measure the stretch of the wire and fill in the ΔL *column* of the table in WebAssign. The big dial reads in 10^{-5}m and the small one reads in 10^{-3}m.
- Calculate the *stress* and *strain* for each measurement to complete the table in WebAssign.
- Plot those points (Strain on the horizontal axis, Stress on the vertical axis).
- Use the *slope of the plot* (*not* an individual measurement) to determine the Young's modulus of the wire. (Note that your graph may *not* go through the origin because of an offset in the measurement of the stretch so an individual measurement *won't* give the correct Y.)
- You have determined Young's Modulus, Y, and the interatomic spacing, d; give the expression for the interatomic spring constant, k_{is}, in terms of these.
- Determine the interatomic spring constant for copper based on your measurements.

Speed of Sound

A. Theoretical Prediction

An approximate prediction of the speed of sound in a copper can be made using the previous measurements.

- Given the atomic mass, m_a, and interatomic distance, d, that you'd previously calculated and the interatomic spring constant, $k_{s,i}$, that you've just calculated, calculate the theoretical speed of sound in copper based on your measurements.

B. Experiment

Now that you've determined a theoretical value for the speed of sound in copper, it's time to measure it directly by determining the time it takes a sound pulse to travel a known distance through copper. The speed of sound will be determined by measuring the time that it takes for a pulse to travel down a metal pipe *and* back.

- Measure the length of the copper pipe.
- Since the distance that the sound pulse will travel is twice that, you know the distance it will travel.
- The microphone should be inserted firmly in the far end of the pipe.

- Open Speed of Sound.cmbl from the PHYS231 folder in Physics Experiments on the desktop.
- Press “Collect,” then use a hammer to tap the end of the pipe with the microphone.
- Repeat until there are several evenly spaced packets of waves (about 1 ms apart) – each is due to a set of ‘echoes’ of the tap traveling down the length of the pipe and back to the microphone again. Note: The microphone may be maxed-out for the first second or so of data (all the peaks in the data plot hit and plateau at the same value)– so look a little later in your data. This apparatus (if you can call a pipe and a screwdriver “apparatus”) is very finicky – your instructor can help you get good data.
- Zoom in to a 10~20 ms region where you can see a few sets of echoes clearly.
- To get numbers from the graph, select “Examine” under the “Analyze” menu.
- Calculate the time between the pulses by averaging the time between several pulses (as many as possible). Mark on your graph the points used and label their times.

- Based on these measurements of time and distance with the pipe, calculate the speed of sound in copper.

  Note: Published values vary by almost a factor of 1.5 (presumably, how the copper was processed significantly effects the material properties) so you should be happy if your theoretical and experimental values for the speed are within this range of each other.