

## Energy & Momentum for a ‘multi-particle’ system (you)

Equipment: Force Plate, Motion Sensor mounted on high rod, hydrogen emission tubes, hand-held spectrometers.

### Objectives

You’ll do two things in this lab. Relating to Chapter 8, you’ll take a look at the Hydrogen spectrum and use the mathematical model of its energy-level structure to predict the levels visible to the unaided eye. Relating to Chapter 9, you will analyze a jump from a crouching position. You will use the Momentum Principle and the Energy Principle for both the real system and the point-particle / center-of-mass system.

## Hydrogen Spectrum

### 1. Background

To quote problem 8.22, “The eye is sensitive to photons with energies in the range from about 1.8eV, corresponding to red light, to about 3.1 eV, corresponding to violet light. White light is a mixture of all the energies in the visible region. If you shine white light through a slit onto a glass prism [or through a diffraction grating], you can produce a rainbow spectrum on a screen, because the prism bends different colors of light by different amounts. If you replace the source of white light with an electric-discharge lamp containing excited atomic hydrogen, you will see only a few lines in the spectrum, rather than a continuous rainbow.” These lines are produced when atoms transition down from excited states (defined by different K+U values). The energy of the photon emitted equals the energy lost by the atom when it transitions between states:

$E_{light} = -\Delta E_{atom} = -E_{atom.f} - E_{atom.i}$ . For the Hydrogen atom, the energy (K+U) of the  $n^{th}$  state is  $E_{H_n} = \frac{-13.6eV}{n^2}$  where  $n$  can be any positive integer.

### 2. Theory

Predict the energies of *visible* lines in Hydrogen’s emission spectrum. For these, give both the energy and the “n” values of the states between which the Hydrogen transitioned in order to produce that light.

Initial n	Final n	Photon energy (eV)



For the sake of comparison late, calculate the corresponding wavelengths of light seen for each visible transition. The conversion between light's wavelength and its energy is

$$\lambda_{light} = \frac{hc}{E_{light}}$$

where  $\lambda$  stands for 'wavelength' and  $hc = 1239.8 eV \cdot nm$ . So, what are the

wavelengths of the light visible in hydrogen's spectrum?



Initial n	Final n	Wavelength (nm)

**Double Check your work in Lab 8 in WebAssign.**

### 3. Experiment

We've got hand-held spectrometers and the equivalent of neon lights for hydrogen; turn on the hydrogen light and view it through the spectrometer. To do this, you look through the small end of the spectrometer and align the slit (at the other end) with the hydrogen tube. Within the spectrometer, you should see the hydrogen's spectrum against a scale off to the right. Now, the scale is in terms of not the light's *energy* but its *wavelength* measured in 100 nano-meters ( $nm=10^{-9}m$ ); for example, when it reads "4" it means 400nm. We'll convert between energy and wavelength in a moment.

You should see one less line in the spectrum than you predicted (more about that later.) Is that correct (if not, you might want to look more carefully / return to the theory section)?

What wavelengths of light are visible in Hydrogen's spectrum?

## 4. Comparison

Qualitatively, how do these compare with the energies you predicted? (Note: these hand-held spectrometers aren't calibrated well enough to make really accurate measurements, so we're just looking for the measurements to agree moderately well with the theory.)

Which predicted line do you *not* see? This is produced by a transition down from which (what  $n$ ) initial state? Apparently the electron beam flowing through the gas discharge tube isn't energetic enough to knock many atoms up to that initial state.

## Jump

### I. Background

The work-energy relation applied to a real system says that the sum of the works done by each external force (which is that force integrated over the distance of its application) equals the change in the system's total energy, which it's often convenient to break up into the translational kinetic energy and the internal energy.

$$W_{net} = \Delta E_{system}$$
$$\sum_i \int \vec{F}_i \cdot d\vec{r}_i = \Delta K_{trans} + \Delta E_{int}$$

Meanwhile, if you just integrate the net force applied to the system over the *displacement of its center of mass*, you get the change in the system's translational kinetic energy.

$$\int \vec{F}_{net} \cdot d\vec{r}_{CM} = \Delta K_{trans} = \Delta \left( \frac{1}{2} M v_{CM}^2 \right)$$

As for force-momentum relation applied to a real system, the sum of forces multiplied by the times over which they are applied, i.e., the impulses, gives the change in the net momentum of the system which is approximately (for speeds  $\ll c$ ) the total mass times the change in its center of mass velocity.

$$\vec{I}_{net} = \Delta \vec{P}_{system}$$
$$\int \vec{F}_{net}(t) \cdot dt \approx \Delta (M \vec{v}_{CM})$$

(I'm explicitly noting that the net force can be time dependent; as will happen in this lab, one individual force may remain constant while another varies.)

## II. Experiment

A force plate will measure the upward force exerted by the floor on the jumper. A motion sensor will keep track of the location of the jumper.

### A. Set Up

- Open the file “jump.cmb1”.
- “Zero” the motion sensor so it considers the top of the force plate the origin. It will measure positions above that as positive.
- “Zero” the force plate *with the jumper standing on it*.
- Determine the weight and mass of the jumper using the force plate (thusly zeroed, the plate will report the jumper’s weight when he/she’s *not* standing on it.)

$$mg = \underline{\hspace{2cm}} \quad m = \underline{\hspace{2cm}}$$

### B. Data Collection

- With the jumper crouched on the force plate and *not* moving (and holding the cardboard box on his/her head to give the motion sensor a nice flat target), press the “Collect” button in LabPro. After the motion sensor starts clicking rapidly, the person should jump without bending the upper body (so that measured displacement of his/her head will be roughly the same as the displacement of his/her center of mass.)
- From the plots, read off the height of the jumper when *crouched* down before jumping and the corresponding time. It may be handy to use the “examine” function under the “analyze” menu.

$$y_c = \underline{\hspace{2cm}} \quad t_c = \underline{\hspace{2cm}}$$

- From the plots, read off the height of the jumper when just *leaving* the force plate and the corresponding time. To do this, identify the time when the force plot levels off because the jumper’s leaving the plate, then read the corresponding height from the position plot.

$$y_L = \underline{\hspace{2cm}} \quad t_L = \underline{\hspace{2cm}}$$

- From the plots, read off the *peak* / maximum height of the jumper and the corresponding time.

$$y_p = \underline{\hspace{2cm}} \quad t_p = \underline{\hspace{2cm}}$$

- From the Force vs. time graph, you can determine the net impulse during the launch, i.e.,  $\vec{I} = \int_{t_c}^{t_L} \vec{F}_{net} dt$ , graphically by finding the area under the graph from  $t_c$  to  $t_L$ .
  - To do this, highlight the force vs. time graph between these two times and then select “integral” under the “analyze” menu.

$$\vec{I} = \int_{t_c}^{t_L} \vec{F}_{net} dt = \underline{\hspace{2cm}}$$

- One last value that you need from your data is the “work” that would be done to the center-of-mass system during the launch,  $\int_{y_c}^{y_L} \vec{F}_{net} \cdot d\vec{r}_{CM}$ . Graphically, that would be the area under an  $F_{net}$  vs. center-of-mass position plot, evaluated from  $y_L$  to  $y_c$ .
  - To do this, first go to the data table and highlight all the data from *before* time  $t_c$ , and then, under the “edit” menu, select “strike through.” Do the same for all the data *after* time  $t_L$ . Now, on your force vs. time plot, click on the bottom axis label, “time” to get a menu of other options and select “position.” Now, you can use the “integral” function on the “analyze” menu.

$$\int_{y_c}^{y_L} \vec{F}_{net} \cdot d\vec{r}_{CM} = \underline{\hspace{10cm}}$$

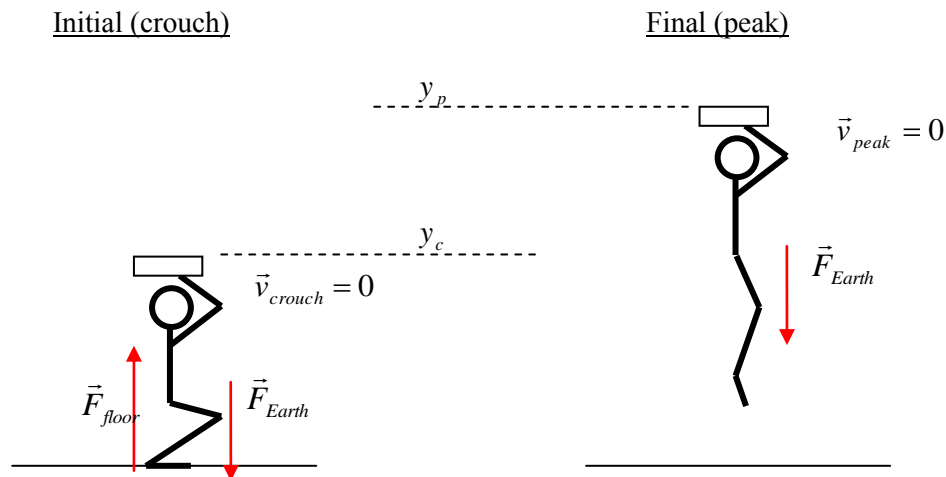
### III. Analysis

For each part, read very carefully what to use as the system, the initial state, and the final state.

#### A. Energy Principle for the **Real System** – crouch to peak

Following the steps below, use the energy principle for the real system of *only the jumper* going from *crouch* to the *peak* of the jump, to determine the approximate change of internal energy of the jumper ( $\Delta E_{\text{int}}$ ). Factors that you need to consider are:

1. Translational Kinetic Energy of the jumper
  2. Forces exerted on the jumper by the Earth and the Floor
  3. Net displacement of the point of application of each force (the gravitational force can be thought of as acting at the center of mass)
  4. Work done by each force
- Consider the initial and final situations illustrated below.



- Write the energy principle symbolically (no numbers!) for this situation and choice of real system. Be sure to include the internal energy of the jumper. Get  $\Delta E_{\text{int}}$  in terms of  $m$ ,  $g$ ,  $y_p$  and  $y_c$ . Be careful about signs. (Are all quantities used labeled in the diagram above?)



Check your equation in WebAssign.

**Question:** How much work was done on the real system by the floor? Explain.

- Solve for a numerical value of the change of internal energy of the jumper (which we'll define to include kinetic energies of arm flailing, etc.) Show *all* of your work.

$$\Delta E_{\text{int}} = \underline{\hspace{2cm}}$$

**Question:** What is the sign of  $\Delta E_{\text{int}}$ ? Explain why this makes sense.

**Questions:** If the jumper didn't flail much, then most of that change in internal energy was on the microscopic scale; we could conceptually break that into the energy associated with flexing interatomic bonds,  $E_{\text{thermal}}$ , and that associated with making and breaking such bonds,  $E_{\text{chemical}}$ . Suppose that the change of internal energy can be attributed to just these two,  $\Delta E_{\text{int}} = \Delta E_{\text{therm}} + \Delta E_{\text{chem}}$ . What is the sign of the change in thermal energy? Explain. What can you say about the size of the change in chemical energy?

**B. Energy Principle for *Center-of-Mass* – Launch to Peak**

Using the energy principle for the *jumper's Center-of-Mass*, going from liftoff (when feet just leave the ground) to the peak of the jump to determine speed of the jumper's center of mass at liftoff,  $v_L$ .

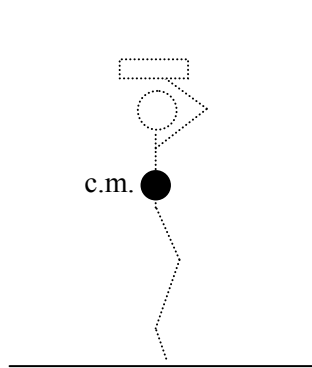
Factors that you need to consider are:

1. Translational Kinetic Energy of the jumper
2. Net force exerted on the point particle system
3. Net displacement of the point of particle system
4. Work done on the point particle system by the net force

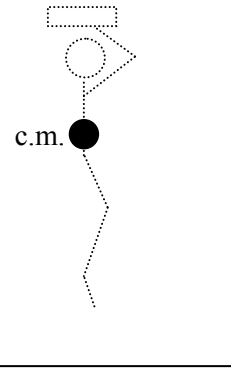
**Question:** Why don't you have to consider the internal energy of the jumper in this case?

- For the initial and final situations, clearly label the important quantities in the diagram. (Remember to use *different* labels for *different* quantities.)

Initial (launch)



Final (peak)



- Apply the energy principle symbolically (no numbers!) for this situation and choice of point-particle system. Get an expression for  $v_L$  in terms of  $m$ ,  $g$ ,  $y_p$ , and  $y_L$ . Be careful about signs.



**Check your equation in WebAssign.**

- Solve for a numerical value the speed  $v_L$  of the jumper's center of mass at lift-off. Show *all* of your work.

$v_L = \text{_____} \text{ (from C.M. launch to peak analysis)}$
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C. Energy Principle for the **Center-of-Mass System** – Crouch to Launch

Following the steps below, you'll use the energy principle for the *center-of-mass / point particle system* describing the jumper, going from the crouched position to liftoff (when feet just leave the ground) in order to determine speed  $v_L$  of the jumper's center of mass at liftoff. Factors that you need to consider are:

1. Kinetic energy of the jumper
  2. Net force exerted on the point particle system
  3. Work done on the point particle system by the net force (*not* constant in this case!)
- Draw the initial and final situations and clearly label the important quantities in the diagram. (Remember to use *different* labels for *different* quantities.)

Initial (crouched)

Final (lift off)



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- From page 5, what was the value you got for  $\int_{y_c}^{y_L} \vec{F}_{net} \cdot d\vec{r}_{CM} = \underline{\hspace{2cm}}$
- Write the energy principle symbolically (no numbers!) for this situation and choice of point-particle system. Get an expression for  $v_L$  in terms of  $W_{cm}$  and  $m$ . Be careful about signs.

**Check your equation in WebAssign.**

- Solve for a numerical value the speed  $v_L$  of the jumper's center of mass at lift-off. Show *all* of your work.

$v_L = \underline{\hspace{4cm}}$ (from C.M. crouch to launch analysis)
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**D. Momentum Principle – Crouch to Launch**

By following the steps below, you'll use the momentum principle for the system of *only the jumper*, going from the crouch to liftoff, to determine the speed  $v_L$  of the jumper at liftoff.

- Draw the initial and final situations. Clearly label the important quantities in the diagram; recall that, while the energy principle relates forces, distances, and speeds, the momentum principle relates forces, *times*, and *velocities*. (Remember to use *different* labels for *different* quantities.)

Initial (crouched)

Final (liftoff)

From page 4, what was the value you got for  $\vec{I} = \int_{t_c}^{t_L} \vec{F}_{net} dt =$  \_\_\_\_\_

Write the momentum – impulse relation symbolically (no numbers!) for this situation and give an expression for  $v_L$  in terms of  $I_{net}$  and  $m$ . Be careful about signs.



**Check your equation in WebAssign.**

Solve for a numerical value the speed  $v_L$  of the jumper at lift-off. Show *all* of your work.

$v_L =$  \_\_\_\_\_ (from momentum crouch to launch analysis)

**Questions:** You've determined the launch speed three different ways: Energy Principle from crouch to launch, Energy Principle from launch to peak, and Momentum Principle from crouch to launch. What's the 4<sup>th</sup> way?

Draw the appropriate pictures, write out the appropriate symbolic relation, and solve for the launch speed.

Initial (crouched)

Final (liftoff)

\_\_\_\_\_

\_\_\_\_\_

$$v_L = \underline{\hspace{2cm}} \text{ (from this analysis)}$$

**Question:** Are the speeds at liftoff calculated in the four different ways fairly consistent (won't be perfect)?