Name:	
Partners:	

# **PHYSICS 221 LAB #1: HARMONIC MOTION**



An antique grandfather clock can be a very reliable timekeeper. The swing of a clock pendulum is just one of many examples of what physicists call harmonic motion. Modern quartz wristwatches rely on electronic oscillations in a crystal. By the end of this lab you should know how to adjust a clock with a pendulum if it is running slow or fast.

Equipment: Computer & Lab Pro, motion detector, force probe, spring, mass set, tall beam & foot, right-angle clamp, pendulum beam & string, pendulum mass, stopwatch.

- 1. To learn about the basic characteristics of periodic motion period, frequency, and amplitude.
- 2. To study what affects the motion of a mass oscillating on a spring.
- 3. To explore the harmonic oscillations of the simple pendulum and the relationship between period, mass, and length of the pendulum.

# **OVERVIEW**

Any motion that repeats itself regularly is known as *harmonic* or *periodic* motion. The motion of the pendulum in a grandfather clock, the vibration of molecules in a crystal, and the orbiting of the earth around the sun are examples of periodic motion. Two of the most important definitions related to harmonic motion are the period (*T*) of an oscillation, which is the time that it takes for the motion to repeat itself, and the frequency (f), which is how often an oscillation occurs per unit time. These two are related by f=1/T. For example,

if it takes two seconds for a clock pendulum to swing out and back, T = 2 s, then one full swing is executed every 2 seconds, f = 1/(2 s). Note that the unit of frequency is 1/s or s<sup>-1</sup>, also called a Hertz, (Hz).

In this lab we will be especially interested in a type of periodic motion known as Simple Harmonic Motion (often abbreviated SHM). SHM involves a displacement that changes sinusoidally in time. You will study the behavior of two simple, classical systems: the mass on a spring and the simple pendulum. Common experience tells us that the force exerted by a stretched or compressed spring a) opposes the stretch or compression and b) gets larger the more the spring is stretched or compressed. A good mathematical approximation for the relation between the force and length of compression or stretching is  $\vec{F} = -k\Delta \vec{x}$ . The minus means that the force is in the opposite direction of the stretch or compression; the spring constant (k) is a measure of how strong a spring is; and  $\Delta x$  is how far the spring is stretched from its equilibrium length. Real springs will approximate this behavior as long as they are not stretched too far. Considering an object of mass m dangling on the end of a spring, Newton's 2<sup>nd</sup> law can be applied to find

$$\sum \vec{F} = m\vec{a}$$
$$m\vec{g} - k\Delta\vec{x} = m\frac{d^2}{dt^2}\vec{x}$$

This relationship can be solved for x, and it is found that  $x = x_0 \cos(2\mathbf{p}t/T + \mathbf{f})$ .  $x_0$  is the amplitude or maximum distance from equilibrium,  $T=2\mathbf{p}\sqrt{m/k}$  is the period, and  $\phi$ , known as the phase, accounts for the possibility of the time measurement starting when the position is not maximum.

A simple pendulum has most of the mass concentrated at one place. It can be found that when a pendulum is set into motion with a small maximum angle, the period is approximately  $T=2p\sqrt{L/g}$ , where L is the length of the pendulum. If the maximum angle  $\theta_0$  is "large", the period can be found using a more complicated expression,  $T=2p\sqrt{\frac{L}{g}}\left[1+\frac{1}{4}\sin^2\left(\frac{q_0}{2}\right)+\frac{9}{64}\sin^4\left(\frac{q_0}{2}\right)+\mathsf{K}\right]$ .

#### PART ONE: Mass on a Spring

- 1. Open **Mass on Spring** (in Physics Experiments / Physics 220 221/ Harmonic Motion). Connect the detectors according to the directions on the screen.
- 2. Calibrate the Force Probe with no mass and a 100-g mass (use 0 N for the weight on an unloaded probe and 0.98 N for the weight on a probe loaded with 100g.)

m

#### **Experimentally Determine Spring Constant, k**

3. Hang a spring from a hook. Use the force probe and a meter stick to measure the force needed to stretch the spring for several distances between 5 and 20 cm. Be sure to measure how far the spring is stretched from its equilibrium length, <u>not</u> the total length of the spring. Be sure to **Zero** the force probe before each time a force is applied to it. To read a value from the plot, Hit the STAT button below the menu bar and hold the mouse button while dragging across the data of interest.

Distance (m)	Force (N)
0.05	
0.20	

4. Graph the force vs. the distance stretched.





5. Determine the spring constant (k) of your spring using your graph. Show your work and write your result below <u>including units</u>.

#### **Experimentally Determine the Period, T**

- 6. The setup used to take data for oscillations of a mass attached to the spring is shown in the figure below. Attach a hooked mass (start with 100 grams) to the end of the spring and position the motion detector directly below it as shown in the diagram.
- 7. To get a sense for how much mass you'll need and how far to pull it, do a dry run first. Pull down on your spring to obtain reasonable amplitude. а (Do not overstretch the spring so much that it remains permanently distorted. Also don't get the mass closer than 0.1 m from the detector since it doesn't work at that distance.) Let the mass go. The mass should oscillate smoothly up and down. If the mass oscillates too violently, replace it with a larger one.



8. Stop the mass and use the motion detector to measure the equilibrium position  $(d_{equil})$  of the mass. Also record the value of the mass used below.



9. Give your mass approximately the same amplitude you gave it for your dry run, then push "Collect" to collect data. Carefully sketch your results below.



10. For the first two cycles, label your graph from step 8 as follows:

- A. Points where the mass is farthest from the motion detector.
- B. Points where the mass is closest to the motion detector.
- C. Points were the mass is standing still.
- D. Points where the mass is moving the fastest.
- 11. Use the graph to find the period  $(T_{exp})$  and frequency  $(f_{exp})$  of the oscillations. Also find the amplitude of the motion  $(x_0)$ .



**Question:** How does the experimentally measured value of the period compare with the theoretically expected value? Show your work.





13. Switch to page 2 to display the velocity vs. time for the oscillations. Use a dotted line or a different color of pen or pencil to sketch this on the velocity graph above. (Clearly indicate which curve is which.)

**Questions:** Where is the mass when the speed of the oscillating mass is at its maximum value? Where is it when the speed is at its minimum value? (Note: we're talking about speed = velocity's magnitude).

14. Measure the periods with a slightly smaller mass and a slightly larger mass hanging from the spring. Record the masses and periods.

m <sub>light</sub> =	Tlight =
m <sub>heavy</sub> =	T <sub>heavy</sub> =
T	

**Question:** Express  $\frac{T_{heavy}}{T_{light}}$  in terms of  $m_{light}$  and  $m_{heavy}$ , according to the theoretical relationship between *T* and *m*.

How does the theoretical value of this ratio compare with the experimental value of it?

# PART TWO: The Simple Pendulum

1. Set up a pendulum as shown below. The string going through the ball should be adjusted so that it is the same length on each side. The length (L) of the pendulum is the shortest distance between the center of the ball and the axis around which it swings, <u>not</u> the length of the strings. Use the wing nut to attach a protractor to the horizontal post so that it reads 0° when the pendulum is at rest.



2. Measure the length of the pendulum.

3. Release the pendulum from an angle of 10°. Measure the time required for 10 periods. Repeat this five times. Divide the measured times by 10 to get the period and take an average of the trials.

Time (s)	Avg.				
Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Period (s)

**Question:** How does the experimentally measured value of the period compare with the prediction for small angles? Show your work.

#### Length Dependence of Period

4. Use the wing nut on the end of the post to adjust the length of the pendulum to 5 different values. For each, measure the period using the same method as before when the pendulum is started from  $10^{\circ}$ .

Length (m)	Time (s) Trial 1	Time (s) Trial 2	Time (s) Trial 3	Time (s) Trial 4	Time (s) Trial 5	Avg. Period (s)

5. Graph the square of the period  $(T^2)$  versus the length (L) below.



6. Calculate the slope of a line that fits your data. Show your work and write your result below including units.

**Question:** How does the value of the plot's slope compare with the theoretically expected value? (Hint: What relation do you expect between  $T^2$  and L?)

# **Mass Dependence of Period**

7. Reel-out the string for the longest pendulum length. Maintaining the same length, change the mass of the pendulum: one run with the regular mass, one hanging the 100 gm mass with it. Measure the period for an amplitude of 10 degrees.

	Time (s)	Avg.				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Period (s)
m m+100 gm						

**Question:** Does the period seem to depend on the mass? (To get a definitive answer, you would have to take more than two measurements.)

# **Breakdown of Small Angle Approximation**

8. Using a fairly long, constant pendulum length of  $L = \_$ , measure the period using the same method as before when the pendulum is started from angles of 10, 20, ... and 60 degrees. Note: you'll want 3 sig. figs. so you can see when the value grows by just 5%.

Amplitude (degs.)	Time (s) Trial 1	Time (s) Trial 2	Time (s) Trial 3	Time (s) Trial 4	Time (s) Trial 5	Avg. Period (s)
10						
20						
30						
40						
50						
60						

**Question:** How does the period change with the amplitude (maximum angle)? At about what angle is the period 5 percent different from the prediction of the small angle approximation?

**Extra Credit:** Use MS Excel to plot your data and the more complicated expression for the period (on p. 2). How do the measurements and theory compare?