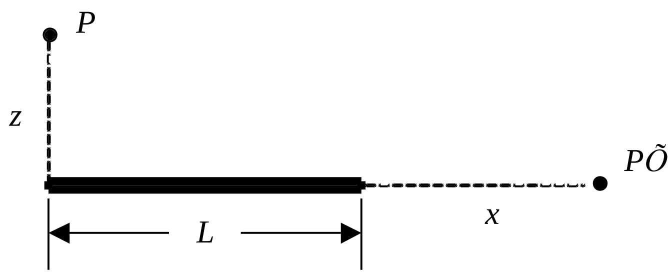
Electricity & Magnetism Homework

Homework 1

EM 1: The Empty Corner (★)

Point charges of 4.0×10^{-9} C are located at three corners of a square whose sides are 15 cm long. Find the magnitude and direction of the electric field at the vacant corner of the square.

EM 2: The Electric Field of a Line Segment (★★)



Suppose a straight line segment of length L carries a uniform line charge λ.

(a) Find the electric field at point P' a horizontal distance x from the right end of the line segment? Check that your formula is consistent with what you expect for the case $x >> L$.

(b) Find the electric field at point P which is a distance z above one end (Problem 2.3 from Griffiths). Check that your formula is consistent with what you expect for the case $z >> L$.

EM 3: The Electric Field of a Disk (★★)

Find the force on a charge *q* located a distance *z* above the center of a flat, uniformly charged disc of radius *R* and charge density σ. Check the limits for $R \to \infty$ and $z >> R$ (hint: for the latter, you should get something familiar if you rephrase σ in terms of total charge, Q and surface area.)

EM 4: From Electric Field to Charge Density (★★)

Suppose the electric field in spherical coordinates is

$$\vec{E} = kr^a \hat{r}.$$

(a) Find the charge density.

(b) For what values of *a* does the charge density go to zero when *r* gets large.

EM 5: Plotting the Electric Field on the Axis of a Ring (★★★ *Computer*)

The electric field a distance *z* above the center of a circular loop of radius *r* which carries a uniform charge per length of λ is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda(2\pi r)z}{\left(z^2 + r^2\right)^{3/2}} \hat{z}.$$

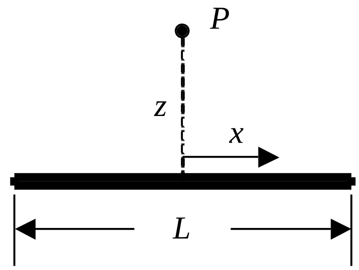
(a) Let $k = 1/4\pi\varepsilon_0$. Show that the quantity $Er/k\lambda$ is unitless, where *E* is the magnitude of the electric field. (Note that it is often useful to make plots with unitless axes if the given quantities are variables.)

(b) Make a plot of $Er/k\lambda$ vs. z/r for values of z/r from -5 to +5. Be sure to label the axes of the plot. Warning: need *from __future__ import division* to divide integers properly.

Homework 2

EM 6: Add It Up (★★★ Computer)

Suppose a thin rod of length *L* has a linear charge density $\lambda(x) = \lambda_0 \cos(3\pi x/L)$, where *x* is measured from the center (positive to the right and negative to the left).



(a) What direction is the electric field at a point above the center of the rod? Explain.

(b) Set up an integral for finding the magnitude of the electric field at a point *P* which is a distance *z* above the center of the rod. Do *not* attempt to solve the integral.

(c) The quantities x/L and z/L are unitless. Find a unitless quantity related to the magnitude of the electric field, the integral from part (b). Don't forget to change the limit.

(d) Write a program to perform the integral in part (c) and to make a plot of the result vs. z/L for values of z/L in the range 0.3 to 5. Be sure to label the axes of the plot.

Note: don't forget about starting your program with *from __future__ import division.*

EM 7: The Electric Field of a Hollow Shell (★★★ Computer)

A spherical shell that of inner radius *a* and outer radius *b* has a volume charge density that varies with radius as

$$\rho = \rho_o \left(\frac{a}{r}\right)^3 \text{ for } a < r < b$$

Find an expression for the electric field in the three regions:

(a) Inside the sphere (*r*<*a*)

(b) Within the shell (*a*<*r*<*b*)

(c) Outside the sphere (*b*<*r*)

(d) On a computer plot the magnitude of the field as a function of *r* across these three regions for the case of *b = 2a;* take a =1. Notes: in Python, "log" is natural log; you can define your E function piecewise with if statements. For example:

```
Def bob(x):
    if x<1:
        return x**2
    else:
        return x
```

EM 8: The Atmosphere and the Earth (★★)

The electric field just above the Earth's surface is approximately 200 V/m, directed downward. At 1400 m above the Earth's surface, the electric field is only 20 V/m, again directed downward. The average radius of the Earth is 6.4×10^6 m.

(c) What is the average charge density of the atmosphere below 1400 m? Are there more positive or negative ions in this part of the atmosphere?

(d) What is the net charge of the Earth?

EM 9: The Electric Field of a Cylinder (★★★)

Find the electric field inside and outside of an infinite cylinder of radius *R* and charge density $\rho(s) = k/s$, where *k* is a constant and *s* is the distance from the axis.

EM 10: The Possible and the Impossible (★★)

Here are two candidates for electostatic field expressions:

i) $\vec{E} = k\{x^2 yz\hat{x} + x^3 z\hat{y} + x^3 y\hat{z}\}$

ii) $\vec{E} = k\{yz\hat{x} + xz\hat{y} + xy\hat{z}\}$

a) Test whether they are physically possible (don't worry about whether they are *plausible*).

b) For one that *is* possible, find the corresponding expression for the electric potential, *V* relative to V(0,0,0). Note: While your integration path shouldn't matter, you do have to choose *some* path to do the integral.

c) To make sure you did it correctly, find -$-\vec{\nabla}V$ and make sure it's consistent with the original expression for E.

Homework 3

EM 11: The Potential of Finite Line Charge (★★★ Computer)

Suppose a line segment of length 2*L* has a uniform charge per length of λ. If the line segment lies on the *z* axis between $z=-L$ and $z=+L$, the potential is

$$V(x,y,z)=\frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{z+L+\sqrt{x^2+y^2+(z+L)^2}}{z-L+\sqrt{x^2+y^2+(z-L)^2}}\right].$$

Make the following contour plots of the unitless quantity $V/k\lambda$, where $k=1/4\pi\varepsilon_0$. Use the unitless quantities x/L, y/L, and z/L for the distances.

(a) In the *xz* plane (*y* = 0) for x/L and z/L from -1.5 to +1.5.

(b) In the *xy* plane (*z* = 0) for x/L and y/L from -1.5 to +1.5.

EM 12: The Potential of a Uniformly Charged Solid Sphere (★★computer)

For a solid sphere of radius *R* and uniform volume-charge density and total charge *q*, the electric field inside and out is

$$\vec{E}=\begin{cases}\dfrac{1}{4\pi\varepsilon_0}\dfrac{qr}{R^3}\hat{r} & r<R\\[2ex] \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r} & r>R\end{cases}$$

(You can check this using Gauss's Law for practice, but that is not required for this problem.) With $r=\infty$ as your reference point, find the electric potential in both regions and plot it as a function of *r* – more specifically, plot V $4\pi\varepsilon_o R/q$ (a nicely unitless property) as a function of r/R.

EM 13: The Potential of a Uniformly Charged Disk (★★)

Using equation 2.30, find the electric potential a distance *z* along the axis of a disc of radius *R* and uniform charge density σ.

Check that you get the point-charge expression for R<<z given that $Q=\sigma\pi R^2$ (note: *not* that your get 0 at infinity, but that you get the actual point-charge expression for R<<z.)

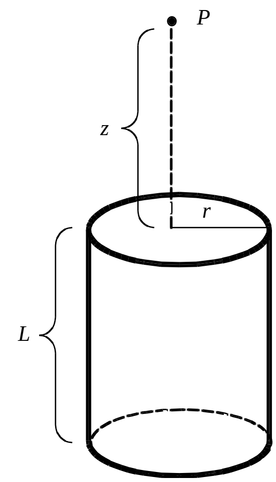
Then use $\vec{E}=-\vec{\nabla}V$ to make sure you the same electric field you'd found in problem EM 3.

EM 14: Bringing Charges Together (★★)

Four positive charges, +*q,* occupy the corners of a square of length *a.*

a. How much work does it take to bring a fifth charge, also +q, from very far away to the center of the square?
b. How much work would it have taken to bring each of the five charges in from very far away and assembled the whole configuration?

EM 15: Some Review (★★★)



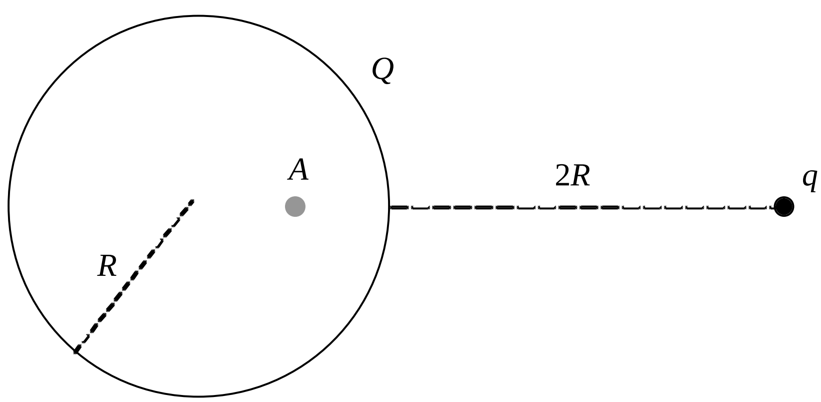
A hollow cylindrical shell of radius r and length L has a uniform surface charge σ on it. There are <u>not</u> end caps on the top and bottom of the cylinder. Point P is on the axis of the cylinder a distance z from top of the cylinder.

(a) Find the electric field at point P.

(b) Find the electrical potential at point P. Use infinity as your reference point.

EC 1: The Potential of a Metal Sphere and a Charge (EXTRA CREDIT ★★★)

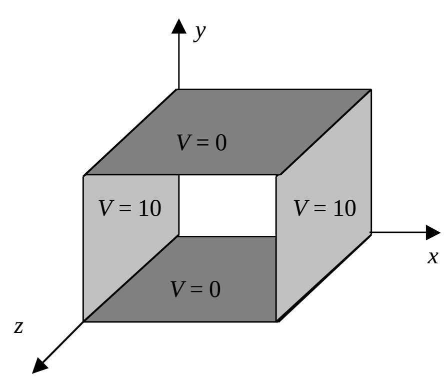
A metal sphere of radius R has a charge Q. A particle with a charge q is placed a distance $2R$ from the surface of the sphere. Find the potential at point A located a distance $R/2$ from the center of the sphere on the line connecting the center of the sphere and particle q. Note that the charge on the sphere is not symmetrically distributed due to the influence of particle q.



Homework 4

EM 16: Just Relax (★★★ Computer)

Two long grounded metal plates lie parallel to the xz plane, one at y =0 and the other at y = 4 cm. Two long metal plates maintained at a constant potential V = 10 V lie parallel to the yz plane, one at x = 0 and the other at x = 6 cm. The diagram below shows a section of the rectangular pipe.



Write a program to use the relaxation method to calculate the potential on the *xy* plane inside the rectangular pipe and make a contour plot of the results.

EM 17: A shell within a Shell (★★)

Say you have a charged, hollow metal spherical shell of inner radius *a* and outer radius *b* and charge *q*. Then you place a *neutral,* hollow metal shell around it, of inner radius c (c>b) and outer radius *d*.

(a) Find the charge density along each of the four surfaces.
(b) Find the electric potential at the center of these two shells (taking as $r = \infty$ your reference.)
(c) Imagine grounding the outer surface (at radius *d*) so now *V*(*d*) = 0 and σ(*d*) = 0.
    a. What's the charge density along each of the other surfaces now (same, different, if different, what)?
    b. What is the electric potential at the center now (same, different, if different, what)?

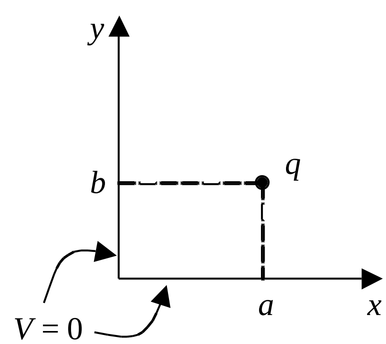
EM 18: Capacitance of Coaxial Cable (★★)

(a) A coaxial cable is essentially a 'signal' wire wrapped with 'ground' sheath. Ideally the resistance between the two is infinite, but there's no escaping that there's a capacitance. Consider two coaxial cylinders, inner of radius *a,* and outer of radius *b.* Derive an expression for their capacitance.

(b) RG-58, a type of coaxial cable, has a metal core with a 0.9-mm diamter separated by insulating material from a conducting sheath with a 5.0-mm diameter. Calculate the capacitance per length in farads per meter for RG-58 cable. (This is an estimate because there is dielectric material, not empty space inside.)

EM 19: Image Problems (★★)

Two semi-infinite grounded conducting planes meet at right angle. A point charge *q* is located in the *xy* plane (*z* = 0) as shown in the diagram below.



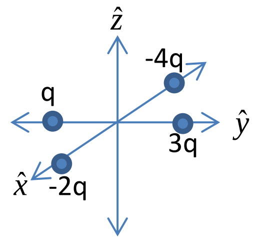
(a) What configuration of image charges will satisfy the boundary conditions of the original problem? Draw and label it.

(b) What is the electric potential $V(x,y,z)$ at an arbitrary point (*x*, *y*, *z*) in the region where *x* > 0 and *y* > 0?

(c) What is the force on the point charge *q*?

EM 20: Multipole Expansions (★★★)

(a) Consider the charge configuration illustrated below; each is a distance *a* from the origin. What are the monopole and dipole terms in the electric potential (which would be a fairly good approximation for the potential ad distances far from the origin.)



(b) A hollow cylindrical shell (with no end caps) of radius *R* and length *L* is centered on the *z* axis. Half of the cylinder is above the *xy* plane and half is below it. The surface charge on the cylinder is $\sigma(z) = k\sin(\pi z/L)$. Find an approximation for the potential far from the origin.

EM 21: Multipoles Again (★★)

The charge per area on the surface of a sphere of radius *R* is $\sigma(\theta) = \sigma_0 \cos(\theta/2)$, where θ is the angle from the *z* axis. Find the first two terms in the approximation of the electric potential.

EC 2: More Image Problems (★★ EXTRA CREDIT)

Consider the situation in problem EM 19.

(a) How much work was required to bring the charge *q* in from infinity (far from both planes) to a point where *x* = *y* = *d*?

(b) Compare the answer for part (a) to the one in Section 3.2.3.

EC 3: Even More Image Problems (★★★ EXTRA CREDIT)

Consider the situation in problem EM 19 in the special case when *a* = *b*.

(a) Find the induced surface charge on the two semi-infinite grounded conducting planes.

(b) Make a contour plot of a unitless quantity related to the surface charge on the *xz* plane for x/a from 0 to 4.0 and z/a from -2.0 to 2.0.

(c) Show that the total induced surface charge (on both surfaces) is –*q*. (The trick used in section 3.2.2 does not work for this problem.)

Homework 5

EM 22: Attracting an Atom (★★)

What is the magnitude of the force between a neutral atom of polarizability α and a point charge *q* a distance *r* away. Hint: The electric field of a dipole is given in Chapter 3.

EM 23: Torque on a Dipole (★★)

What is the torque on a perfect dipole (about itself) of moment ***p*** facing in the *z* direction if it's a distance *r* along the *x* axis from a monopole of charge *q*? What's the force on the dipole? See the hint for the previous problem.

EM 24: A Polarized Sphere (★★)

A sphere of radius R has a polarization that varies with the distance from its center, *r*, as

$$\vec{P}(\vec{r}) = -k\frac{r^2}{R}\hat{r}$$

Where *k* is a constant.

a. Find the bound charges σ_b and ρ_b.
b. Find the electric field inside and outside the sphere

Homework 6

EM 25: Dielectrics in a Capacitor (★★★)

Solve Problem 4.18 from Griffiths. For part (f), recalculate the electric using just the charges (free and bound).

EM 26: A Charged Dielectric Sphere (★★★ Computer)

A dielectric sphere of radius *R* and dielectric constant of 3 has a free charge of *Q* spread uniformly throughout its volume.

(a) Find the electric field as a function of the distance *r* from the center. Hint: Start by finding the electric displacement.

(b) Find the electric potential as a function of the distance *r* from the center using $r = \infty$ as the reference point.

(c) Use the computer to make plots of $E/\left(Q/4\pi\varepsilon_0 R^2\right)$ vs. r/R and $V/\left(Q/4\pi\varepsilon_0 R\right)$ vs. r/R (all unitless quantities) for r/R from 0 to 3. Make two separate plots. Hint: Calculate the quantities separately for r/R from 0 to 1 and from 1 to 3, but put both ranges on the same plot.

EM 27: Faster than a speeding photo torpedo? (★★)

The Enterprise is making its getaway at 2/3 c while a Klingon bird of prey, chases it at 1/2 c when it (the bird of prey) fires a torpedo at 1/3 c (relative to it). Will the torpedo strike the Enterprise (baring any fancy evasive maneuvers)?

EM 28: Honey, I shrunk the trains! (★★)

Two super-super-super sonic trains are traveling in opposite directions down parallel tracks. When both are stationary, you'd say the east-bound was a third longer than the west-bound, but when they cross before you, they appear to be exactly the same length. If the east-bound is going a quarter of the speed of light, then how fast is the west-bound going?

EM 29: A Trajectory (★★★ Computer)

(a) Considering the particle in example 5.2, solve for and sketch its trajectory if its initial velocity were $\vec{v}(0) = \frac{2E}{B}\hat{y}$

(b) Find unitless quantities related to the positions *y* and *z*. Make a plot (not just a sketch) of the trajectory using the unitless quantities from part (a). Hint: In the program, make a list of values of ωt, then use that list to calculates the quantities related to *y* and *z*.

EM 30: Charge to Mass Ratio (★★)

You may remember this from Phys 232 or 233 – a hot filament 'boils off' electrons which get accelerated by the electric field generated by two capacitor plates, with potential difference *V*, and then it shoots into a region with constant magnetic field and so it arcs up to circle around the direction of the field. Knowing the strength of the magnetic field and the voltage and measuring the resulting radius of the electron's orbit, you could determine the electron's charge-to-mass ratio, q/m. Work that out.

EM 31: Force on a Circle (★★)

Say you have a loop of radius *R*, centered on the origin, with current *I* running clockwise around it. If it lies in the x-y plane and is in the presence of a magnetic field who's strength and direction varies like

$$\vec{B} = B_o \frac{x}{R}\hat{z}$$

Then what would be the net force (including direction) on the current-carrying loop? Hint: rephrase $\hat{\phi}$ in terms of $\hat{x}$ and $\hat{y}$ before attempting to integrate.

EM 32: Current Densities of Charged, Spinning Disc and Ball (★)

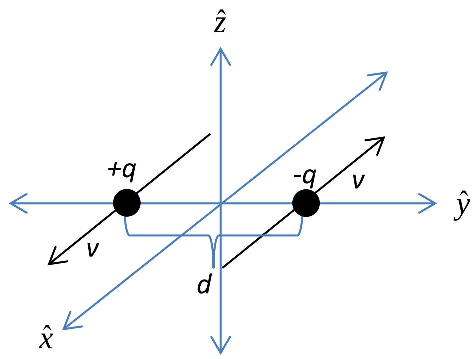
a.) Say you have an old fashioned / new-retro vinyl record of radius *R*. If it had a surface *charge* density that varied with radius like $\sigma(r) = \sigma_o r / R$, and it spun with angular frequency ω, then find the expression for the surface *current* density, *K*(*r*).

b.) How about a spherical balloon that you'd rubbed on your hair – say you rubbed the around the middle of the balloon so it was most charged there, $\sigma(\theta) = \sigma_o \sin(\theta)$, then if you spun it with angular frequency ω, what would be the surface current density, $K(\theta)$?

EM 33: Different Perspectives on Fields & Forces (★★)

Note: This is really a Chapter 12 problem, but I've listed it *after* the Ch 5 problems. You can do the bulk of the work on this problem (finding the fields) right after reading the Ch 12 sections, but you use the Lorentz Force expression, which is covered in Ch 5, to find the associated forces.

i. As is illustrated, oppositely charged particles are traveling with opposite velocities along parallel paths that are a distance *d* apart. The +q charge is moving in the +x direction while the -q charge is moving in the –x direction. At the moment of their closest approach (when the two particles are distance *d* apart, find expressions for the fields and force of charge +q experienced by charge –q in three different reference frames.



  i. Find the electric field vector (magnitude and direction) at the location of charge +q due to charge –q.
  ii. Find the magnetic field vector (magnitude and direction) at the location of charge +q due to charge –q.
  iii. Find the force vector (magnitude and direction) acting on –q due to +q.

ii. In the case that +q is at rest, repeat the above (finding electric and magnetic fields and the force.)

iii. In the case that –q is at rest, repeat the above (finding electric and magnetic fields and the force.)

EC 4: Avoiding a Collision (★★★ EXTRA CREDIT)

Two particles have equal masses *m* and electric charges of equal magnitude and opposite sign (+*q* and –*q*). The particles are held at rest in uniform magnetic field with a magnitude of *B*. The direction of the field is perpendicular to the line connecting the charges. The particles are released simultaneously. What is the minimum initial separation *L* so that the particles do not collide after they are released? Neglect the magnetic fields produced by the particles themselves and the effects of gravity.

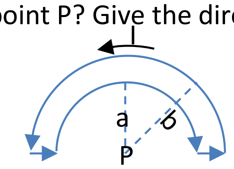
Homework 7

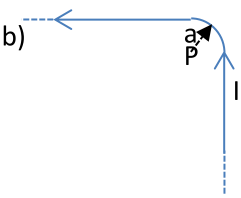
EM 34: Magnetic Field of a Rectangular Loop (★)

Imagine a rectangular loop of height *a* and width *b* laying in the plane of the page, with a constant current *I* flowing around it (what's making it flow? Let's say it's a superconductor so the current, once established, just keeps going). What's the magnitude of the magnetic field at its center?

EM 35: Magnetic Fields for Two More Shapes (★★)

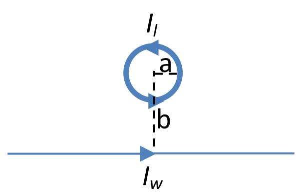
Now say that the steady current is flowing around these two shapes, what is the magnetic field at point P? Give the direction in terms of 'into' or 'out of' the page.

a) 

b) 

EM 36: Force on a Square Loop (★★)

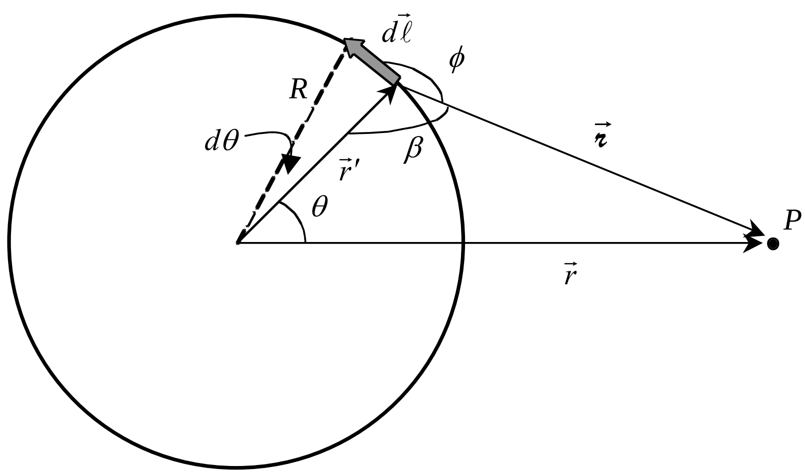
Consider the scenario illustrated below: an infinite wire carrying current I_w and a circular loop of radius a, laying in a plane with the wire, centered a distance *b* away from the wire, and carrying a current I_l. What is the magnetic force on the loop of wire due to the infinite wire? Give the direction in terms of 'up' or 'down' the page.



EM 37: The Magnetic Field in the Plane of a Loop (★★★)

Current *I* is flowing counterclockwise around a circular loop of radius *R*. The loop is centered on the *z* axis which points out of the page. The goal is to find the magnetic field as a function of the distance *r* from the center of the loop (only the distance matters because of the symmetry of the problem).

This is an involved problem so it is broken into several smaller steps. Be sure to answer all of the questions in each part. Consider a short segment of the loop $d\vec{\ell}$ between the angles θ and $\theta + d\theta$ as shown below.



(a) Explain why the magnetic field of the loop only has a *z* component in this plane. Where (for what values of *r*) is B_z (for the entire loop) positive? Where is it negative?

(b) Write an expression for the *z* component of the magnetic field (dB_z) at point *P* due to the segment in terms of the separation $\mathscr{r}$, the angle ϕ between $d\vec{\ell}$ and $\vec{\mathscr{r}}$, and the length of the segment $d\ell$

(c) What is the length of the segment?

(d) How are the angles ϕ and β related? Relate $\sin\phi$ to a simple trigonometric function of β.

(e) Relate the lengths *R*, *r*, and $\mathscr{r}$ to θ and to β (two separate equations) using the law of cosines. (*note: there's one more part to the problem on the next page*)

(f) Combine the answers to parts (b)-(e) to express dB_z in terms of *R*, *r*, θ, and $d\theta$. Write an integral for the *z* component of the magnetic field (B_z) due to the entire loop. The integral does not have an analytical solution, so do not try to solve it! You should ask me if you have the correct integral before starting problem your simulation in the next problem.

EM 38: Plotting the Magnetic Field in the Plane of a Loop (★★★ Computer)

(a) Let B_0 be the magnitude of the magnetic field at the center of the loop. Using the answer to the previous problems part (f), express B_z/B_0 as a function of r/R (both quantities are obviously unitless). Hint: Factor $(R/r)^3 = 1/(r/R)^3$ out of the integrand.

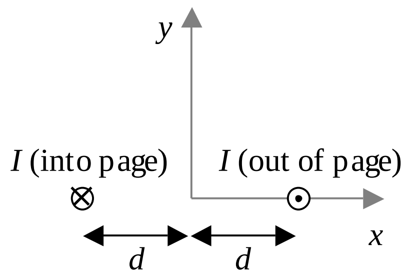
(b) Write a Python program to calculate the integral from part (a) as a function of r/R and plot B_z/B_0 vs. r/R. The magnetic field is not defined at the location of the loop ($r/R = 1$) and there are difficulties with using the expression from part (a) at the center ($r/R = 0$), so separately calculate B_z/B_0 for r/R from 0.1 to 0.9 and from 1.1 to 3.

(c) Does B_z/B_0 have the proper sign in each region? What should the function look like at $r/R = 0$, near $r/R = 1$, and as r/R gets large? (Check that your results in part (b) are reasonable.)

Homework 8

IE 39: Vector Field Plot of a Magnetic Field (★★★Computer)

Two long parallel wires carry currents of *I* = 1 A in opposite directions. The wires are on the *x* axis and each is a distance of *d* = 4 cm from the origin as shown below.



(a) For just the wire on the right, find an expression for the components of the magnetic field at the point (*x*, *y*). A diagram will help.

(b) Find an expression for the components of the total magnetic field due to both wires at the point (*x*, *y*).

(c) Make a quiver plot of the magnetic field in the *xy* plane. Use the range –6 cm to +6 cm for both *x* and *y*. Choose a number of points on the grid that makes the diagram look good. Be sure to include labels and a key on the plot.

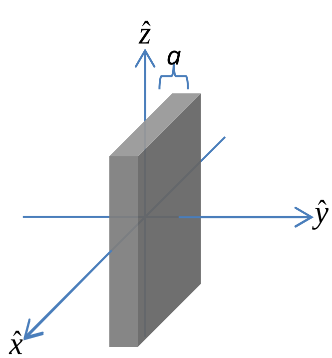
EM 40: Magnetic Field of a Thick Wire (★★)

A steady current *I* flows in the +z direction down a long cylindrical wire of radius *a*. Find an expression for the magnetic field (direction as well as magnitude) both inside and outside the wire, if

a) The current is uniformly distributed over the outside surface of the wire.

b) The current runs through the body of the wire such that the volume-current density is $\vec{J} = ks^2\hat{z}$, where *k* is a constant that you must determine such that the total current down the wire is *I*.

EM 41: Magnetic Field of a Thick Slab (★★)



Consider a slab of conductor that is a thick, extending from y = 0 to y = a, and infinitely tall (z) and wide (x), with a uniform volume-current density of $\vec{J} = J\hat{z}$. Find expressions for the magnetic field (magnitude and direction) as a function of *y*, to the left, right, and within the slab.

EM 42: Vector Potential for a Surface Current (★★)

(a) Find the vector potential above and below the plane surface-current in Example 5.8 two ways.

i. Use an Amperian loop to find the difference between $\vec{A}$ at two distances from the sheet.

ii. Use equation 5.66 twice to find an expression for the difference between $\vec{A}$ (as a function of distance from the sheet) at two different distances from the sheet. Note: this latter approach will give you infinities; however, the infinities will cancel if you use that expression to again find the difference between $\vec{A}$'s at two distances from the sheet.

(b) Check that $\vec{\nabla} \cdot \vec{A} = 0$.

EM 43: A Charged Record (★★★)

(a) A disc of radius R, carrying a surface charge density $\sigma(r') = \sigma_o \dfrac{r'}{R}$, is rotating at constant angular velocity ω counterclockwise about the *z* axis as viewed from a point in the positive *z* direction. Find its magnetic dipole moment.

(b) What is the approximate vector potential far from the origin?

EC 5: Vector Field Plot of an Electric Field (★★ Computer / EXTRA CREDIT)

In Problem EM 16, you computed the approximate electric potential *V* on a grid. Suppose you also want to calculate the electric field for that problem, which for a 2-D problem is

$$\vec{E} = -\vec{\nabla} V = -(\partial V / \partial x)\hat{x} - (\partial V / \partial y)\hat{y}.$$

The partial derivative of *V* with respect to *x* at the grid point `[i,j]` can be approximated using the values on each side. Since those points are separated by twice the grid spacing (*d*),

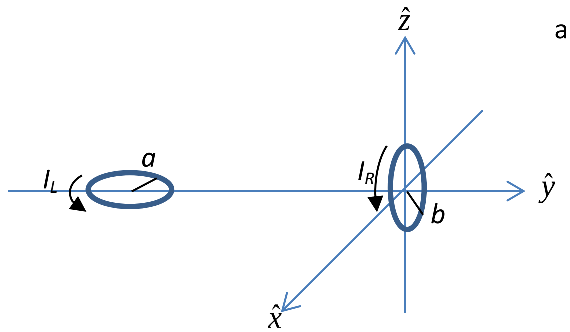
$$\frac{\partial V}{\partial x}[i,j] \approx \frac{V[i,j+1] - V[i,j-1]}{2d}.$$

This is known as the three-point method for calculating the derivative because the value of the function at two points is used to find the derivative at a third point. The partial derivative with respect to *y* is approximated in a similar way.

Overlay a filled contour plot and a quiver plot on the same figure. Use a grid with enough points so that the contour plot looks good, but not too many point because that makes the vectors too close together and small. Be sure to include labels for the axes and scales (a colorbar for the contour plot and a key for the quiver plot).

Homework 9

EM 44: Torque of a Loop on a Loop (★★)



a) Calculate the torque exerted on the loop to the right (about its own center) due to the loop to the left. Assume that their separation, *r*, is much greater than either of their radii, *a* and *b*. The left loop lies in the *x*-*y* plane with current flowing counter-clockwise about the *z* direction while the right loop lies in the *x*-*z* plane and its current flows counter-clockwise about the *y* direction.

b) If the right loop is free to rotate (but the left one isn't), what would be its stable equilibrium orientation?

EM 45: Paramagnetic or Diamagnetic? (★)

Answer Problem 6.6 from Griffiths. Be sure to explain your answers!

EM 46: A Magnetized Cylinder (★★)

A very long cylinder of radius *R* has a magnetization $\vec{M} = M_o \frac{s}{R}\hat{\phi}$ (using physics' traditional cylindrical coordinates). Find expressions for the magnetic field (magnitude and direction) at points both inside and outside the cylinder (for s < R and for s > R.)

EM 47: Another Magnetized Cylinder - the hard way and the easy way (★★★)

Now, what if the cylinder has magnetization $\vec{M} = M_o \frac{s}{R}\hat{z}$ instead.

a) Find all bound current densities (a la equations 6.13 and 6. 14) and use those to determine the field.

b) Use equation 6.20 to find $\vec{H}$ and then 6.18 to find $\vec{B}$.

EM 48 (65): Current in a Dielectric Wire (★★★)

Current *I* flows along a straight wire of some linear material with susceptibility χ_m. If the wire has radius *a* and the current is uniformly distributed then

a) What is the magnetic field a distance s from the wire's axis (inside and outside)?

b) Find all bound currents.

c) What is the *net* bound current flowing down the wire?

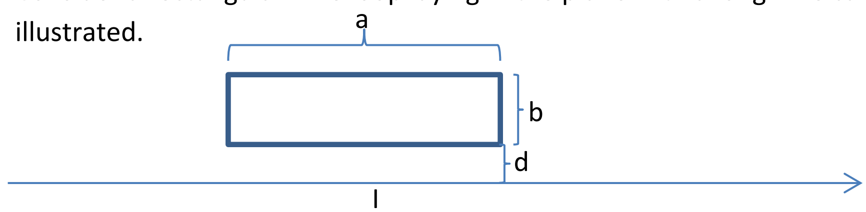
Homework 10

EM 49 (39): Measuring Conductivity of a Liquid (★★)

Solve Problem 7.1 from Griffiths. Be sure to explain your reasoning in part (c).

EM 50: Inducing current (★★)

Consider a rectangular wire loop laying in the plane with a long wire carrying current I, as illustrated.



a) First, what is the magnetic flux through the loop?
b) Next, imagine pulling the loop straight away from the wire at rate *v*, if it has resistance R, what is the current that must be induced in it?

EM 51 (41): Electric Field around a Solenoid (★★)

First, use Ampere's law to remind yourself of the magnetic field inside and outside a very long solenoid along the z axis, with *n* turns per unit length, radius R, and current *I* running counter-clockwise, as viewed from above, i.e., in the ϕ direction.

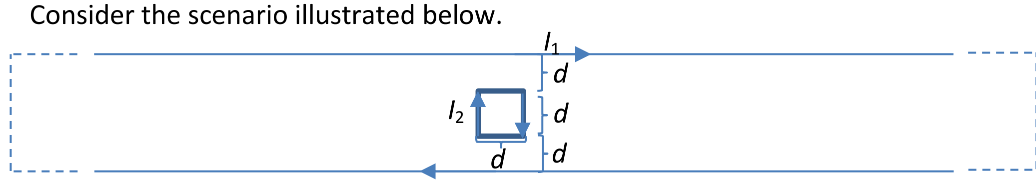
Next, if that current is slowly varying, what is the electric field everywhere – inside and outside the solenoid?

EM 52: Measuring a Magnetic Field (★★)

Rotating coils are often used to measure magnetic fields. A circular coil with a radius of 1 cm and 100 turn is rotated at 60 revolutions per second in a magnetic field. If the maximum value of the *emf* induce in the coil is 12.3 volts, what is the magnitude of the magnetic field?

EM 53: Mutual Inductance (★★★)

Consider the scenario illustrated below.



a) First, what is the mutual inductance using a current I_1 flowing around the large, rectangular loop (<u>not</u> the small, square loop).
b) Now imagine I_2 increasing at a slow and constant rate, $dI_2/dt = k$. Find the *emf* that would induce in the larger loop and determine which way the induced current would flow (clockwise as illustrated or counter-clockwise).

EM 54 (44): Steady Currents & Charge Density (★★)

Solve Problem 7.60. Hint: Follow Griffith's argument on pp. 232-233; when you put back together the three components of the term with $\vec{\nabla} \cdot \vec{J}$ (which isn't 0 anymore), you'll see something reminiscent of Coulomb's law.

EM 55: Betatron I (★★ Computer EXTRA CREDIT)

Write a simulation that demonstrates what Griffiths Problem 7.50 claims. If rusty on writing a simulation that evolves a system by taking little time steps, you might want to look back at similar simulations you wrote in Phys 231, 232, or 331.

Homework 11

EM 57: Gauge Transformation (★★)

Perform Griffith's problem 10.3

EM 58: Turn it on (★★ computer)

Write a simulation to illustrate the time-evolving Electric and Magnetic fields in Example 10.2. As you did in Phys 232, I recommend creating a grid of arrows whose length and direction represent the electric and magnetic fields at their locations (you may wish to borrow from code you'd written for Phys 232, and you'll certainly want to fiddle you're your arbitrary scaling factors for the lengths of the arrows representing E and B fields so they're visible but not overwhelming.) You'll want to make sure your time steps are small enough so you see the interesting behavior while the current's being turned on.

EM 59: Field of time-varying current (★★)

Perform Griffith's problem 10.12

EM 60: Coulomb's Law – better than you'd think (★)

Perform Griffith's problem 10.13

EM 61: What makes the electric field circulate (★★)

Take the curl of equation 10.36 and get a form that involves only time derivatives rather than spatial derivatives. (no shortcuts involving 10.38). This can be done because sources' retardation times are themselves position dependent. Some very similar work was done in the proof on page 446. Note that deriving equation 10.38 from 10.33 essentially demonstrated that $\nabla \times \left(\frac{\vec{J}(\vec{r}\,', t_r)}{\mathscr{r}} \right) = \left(\frac{\vec{J}(\vec{r}\,', t_r)}{\mathscr{r}^2} + \frac{\dot{\vec{J}}(\vec{r}\,', t_r)}{c\mathscr{r}} \right) \times \hat{\mathscr{r}}$. You can then take it as a given that $\nabla \times \left(\frac{\dot{\vec{J}}(\vec{r}\,', t_r)}{\mathscr{r}} \right) = \left(\frac{\dot{\vec{J}}(\vec{r}\,', t_r)}{\mathscr{r}^2} + \frac{\ddot{\vec{J}}(\vec{r}\,', t_r)}{c\mathscr{r}} \right) \times \hat{\mathscr{r}}$.

EM 64: Scalar Potential of constant-velocity point charge (★★)

Perform Problem 10.16 of Griffiths

EM 65: Field of a moving charge (★★ Computer)

Write a simulation to illustrate the electric field for Example 10.4 (Equation 10.75.) To be concrete, let's say the moving charge is a proton (so $q = +e$.) Try different values of v, like 0.01c, 0.1c, 0.5c, and 0.9c. As in EM48, you'll want to define a grid of arrows representing the field; indeed, you may want to make a copy of your code for EM48 and then modify the field expressions to suite this problem.

Analytically perform Problem 10.20 of Griffiths.