Facility Location/Placement in the Presence of Congested Regions

by

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1 Introduction

Location problems in which regions are prohibited from locating new facilities but traveling through is allowed are typically referred to as *constrained* or *restricted* location problems. Such problems have the following two topographical properties. (1) The new facilities cannot be located within certain predescribed restricted areas in the plane. (2) It is not always necessary that any two points in the plane would be "simply communicating", i.e. the minimum travel distance between any two points in the plane may be made longer by the presence of the restricted regions.

The existing literature consists of three types of regions that are encountered in Restricted Facility Location problems. All are closed and bounded regions in $\Re^2$: *Barriers* through which travel is not permitted and facility location is also prohibited, *Forbidden Regions* in which facility location is prohibited but travel through is permitted, and *Congested Regions* in which facility location is prohibited but thorough which travel is permitted at an additional cost per unit distance. Restricted location problems have been solved for two objectives in the literature, viz. *median* and *center*. Larson and Sadiq [3] examine the p-median problem in with arbitrarily shaped barriers under the rectilinear distance metric. Batta *et al.* [1] examine the p-median problem in the presence of arbitrarily shaped barriers and convex forbidden regions assuming that all distances are rectilinear. The primary objective of this work is to examine the facility location and placement problem in the presence of congested regions, where all distances are measured with the rectilinear distance metric.

2 Planar Facility Location Problem with Congested Regions

2.1 Problem Definition

To aid in our problem definition, we define a congested region as a closed and bounded area in $\Re^2$ in which a new facility cannot be located but traveling through is allowed at an additional cost per unit distance. The following are assumed in this work:

- A congested region is the interior of a convex polygon that is defined by a finite number of vertices. This implies that there is no congestion along the boundary of the congested region. Thus traveling along the boundary of a congested region would not result in an increase in the cost per unit distance.
- The congested regions are non-intersecting and share no common boundaries.
- No existing facility is located inside a congested region.

Our problem statement is developed as follows. There exists a finite number of congested regions where facility location is not permitted but travel is permitted at a possible extra cost. The additional cost per unit distance is called the congestion factor of the congested region and is denoted by α, $0 \leq \alpha < \infty$. Thus if w is the cost of travel per unit distance between two points lying outside the congested region, then the cost of travel between the same two points when lying inside the congested region would be $(1 + \alpha)w$. (It is to be noted that congested regions can be considered to be a generalization of barriers (allow no travel through, hence $\alpha = \infty$) and forbidden regions (allow travel through at no extra cost, hence $\alpha = 0$)). The existing ***users*** are distributed over a finite set of demand and/or supply points located anywhere in the plane outside the congested regions. A new facility, assumed to be infinitesimal, is to be located in the presence of the congested regions and the existing users.

2.2 Solution Methodology and Preliminary Results

In order to solve the facility location problem in the presence of congested regions, it is necessary to determine the rectilinear least cost path between two existing users or an existing user and a potential facility location in the presence of congested regions. This is because, considering the rectilinear distance metric, there exist an infinite number of shortest paths between any two points in $\Re^2$ and hence an infinite number of least cost paths. However in the presence of congested regions, the least cost path between two points may no longer be the path of shortest length. It is also pertinent to point out that as the congestion factor of a congested region increases, the least cost path between two points will be gradually forced out of a congested region

In this work, an ***entry point*** and an ***exit point*** are defined as points where a least cost path enters and leaves a congested region. A least cost path may also want to enter and exit a congested region more than once, depending upon the location of the origin and destination points, their location relative to the congested region and also the shape of the congested region. A Mixed Integer Linear Programming (MILP) formulation has been developed to calculate the "cost" of a least cost path. Considering that the congested regions are convex polyhedra, the formulation chooses the side on which the entry and exit points are located, whether there are multiple entry/exit points and also their optimal location(s). It has been found that:

- The objective function of the minimum coat path between two points in the presence of a congested region with congestion factor α is piecewise linear and concave in α.

- The objective function of the minimum cost path between two points in the presence of several congested regions with congestion factors $\alpha_1, \alpha_2, \ldots, \alpha_n$ is concave in α space.

This gives rise to the definition of α *break points*, i.e. threshold values of α beyond which a least cost path would tend to bypass (rather than enter) a congested region. e.g., the break point of α for a single rectangular congested region with length l and width a, $(l > a)$ is given by $\alpha > \frac{2max(a_1,a_2)}{l}$, where $a_1 + a_2 = a$. Some other results obtained, however differ from the results by Butt and Cavalier [2]. Butt and Cavalier [2] suggest that a grid structure to solve this problem that involves extending the node traversal lines of Larson and Sadiq [3] to pass through the congested regions. (In [3], the authors develop a grid construction procedure for barriers. They pass horizontal and vertical X and Y lines through the existing facilities and the barrier vertices.) Our work establishes that such a straightforward "barrier" extension of grid construction for congested regions is incorrect. When the congested regions are convex polyhedra, we conjecture that such a grid would have an extended set of node traversal lines that are perpendicular to a node traversal line at the point of its incidence to a congested region with $\alpha < \infty$. The completeness of this, however, needs to be proven. Based on this, a generalized optimal grid construction procedure for finding the least cost path between two points in the presence of congested regions can be established. The solution to the infinitesimal facility location problem will follow. Construction of the grid, proving its optimality and solving the facility location problem is part of our future work. It is also our endeavor to develop new theory for non-convex congested regions.

3 Finite-size Facility Placement in the Presence of Congested Regions

3.1 Problem Definition

In the problem discussed above, we assume that the new facility is infinitesimal. This representation of the problem may not be accurate when the physical aspects of the new facility are comparable with those of the existing facilities or congested regions. This serves as our motivation to solve the finite-sized facility "placement" problem in the presence of congested regions. A finite-sized facility has a server located on its boundary through which it communicates with the users. The problem is to find the optimal placement(s) for a finite-sized facility such that the facility does not overlap with any of the congested regions and the sum of server-user and user-user interaction is minimized. The term "placement" is more appropriate in this case, because placing a finite-sized facility involves finding an optimal location for its server as well as determining the orientation of the new facility. Thus location and orientation together determine the placement of a facility.

The finite size facility "placement" problem presents the following complications:

- The *orientation* of the facility in addition to its server's *location* need to be known.
- The facility may itself act as a barrier to travel between the existing users and the server.
- The facility may increase the travel distances between users. Therefore an interactive model, that considers user-user interaction in addition to the traditional user-server interaction need to be developed.
- Determining the set of feasible placements of the facility is a challenging task. For a given server location, the facility can have infinitely many orientations. Conversely, for a given orientation, infinitely many server locations can be conceived.

The ***finite-size facility placement problem*** can thus be stated as follows: determine the optimal placement(s) for a finite-size facility such that the facility does not overlap with any of the existing facilities and congested regions, and the sum of user-server and user-user interaction is minimized. A mathematical model for the problem and a solution methodology are yet to be developed.

References

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